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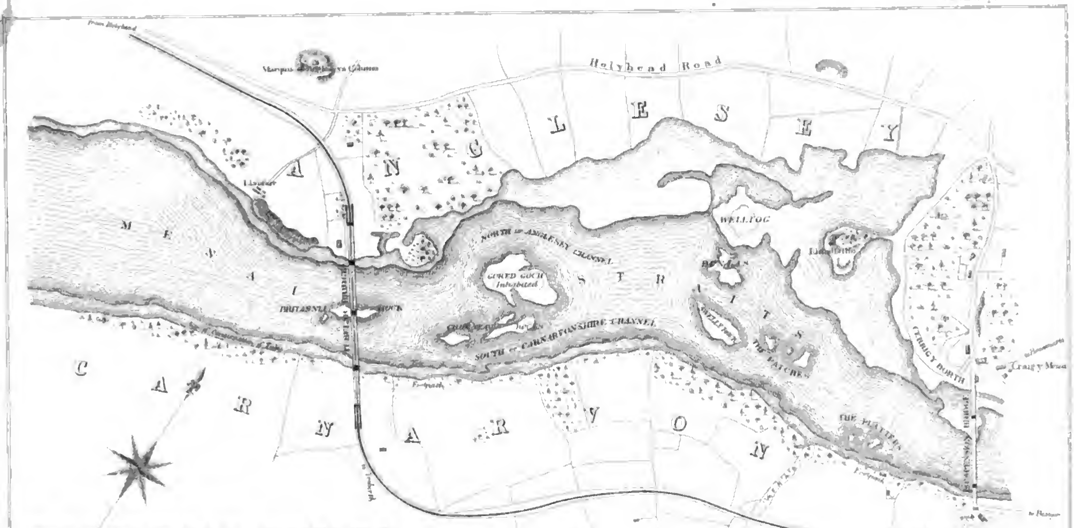

BRITANNIA BRIDGE.

Scale 17 Chains to the Inch

Engraved by Davy & Son.

THE

BRITANNIA AND CONWAY

TUBULAR BRIDGES.

WITH

GENERAL INQUIRIES ON BEAMS

AND ON THE

PROPERTIES OF MATERIALS USED IN CONSTRUCTION.

BY

EDWIN CLARK,

RESIDENT ENGINEER.

PUBLISHED WITH THE SANCTION, AND UNDER THE SUPERVISION, OF

ROBERT STEPHENSON.

IN TWO VOLUMES.

(WITH PLATES IN FOLIO.)

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PREFATORY NOTICE.

THE original object of the following pages was the preservation of the history of a conception as remarkable for its originality, as for the bold and gigantic character of its application. The general reader cannot fail to be interested in a popular record of an engineering achievement which stands confessedly unrivalled in daring and success. The singular circumstances which led to these magnificent structures, the elegant reasoning from which they emanated, and the early developement of their design, are given in Section I. by Mr. Stephenson himself. Section II. contains an account of the Preliminary Experiments made for testing the general principles of the design, with such practical deductions as followed immediately from the results obtained; the important specific inquiries which originated from these experiments form a portion of Section IV. The elaboration of the detail is continued in Section V.; and a minute description of the structures themselves, of the floating and raising, and of the many interesting incidents peculiar to such critical operations and to works of such

unusual magnitude, will be found in Sections VI. and VII. The descriptions are completely illustrated by explanatory sketches, and a folio volume of plates of a novel character as applied to works on engineering; and the process of construction is faithfully perpetuated in a series of tinted lithographs from sketches with the camera lucida.

These were the limits originally intended for this work.

It was impossible, however, to proceed even thus far without some explanation of the principles applied in the investigation of the strength of beams.

At the suggestion, therefore, of many excellent friends, the Author was subsequently induced to attempt a more complete explanation of the nature of transverse strain, embodying, as far as he was able, the views entertained by Mr. Stephenson on this subject, and the practical information accumulated during a period of four years in the superintendence of a work so entirely novel, and in which beams of every variety, and of unprecedented magnitude, have been so extensively employed.

In accordance with these views, a general exposition of the Theory of Beams is given in Section III.; and, in continuation of the subject, much new, and it is hoped valuable, information has been added in Section IV., in which an extensive experimental investigation of the strength of materials, as employed in construction, will be found.

Section VIII. is devoted to the application of the general reasoning contained in the preceding chap-

ters,—to the calculation of the strength and deflection of these bridges. The close confirmation of theory by practical results obtained on so magnificent a scale will be observed with much interest.

The last Section contains an account of a long series of tidal observations made at the works. The durable datum afforded by the massive masonry of the piers may furnish a valuable record of the present mean level of the ocean.

An acknowledgment of all sources of information has been everywhere made. The elaborate experimental researches of Eaton Hodgkinson, Esq., F.R.S., form the basis of most of the deductions. The experiments performed by that gentleman for Mr. Stephenson have, with his sanction, been already published in the "Report of the Commissioners on the Application of Iron to Railway Structures." Full use has in return been made of that treasury of information.

All calculations have, as far as practicable, been conducted in language strictly popular and devoid of theoretical technicality. A more general investigation of the subject was, however, indispensable, for the solution of many problems incident to such structures. At the suggestion of Mr. Stephenson, the Author has been kindly assisted in this respect by his friend, Mr. William Pole, by whom the valuable analytical investigations contained in Sections III. and VIII. have been contributed.

The subject of deflection and of the relative strains in single and continuous beams, in Section III.,

SECTION III.

GENERAL PRINCIPLES OF BEAMS.

SECTION IV.

SPECIFIC EXPERIMENTAL INQUIRIES.

LIST OF PLATES.

SECTION I.

EARLY HISTORY OF THE DESIGN.

INTRODUCTORY OBSERVATIONS.

It is a national characteristic, in which we may be said to stand almost alone, that our greatest public works are conceived and developed by private enterprise: the peculiar sagacity of a commercial people appears, indeed, most conspicuous in their immediate appreciation of the important principle, that whatever is conducive to the general weal is also to the promoters a certain source of benefit; and, conversely, that the richest harvests of individual enterprise will be always reaped in the broad and fertile field of public philanthropy. No sooner was the steam-engine seen to be an element of national prosperity, than the rapid progress of its improvements and applications outstripped all historical record. On the introduction of railroads, a similar appreciation of their public importance enlisted at once the whole commercial community in their construction; and without, as among our Continental neighbours, the assistance of Government patronage, nay, even in the face of Government opposition, the land was speedily covered with a net-work of elaborate intricacy; indeed, the miraculous rapidity of their increase has even been temporarily detrimental to the interests they were designed to foster.

Among the great works to which this enthusiasm gave rise, there are few which have, either in public importance

or in natural difficulties, equalled the Chester and Holyhead Railway. The connecting link between the capitals of England and Ireland was pre-eminently a legitimate object for commercial enterprise; a proposition to reduce so important a route within the limits of a twelve-hours journey met at once with both national and private encouragement, and the prodigious capital requisite for this extension of the North-western line through the mountains of North Wales was speedily raised.

A series of works of unrivalled magnitude characterises its whole length of 84½ miles. It emerges from Chester through a tunnel in the red sandstone 405 yards in length; a viaduct of forty-five arches leads to the bridge by which it crosses the Dee. Following the embanked channel of this river, and the level shores of its estuary, it crosses the River Foryd by a pile and swing bridge, and continues its course along the shore through the Rhyddlan Marshes, and through the limestone promontory of Penmaen Rhos, by a tunnel 530 yards long, until stopped by the bold headlands of the Great and Little Orme's Head. It now for the first time leaves the coast, and, passing through the narrow valley that separates these headlands from the mainland, crosses the River Conway beneath the Castle walls by means of the tubular bridge. Passing through the town of Conway and under the walls by a tunnel 90 yards long, it again reaches the coast at the Conway Marshes, and continues its course along the shore through the greenstone and basaltic promontories of Penmaen Bach and Penmaen Mawr,—the terminating spurs of the Snowdon range, by tunnels 630 and 220 yards long respectively, being carried, for some distance after leaving Penmaen Mawr, on a cast-iron-girder viaduct over the beach. The sea-walls and defences, on the one hand, along this exposed coast, are all on a magnificent scale; whilst, on the other, a timber gallery, similar to the avalanche galleries on the

Alpine roads, protects the road line from the *débris* that rolls down from the lofty and almost overhanging precipices above it.

The Ogwen River and valley are then crossed by a stone viaduct 246 yards in length; and between this and the Britannia Bridge the line passes through three ridges of hills perforated by tunnels, 440, 920, and 726 yards in length, through slate, greenstone, and primary sandstone; the River Cegyn, with its beautiful valley, being crossed by a viaduct 132 yards long and 57 feet high. The line thence continues rising to the level of the Britannia Bridge, and entering Anglesey, passes across the Maldraeth Marsh, and through a tunnel, in slate, rock, and clay, 550 yards in length. To enter the Island of Holyhead use is made, to a certain extent, of the embankment of the Holyhead Road Commissioners, called "The Stanley Sands Embankment," for which the Company are required, as at Conway, to make a yearly payment to Her Majesty's Commissioners of Woods and Forests. The amount in this case is 106*l.**

It is the object of the present volume to describe two of the most important works in this magnificent catalogue— the passage of the Conway River and of the Menai Straits.

* The magnificent harbour of refuge, now in course of construction by Her Majesty's Government, under James Rendel, Esq., and towards which the Company contributes a sum of 200,000*l.*, will ultimately form the terminus. The total area of this harbour at low water will be 316 acres, protected by a pier and breakwater 2500 yards long, and about 115 feet wide. The distance of the various stations from Chester is as follows:—

Queensferry, Flintshire		7	Miles.	Conway, Carnarvonshire		45¼	Miles.
Flint	—	12½	,,	Aber	—	54¼	,,
Bagilt	—	14½	,,	Bangor	—	59½	,,
Holywell	—	16¾	,,	Llanfair, Isle of Anglesey		63½	,,
Mostyn	—	20	,,	Gaerwen	—	66½	,,
Prestatyn	—	26¼	,,	Bodorgan	—	72½	,,
Rhyl	—	30	,,	Ty Croes	—	75½	,,
Abergele, Denbighshire		34¼	,,	Valley	—	81	,,
Colwyn	—	40¼	,,	Holyhead Island		84½	,,

THE BRITANNIA BRIDGE.

The construction of the Britannia and Conway Tubular Bridges, more especially of the former, has excited an unusual degree of public interest.

Their national importance, the magnitude of the works themselves, the immense expenditure involved in their execution, and the great interests depending on their success, together with the natural uncertainty that must always characterise so bold an extension of novel principles, almost indeed to their ultimate limit, were all circumstances sufficient to command considerable public attention, while the importance of the principles themselves, which have already become of such general application as to influence materially the whole science of engineering,—and confirmed, as they are, by an elaborate series of experiments unparalleled for their magnitude in experimental philosophy,—has already attracted the general notice of the scientific world, and has given rise to much valuable investigation, which will not fail, in due time, still further to add to our rapidly increasing store of practical knowledge.

It is a remarkable fact, that Mr. Telford in 1820, and Mr. Stephenson in 1849—whose two bridges are only one mile asunder—have each been compelled to effect the passage over the wide and rocky bed of the Menai Straits, not only by new and untried expedients, but also to extend such expedients to limits which will probably seldom have to be exceeded. Hence their history becomes doubly important and interesting.

The reader who is unacquainted with the locality must bear in mind that the Island of Anglesey is separated from the mainland by a rocky and precipitous channel, whose general direction, from Carnarvon Bay to Beaumaris Bay, is from south-

west to north-west, its length being about 11½ miles, and its width of water-way varying from about 1000 feet to three-quarters of a mile. Its course is at the same time tortuous, and the extensive sand-banks at either extremity, together with the numerous rocks which intercept the channel, and the baffling and violent currents and eddies occasioned by the peculiar tides of such a locality, render its navigation exceedingly difficult. Its shelter is, however, so important, and the saving of distance is so considerable in avoiding the journey of 60 miles round the unsheltered and dangerous coast of the island, that the bulk of the coasting vessels, some of them of large tonnage, avail themselves of its advantages, as do also a great number of vessels employed in the carriage of slates from the Penrhyn, Llanberris, and other slate quarries among the Carnarvon hills.

The tides are very peculiar; the main tidal wave, as it advances northward up the Irish Channel, branches off into the Straits over the sand-banks of Carnarvon Bay, and arrives in Beaumaris Bay by this contracted route some time before the main tidal wave has completed the circuit of the island. As soon, however, as the main tidal wave enters Beaumaris Bay, it repels the current that has set in from Carnarvon, and the tide flows into the Straits in opposite directions. This meeting of the waters gradually retires before the Beaumaris wave, and arrives at the Britannia Bridge about twenty minutes before high water there, so that the tide continues to flow, or the water to rise, twenty minutes after the current has changed its direction: this peculiarity, it will be seen hereafter, gave increased security in floating the tubes.

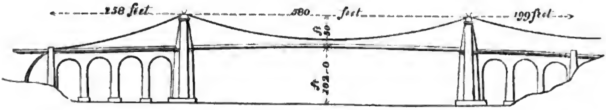
From either extremity the channel narrows, and is also partially obstructed, at the part represented in the accompanying map (*see* Frontispiece) by the numerous rocks in its bed. The current sets through these with fearful rapi-

dity, and in certain states of the tide the Straits resemble the rapids of a great river: the roaring of the tide is heard at a considerable distance, and at these times the navigation is perfectly impracticable.

The baffling winds that prevail between the high hills on either side, moreover, tend to increase the sailor's difficulties in these narrow channels; all this might, however, be remedied to a great extent by the removal of the Swelly and Cribiniau Rocks, and the expense would not be considerable, while the navigation is certainly of sufficient importance to warrant such an outlay.

The rapidity of the tide is assisted by the wind, the direction of which is almost constantly either up or down the Straits, so that even at the Britannia Bridge the current often runs at eight miles an hour: much valuable information on this subject will be found in Captain Vidal's Report to the Admiralty, page 76.

It was the narrowest portion of this channel that Telford selected for the Menai Suspension-bridge, the steep shore on either side being particularly favourable for the construction of a roadway at so great a height. The high-road rises from the town of Bangor to the height of 103 feet at the suspension-bridge, and continues at this level for some distance over the high land of Anglesey. The map before referred to, and the annexed wood-cut, will assist in explaining its proportions and position.



The foundations here, as at the Britannia, are solid rock, and the material also mountain-limestone, from quarries in the Island of Anglesey. The position of the main piers is

at the line of low-water mark, and the maximum depth of the bed of the stream from low-water mark is about 40 feet. This beautiful structure has well stood the test of time, and is still one of the finest monuments of engineering skill, and a worthy relic of the master-mind that conceived it. For complete details, the reader is referred to Mr. Provis's elaborate work on the subject.

It will be seen by the map that there was but little choice in the selection of a site for the Britannia Bridge. By crossing at the Swelly Rock—the only other site possible—it is true, that the maximum spans required would not have exceeded 350 feet, instead of 450; but the total length of the bridge, with the viaduct necessary to reach the retiring high land on the Anglesey side, would have been much increased, while the sharp curve which in that case would have been requisite on the same side, offered serious difficulties, as the viaduct itself would have been considerably curved. This was well considered by Mr. Stephenson; and he preferred the present position, considering the increased span of less importance than the difficulties enumerated.

The tide-way is here contracted as at the Menai Bridge, though in a less degree. The shore is steep on the Carnarvon side, and rises rapidly on the Anglesey coast, the beach on both sides being extremely rocky, as will be seen in the section Plate V. The bed of the river consists of rough, uneven rocks, which rise, as nearly as possible, in the centre of the Straits, about 11 feet above low-water level, and there form an oblong island of chlorite schist, about 350 feet long, and 120 broad, running nearly in the direction of the Straits, and called THE BRITANNIA ROCK, from which the bridge is named.

This island, consequently, divides the channel into two nearly equal portions, either of which is navigable, although the Carnarvon passage is much more used than the Anglesey

one. The depth at high water is 47 feet in the Carnarvon, and 56 feet in the Anglesey channel, the total water-width from shore to shore at high water in the line of the bridge being 1100 feet; and through these contracted channels, as before observed, the tide occasionally attains a velocity of upwards of eight miles an hour.

The natural difficulties to be overcome in crossing such a gulf were, moreover, much increased by the requirements of the Act of Parliament,* by which the dimensions of the central pier were limited, and the roadway, as at the suspension-bridge, was to be 103 feet above the water, this clear height or windway being insisted on throughout the whole span. Thus the arch was rejected; scaffolding from below was impracticable; and the navigation was, under no circumstances, to be interfered with. These were the apparently insurmountable difficulties which the engineer had to encounter and to overcome without delay. No existing kind of insistent structure appeared capable of such fearful extension; and the developement of some new principle became imperative.

The description of the novel and magnificent structure designed by Mr. Stephenson to meet these unusual requirements is a principal subject of the following pages. Hollow beams, 470 feet long, were constructed on the beach, 1500 feet from their permanent site; they were floated upon these rapid tides to their place; and finally, these stupendous fragments of the Holyhead Road, weighing nearly 2000 tons each, were lifted 100 feet into their place.

* *See* the Reports of Sir John Rennie, Captain Vidal, and Mr. Rendel, page 72, *et seq.*

THE CONWAY BRIDGE.

The distance between the Britannia Bridge and the Conway is 17 miles. The difficulties here were but in a small degree less formidable than those at the Straits. The Conway River forms the eastern watershed of the Snowdon range; from its source near Festiniog, down to its extensive and sandy estuary at Conway—a distance of upwards of 20 miles—it passes through some of the finest scenery of North Wales, and receives in its course the numberless streams that descend from its mountain boundary. The quantity of water discharged by it varies, as is the case with all mountain-streams, very much; but in the rainy seasons it is considerable.

For about eight miles above Conway the river is navigable, and its bed widens considerably as the town is approached, being at two miles up one quarter of a mile wide, while in the immediate neighbourhood of the bridge it is three-quarters; it, however, contracts in passing round the base of the steep rock upon which the Castle stands to the width of three furlongs. At this place, too, the channel is intercepted by a small rocky island, about 120 yards from the Castle rock, which is connected with it by the elegant suspension-bridge erected by Telford simultaneously with that over the Menai Strait, the Holyhead Road being carried over the remainder of the river on an embankment, pitched on each side with rubble masonry, to protect it from the effects of the tide. The river is contracted between the Castle rock and this island into a channel only about 240 feet broad. (*See* the Plan, Plate XXXIV.) The rock on either side is very steep, the depth at the centre of this channel being at high water 63 feet; and through this confined space the tide flows and ebbs with a velocity of six or seven miles an hour, as it successively covers and exposes the extensive silty banks of the broad and lakelike estuary. The

ebb of the tide is much increased by the great quantity of fresh water which descends in rainy weather.

Towards the sea, the estuary expands again rapidly, the river at low water continuing its tortuous course for a distance of four miles, through the vast sand-banks which fill up the whole interval of Conway Bay between the Orme's Head and Penmaen Bach.

The total range of tide at Conway is, as at the Straits, about 21 feet.

In carrying the railway across the river, permission was obtained from the Holyhead Road Commissioners to make partial use of Telford's embankment above described; for which privilege an annual payment of 260*l.* is made by the Company to the Commissioners of Woods and Forests. The new bridge is thus only 50 feet from the suspension-bridge at the Chester end, and 65 feet at the Conway end. On reference to Plate XXXIV. the reader will better appreciate the difficulties to be encountered in this locality. The rapidity and depth of the current rendered the construction of scaffolding or centering for an arch totally impracticable, as at the Straits. The span of the suspension-bridge is only 315 feet.

The rock on which the tower at the Chester end of the suspension-bridge is built has fortunately face enough to allow of the construction of the abutment for the new bridge, with its river front in the same line. But the rock on the mainland shelves off so rapidly beneath the Castle walls, that not only was it requisite to fall 85 feet farther back than the line of the tower of the suspension-bridge with the Conway abutment to obtain foundation, but even in this position it was necessary to build a portion of the southern flank on piles driven 15 feet into the beach, to reach the rock beneath. The least span that could thus be obtained was 400 feet, while the height above the water was to be 18 feet,

to correspond with that of the suspension-bridge. The magnitude of this span, and the natural difficulties above enumerated, are now less imposing, as they are naturally compared with the somewhat similar obstacles on a larger scale in the Straits; but they were in reality quite as formidable, and it is here equally difficult to conceive any substitute for the tube. The span was 60 feet less; but 60 feet was no very important deduction from a beam 460 feet long. The height to which the bridge was to be raised was only 18 feet instead of 103; but a mere repetition of the process which would raise such a structure 18 feet was not a very formidable difficulty, while the velocity of the current was equal to that at the Straits, and the depth of the channel at high water rendered the construction of any centering or scaffolding entirely impracticable. It will assist the judgment, in considering the magnitude of this span, to call to mind the dimensions of some of the largest existing insistent structures, premising that a suspension-bridge would be inapplicable to railway purposes.

	Material.	Span of Centre Arch.	Rise of Arch at Centre.	Date of completion.	Engineer.
		Feet.	Feet.		
Sunderland over the Wear	Cast Iron	240	30	1796	Wilson
Southwark Bridge	Ditto	240	24	1818	Rennie
Dee Bridge, Chester	Sandstone Arch	200	42	1833	Hartley
Pont du Carrousel, over the Seine	Cast Iron	187	15½	1836	Polonceau
London Bridge	Granite	152	29½	1831	Rennie
Maidenhead Bridge, over the Thames, on the Great Western Railway	Brick	128	24¼	1835	Brunel

It will be observed how far short all these magnificent works fall of the dimensions required, the tube for the Britannia Bridge being 472 feet long, or nearly double the largest

of them; while the span at Conway is twice as great as that of the Dee bridge, the largest stone arch in existence. The versed sine, or rise, of this arch, moreover, is 42 feet; so that had a similar structure been practicable, it would have been necessary to raise the line considerably above its present level of 18 feet from high water. It must also be borne in mind that, inadequate as all these structures are as precedents for the arch of the dimensions here required, they were all constructed with timber centres, which were here impracticable. The circumstances of the two localities were so similar, that, in the first instance, every inquiry had reference to the construction of a bridge at the Straits, which would evidently, with slight alteration, be equally applicable to Conway; and thus special attention was never given to the detail of the latter bridge, until the general principles experimented on were fully confirmed as applicable to the larger structure. Thus the design of both bridges was simultaneous, and the early records of their progress are too closely interwoven to be separated.

The great experiment, however, was first tried at Conway. The first tube for this bridge was constructed on the beach, about 600 feet from its permanent site. It was commenced in March 1847, tested in January 1848, floated to its place in March, and ultimately raised and in use for railway traffic in April 1848. The rapidity of its execution being as unparalleled as were then its colossal dimensions.

Every detail connected with the early history of these conceptions will be equally interesting with that of their developement and ultimate perfection, which more particularly progressed under the author's personal observation. And that the subject may be the more complete, Mr. Robert Stephenson has himself contributed the following interesting details of the origin and early history of the designs of the Britannia and Conway Tubular Bridges.

CHAPTER I.

INTRODUCTORY OBSERVATIONS ON THE HISTORY OF THE DESIGN, BY MR. ROBERT STEPHENSON.

Shortly after the metropolis of this country and the great commercial port of Liverpool had been connected by means of railways, the public attention began to be directed to the further improvement of the communications with Ireland. This, it was obvious, could only be accomplished by extending the land journey and diminishing the sea voyage, or, in other words, by increasing the comparatively certain, and diminishing the uncertain portion of the journey. The ports of Holyhead and Dynllaen each had their advocates as the most eligible packet station and terminus for a railway destined to curtail the journey from London to Dublin. The relative merits of these two ports as points of departure for Dublin were keenly discussed, and various investigations were entered upon, and reports made both by civil engineers and naval officers.

These discussions terminated in the preponderance of evidence being in favour of Holyhead, which led to the adoption of the line of railway as then designed by my lamented father the late George Stephenson, which, with the exception of about five miles in the neighbourhood of Bangor, is that which has now been brought so near to completion. The first survey for

this was made as early as 1840, but the formal application to Parliament did not take place until the session of 1843–4. The chief engineering work then involved was the bridge over the River Conway, close to the existing suspension-bridge. The passage over the Menai Straits was proposed to be effected by permanently appropriating one of the two roadways of the great suspension-bridge to railway purposes. The steep ascents at each end of the present suspension-bridge it was designed to avoid by elevating the level of the railway to that of the suspended roadway at its highest point. As the strength of the suspension-bridge was deemed inadequate for carrying safely railway trains and ponderous locomotive engines,* it was intended to convey the trains across in a divided state if necessary, by means of horse-power, another locomotive being in readiness to be attached on the opposite side: thus the passage of engines was entirely obviated. To this proposal the Commissioners of Woods and Forests assented, with the condition, however, that the appropriation of the south suspension roadway for railway purposes should only be temporary. Such a stipulation rendered their assent merely nominal, because the expenses, which must necessarily have been incurred in carrying out the proposal, were, although suggested with the view of limiting the cost of crossing the Straits, totally inconsistent with the idea of its being only a temporary expedient.

The Company were thus driven to abandon this part of their plan, and to propose an independent bridge for the railway.

The bill was permitted to pass Parliament with an hiatus of five miles, which were affected by the abandonment of the suspension-bridge as a means of crossing the Menai Straits. There were, however, other objections distinct from that just

* *See* page 38.

alluded to in reference to the suspension-bridge. Some parties urged that this portion of the line was needlessly circuitous, that three-fourths of a mile might be saved by some additional expense; others objected to it, because it approached and interfered with the privacy of the residence of the prelate at Bangor. It would be out of place here to discuss the value of either of these objections, it is sufficient to say that they prevailed, and the Company directed their engineer to deviate the line to avoid them, and to select the best point for crossing the Straits by an independent bridge.

Previous to the erection of the suspension-bridge by Telford, in 1826, various modes and points of crossing had been proposed by Rennie and Telford. Their reports, plans, and opinions, were carefully studied, which led to the adoption of the site known by the name of the Britannia Rock, about a mile to the south of Telford's suspension-bridge. This spot is peculiarly eligible for the purpose, the rock being nearly in the centre of the channel, rising just to high-water mark, and of sufficient area to admit of the easy erection of a pier upon it. The channel is here also entirely free from sunken rocks, and the current unbroken during the ebb and flow of the tide. These peculiarly favourable circumstances were considered highly advantageous, not only for facilitating the erection of a bridge, but for rendering such a structure unobjectionable to the navigation of the Straits. It was proposed to construct the bridge of two cast-iron arches, each 350 feet span, with a versed sine of 50 feet, the roadway being 105 feet above the level of high water at spring-tides. (*See* Plate XXXII.)

The span here proposed was the same as that which had from the first been designed for crossing the Conway River.

Such was the state of the engineering problem in reference to the Conway and Britannia Bridges when the Company obtained the first Act of Parliament in July 1844. It was proposed to construct a bridge consisting of one arch

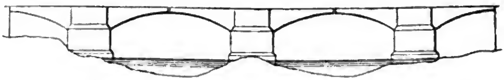
of the unusual span of 350 feet over the Conway River, at 20 feet above high-water mark, and another over the Menai Straits at the Britannia Rock, consisting of two arches, each of similar span, but at the elevation of 105 feet above high-water spring-tides.

The rise of tide in both cases is nearly the same, the channels are also very similar, being from 50 to 60 feet deep, with a rocky bottom, and a rush of tide reaching five miles an hour at Conway, and seven miles an hour in the Straits.

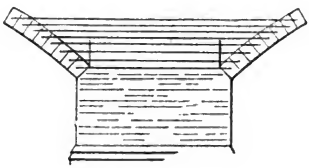
These conditions, together with the necessity of keeping the channels open at all times for the purposes of navigation, rendered it perfectly clear that none of the methods heretofore adopted in the erection of cast-iron arches could be brought to bear in either of these localities. The inordinate cost of centering, even if other arrangements had admitted of its application, was at once fatal to its adoption; and it soon became evident that some means external to the arch should be employed to suspend the voussoirs, or ribs, until the arch was keyed in.

A contrivance of this kind had at one time been considered by Telford for the suspension of centering, upon which he proposed to frame and connect the voussoirs, or ribs, of a cast-iron arch; and a slight drawing of such a project is given in the account of the Menai Bridge. Without going into the merits of this proposal in the form suggested, or into its applicability to the present case, it is sufficient to say that it was discarded, and a modification, as brought forward some years ago by Sir Isambard Brunel, for constructing brick arches without centering, taken up as more suitable. Sir Isambard's idea, which was experimentally carried out to a great extent, appeared unexceptionable, and led to the following design for the erection of the cast-iron arches at the Britannia Rock. Instead of the two arches being erected upon two abutments

and one pier, it was proposed to treat the abutments as piers also, and to complete the iron-work in the form shewn by the following figure.



The erection of the arch was to be proceeded with by placing equal and corresponding voussoirs on opposite sides of the pier at the same time, tying them together by horizontal tie-bolts, as shewn below.



This system, it is confidently believed, may be successfully carried out to a far greater extent than would have been required in the case of the Britannia Bridge.

It will appear evident, on a little reflection, that as every succeeding step of voussoirs is secured by the tie-bolts, the tension of the last bolt, as well as all the previous ones, will be relieved by an amount equal to the whole of the horizontal thrust due from the voussoirs last placed.

If the voussoirs could be constructed or weighted, so that an arch of equilibrium could be formed, all the horizontal tie-bolts might be removed, except the last one, for in such an arch the horizontal thrust is everywhere equal. It is not meant that such a method of proceeding as that of removing all the bolts could be carried out practically—it is merely alluded to here to shew how largely the bolts would have

been relieved from strain as the arch progressed into a form which might appear to endanger the stability of the structure.

Had this plan been carried out, it was not intended to have keyed the arches at the crown, but to have left ample space between the culminating voussoirs to admit of expansion and contraction taking place freely. The bridge would, therefore, have been simply a double-jibbed crane, perfectly balanced on each pier. A connexion at the apex of each arch would be necessary, but so contrived as not to interfere in the least with the expansion and contraction, and yet to counteract any tendency to tilt, consequent upon the variable pressure of the passing loads.

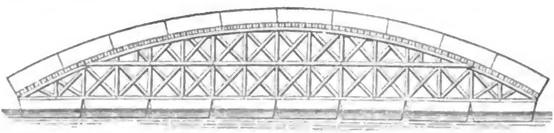
This mode of construction, although decided upon for the Britannia Bridge, was found unsuited for that of Conway. There only one span was required, and the springing of the arch would have been below the high-water line, and from a natural mass of rock on both sides, which at the east extremity rose nearly to the permanent level of the railway.

It was, consequently, impossible conveniently to treat the abutments in the light of piers, as has been just described. Moreover, the great additional expense of this method, where one arch only is required, formed a serious objection to it, as it necessarily involved the use of double the weight of material requisite for one simple arch, the weight of each overhanging wing being equal to half the weight of the arch itself.

The objection on the score of expense did not apply to the Britannia, for there the overhanging wings were a useful portion of the bridge, and formed a substitute for the extension of masonry, which would have been nearly as costly. Both the expense, therefore, and the peculiarity of the site of the Conway Bridge, pointed out the necessity of some other method being devised for the erection of the arch. Various modes for erecting and supporting a fixed centering were

considered, but none appeared satisfactory or safe; whilst the formidable difficulty of stopping the navigation, and seriously interfering with many vested interests for probably two years, remained in all its force.

This state of things led to the idea of building the arch complete on centering supported entirely upon, and framed into, a series of pontoons kept afloat during the whole time of construction. This arrangement, which is shewn in the following sketch, appeared upon the whole by far the most feasible that had been suggested, and well adapted for placing the arch into its permanent position.



The rise and fall of the tide was such as to admit of its being brought immediately above the springings and lowered into its place by the falling tide, or by admitting water into the pontoons at the top of the tide, before the velocity of the ebb stream had increased so as to interfere with the accurate adjustment of the descending mass. This method of fixing arches I have since learned was proposed many years ago by Mr. Dixon, of Darlington. He made designs for a cast-iron bridge across the River Tees at Stockton, and, instead of erecting centres on the permanent site of the arch, he proposed to use pontoons, precisely in the manner which has been described. These plans were not carried out, in consequence of the Stockton and Darlington Railway Company having determined to try a suspension bridge for railway purposes instead of the cast-iron arch. For a brief description of the particulars of the novel proposal of Mr. Dixon I have been favoured with a communi-

cation from Mr. R. B. Dockray, who resided at Darlington at the time when Mr. Dixon made the design.* I have also learned from Sir John Rennie that this was the method adopted for placing the centering of the Waterloo and London Bridges; the centres being constructed on pontoons and floated and lowered into their proper position.

* "*Euston Station*, 25 *May*, 1849.

"My dear Sir,

"In accordance with your wish, I beg to send you the following account of a mode of erecting the ribs of a cast-iron arch proposed by the late Mr. James Dixon, of the Stockton and Darlington Railway.

"The proposed bridge was for the purpose of carrying the Middlesborough branch of the Stockton and Darlington Railway across the River Tees, at a little higher up the river than Stockton. Several plans were laid before the Directors, and at length that of Captain Brown, for a suspension-bridge, was adopted and carried into execution. This bridge, as you will remember, proved insufficient for the weight passing over it, and for several years it was supported by timber geering, carried upon piles driven into the bed of the river, until at length it was entirely removed, and the present cast-iron-girder bridge, built under your own directions, was substituted. One of the designs originally submitted to the Directors was by their resident engineer, Mr. James Dixon; the bridge was of cast-iron, of three openings of about (if I remember right) 80 feet each. There was nothing particular in the construction of the bridge itself; but it was supposed that the Tees Navigation Company, then hostile to the Railway Company, would object to the temporary obstruction of the navigation by the erection of the centres for placing the ribs of the arches. To obviate this difficulty, Mr. Dixon proposed to erect each rib separately, upon a scaffolding or centre placed in a pontoon, at such an elevation that, when floated to the site of the bridge, the ends of the ribs would clear the skew-backs on the pier and abutment, and, when properly moored in this position, the pontoon was to be lowered (until the rib rested upon the skew-backs) by admitting water into the hold; and thus he proposed to proceed with the erection of each individual rib.

"I saw the drawings in the year 1831 or 1832, not only of the bridge, but of the pontoon, with its centre and valve for admitting the water; they were in detail, and beautifully executed, and I have no doubt they are still in existence, probably in the hands of his brother, John or Edward.

"I am, my dear Sir, very truly yours,

"ROBERT B. DOCKRAY.

"*To Robert Stephenson, Esq., M.P.*"

Such were my intentions regarding these two bridges when the general meeting of the Chester and Holyhead Railway Company took place on the 30th of August, 1844.

In the following November, the Company deposited new plans preparatory to an application to Parliament in the ensuing session for the deviation which had been forced upon them by the circumstances already alluded to; extending from near the River Ogwen to Llanfair, in the Island of Anglesey, comprising, in this distance, a series of railway works unparalleled in cost and magnitude, the Britannia Bridge being one of them.

Immediately on its becoming known what description of bridge it was intended to throw over the Menai Straits, a new series of objections was raised, and a violent opposition started on behalf of those interested in the navigation of the Straits. It was urged, that any such bridge as that proposed would seriously injure and fatally aggravate all the evils and dangers which beset the navigation. It was maintained that the difficulties of navigation arising from the great velocity of the tidal currents, the rapid eddies, the number of sunken rocks, and the baffling winds which frequently prevail, demanded the utmost skill on the part of the pilots to avoid accidents of a serious character; hence the necessity of Parliament refusing to sanction the erection of arches, which, in consequence of the great area occupied by the spandrils and piers, would not only restrict vessels to a narrower channel than heretofore on passing near the Britannia Rock, but would also shelter the vessels from the wind in situations where it was of the utmost importance to them.

These objections were deemed by many, deeply interested in the Holyhead Railway, so grave as likely to endanger the success of the Bill then before Parliament, and consequently

the whole undertaking. In this position of affairs I felt the necessity of re-considering the question, whether it was not possible to stiffen the platform of a suspension-bridge so effectually as to make it available for the passage of railway trains at high velocities.

In an attempt of this kind one remarkable failure had taken place some years before at Stockton-upon-Tees, and a professional survey of that structure had sufficiently demonstrated the extreme difficulty of such a task.

At this time Mr. Rendel called my attention to the mode of trussing which he had arranged for preventing oscillation in the platform of suspension-bridges, and afforded me the opportunity of inspecting the working drawings of the method he had pursued in correcting that defect in the Montrose Suspension-bridge, which gave way from the accumulation of a mass of people during a boat-race, on the 19th March, 1830, and again, subsequently, during a hurricane, on the 11th October, 1838. As this latter accident appeared to arise from, or at least to be materially aggravated by, the flexibility of the roadway, Mr. Rendel, being appointed to repair it, devised an excellent system of trussing, which has stood the test of several years: an elaborate and interesting description of the repairs executed under Mr. Rendel's directions is recorded in the "Transactions of the Institute of Civil Engineers for 1841."

The system of trussing here adopted by Mr. Rendel appears to me admirably adapted for a suspension-bridge intended for such weights as pass along ordinary turnpike roads, but the case under consideration was unquestionably very different, and certainly demanded, if trussing were resorted to, a much stronger and more ponderous system than that followed on the Montrose Bridge. Amongst a variety of devices for the accomplishment of this object, the most feasible appeared to be the combination of the suspen-

sion chain with deep trellis trussing, forming vertical sides, traversed by the suspension rods from the chains, with cross bracing frames top and bottom, to retain the sides in their proper position, thus forming a roadway surrounded on all sides by strongly trussed framework.

A structure of this kind would no doubt be exceedingly stiff vertically, and has, indeed, been applied successfully in America on a large canal aqueduct, and is clearly described in the 44th vol. of the "Mechanics' Magazine"* for 1846.

The application, however, of this system to an aqueduct is perhaps one of the most favourable possible; for there the weight is constant and uniformly distributed, and all the strains consequently fixed both in amount and direction, two important conditions in wooden trussing, constructed of numerous parts. In a large railway bridge it is evident, so far from these conditions obtaining under any circumstances, they are ever varying to a very large extent, but when connected with a chain which tends to alter its curvature by every variation in

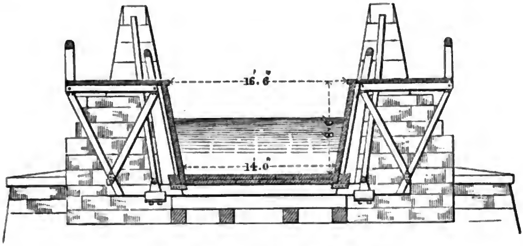
* This work consists of seven spans, of 160 feet each, from centre to centre of pier. The trunk is of wood, and 1140 feet long, 14 feet wide at bottom, 16½ feet on top, the sides 8½ feet deep. These, as well as the bottom, are composed of a *double* course of 2½-inch white pine plank, laid diagonally, the two courses crossing each other at right angles, so as to form a solid lattice-work of great strength and stiffness, sufficient to bear its own weight and to resist the effects of the most violent storms. The bottom of the trunk rests upon transverse beams arranged in pairs, 4 feet apart; between these, the posts which support the sides of the trunk are let in with dove-tailed tenons, secured by bolts. The outside posts, which support the side-walk and tow-path, incline outwards, and are connected with the beams in a similar manner. Each trunk-post is held by two braces; 2½ × 10-inch, and connected with the outside posts by a double joist of 2½ × 10. The trunk-posts are 7 inches square on top, and 7 × 14 at the heel; the transverse beams are 27 feet long, and 16 × 6 inches; the space between two adjoining is 4 inches. It will be observed, that all parts of the framing are double, with the exception of the posts, so as to admit the suspension rods. Each pair of beams is supported on each side of the trunk by a double suspension rod of 1⅛th-inch

the position of any superincumbent weight, the direction and amount of the complicated strains throughout the trussing become incalculable as far as all practical purposes are concerned.

Putting these objections on one side for a moment, the introduction of wood into such works as the Conway and Britannia Bridges seemed inadmissible, both on account of its perishable nature and danger from fire.

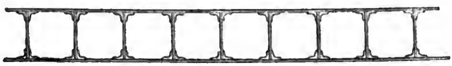
This led to the revival of a design I had made in 1841 for a small bridge on the Hertford and Ware Branch of the Northern and Eastern Railway, where it was necessary—in consequence of certain restrictions in the Act of Parliament authorising the construction of this branch—to construct a

round iron, bent in the shape of a stirrup, and mounted on a small cast-iron saddle, which rests on the cable. These saddles are connected, on the top of the cables, by links, which diminish in size from the pier towards the centre. The sides of the trunk rest solid against the bodies of masonry, which are erected on each pier and abutment as bases for the pyramids which support the cables. These pyramids, which are constructed of three blocks of a durable, coarse, hard-grained sandstone, rise 5 feet above the level of the side-walk and tow-path, and measure 3×5 feet on top, and $4 \times 6\frac{1}{2}$ feet at base. The side-walk and tow-path being 7 feet wide, leave 3 feet space for the passage of the pyramids. The ample width of the tow and foot-path is therefore contracted on every pier, but this arrangement proves no inconvenience, and was necessary for the suspension of the cables next to the trunk.



Seven Spans of 160 *feet each.*

bridge for the purpose of carrying a common road over the River Lea, in the town of Ware, with a certain headway above the towing path, and yet not to raise the street more than a given amount. The span was to be 50 feet, and the conditions only admitted of a platform 18 or 20 inches in thickness. For this purpose a wrought-iron platform was designed, consisting simply of a series of cells, as shewn in section in the following figure, the whole being of boiler plate, riveted



together with angle-iron, as in ordinary boiler building. The bridge was not, however, carried out in conformity with the design. Instead of the platform consisting of wrought-iron plates, riveted together, forming one mass, it was constructed of separate wrought-iron girders, composed of wrought-iron plates riveted together, and arranged as in an ordinary cast-iron-girder bridge.

It was reverting to this bridge that led me to apply wrought-iron with the view of obtaining a stiff platform to a suspension-bridge, and the first form of its application was simply to carry out the principle described in the wooden suspended structure last spoken of, substituting for the vertical wooden trellis trussing, and the top and bottom cross braces, wrought-iron plates riveted together with angle-iron. The form which the idea now assumed was, consequently, simply a huge wrought-iron rectangular tube, so large that railway trains might pass through it, with suspension chains on each side. The first arrangement, therefore, of the tubular structure was exactly similar in form to the trellis trussed wooden design before alluded to; but it was evident that the action of the top and bottom of the tube, composed of thick wrought-iron plates, would be infinitely more efficient than the

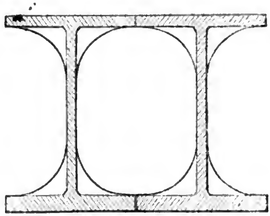
top and bottom braces, whose duty was chiefly to keep the side trusses in their vertical position. The top and bottom plates performed precisely the same duties as those of the top and bottom webs of a common cast-iron girder. It was now that I began to regard the tubular platform as a beam, and that the chains should be looked upon as auxiliaries. The rectangular figure, although it admitted of great facilities for attaching the chains, appeared ill suited for maintaining its form, and liable to become lozenge-shaped without a system of diagonal struts inside. This latter arrangement appeared impracticable, so long as the idea of passing the trains inside was adhered to.

The rectangular figure was also deemed objectionable, from the large surface which it presented to the wind. The side pressure due from a hurricane being very considerable. These circumstances suggested the propriety of circular, or elliptical tubes, which appeared well calculated, if not to remove, certainly greatly to moderate, these difficulties. On the 13th and 14th of March, 1845, I gave instructions to my assistants, Mr. G. Berkley and Mr. W. P. Marshall, to prepare drawings of a tubular bridge in accordance with the last-mentioned views, the tubes being made with a double thickness of plate, top and bottom. All calculations were now made as to the strength of the tube, irrespective of the chains, by following the principle which had been adopted by Mr. Hodgkinson in determining an empirical formula for cast-iron girders. It will be seen hereafter that although this was not strictly correct reasoning, it was for practical purposes a near approximation to the truth; and as it disregarded the sides as an element of strength, it appeared to lead to unquestionably safe results. I could not at that time avail myself of the generally received theories of tubes, viz. that their strength was directly as their sectional area and depth, and inversely as their length, for no experiments had

been then made confirmatory of such theory, or which furnished any data for practical purposes. The results, however, arrived at by assimilating the tube to a cast-iron girder, as has just been mentioned, were so favourable, that I determined at once to make use of this description of bridge, should the opposition to the design with two cast-iron arches, already described, prove formidable in our progress through Parliament with the bill then pending.

At this juncture I was placed in a most difficult position. Those interested in the navigation of the Menai Straits, as well as those who had, prior to this period, strenuously advocated Dynllaen in opposition to Holyhead as the proper terminus for a railway, had succeeded in inducing the Admiralty to give instructions to Sir John Rennie, Mr. J. M. Rendel, and Captain Vidal, to visit the Straits forthwith and report upon the probable injury which might accrue to the navigation of its waters by the erection of the proposed arches at the Britannia Rock. I was too well acquainted with the overwhelming weight which is almost invariably given in such investigations to a long-established public interest, and the extreme jealousy with which any interference with it is watched, not to feel that the fate of my first designs was sealed. I stood, therefore, on the verge of a responsibility from which I confess I had nearly shrunk: the construction of a tubular beam of such gigantic dimensions, on a platform elevated and supported by chains at such a height, did at first present itself as a difficulty of a very formidable nature. Reflection, however, satisfied me that the principles upon which the idea was founded were nothing more than an extension of those daily in use in the profession of the engineer. The method, moreover, of calculating the strength of the structure which I had adopted was of the simplest and most elementary character; and whatever might be the form of the tube, the principle upon which the cal-

culations were founded was equally applicable, and could not fail to lead to equally accurate results. When I began to regard the tube as a beam, one of the forms which the notion took was that of two huge double T cast-iron girders placed alongside of each other thus, sufficiently large for a railway between them :



In such a pair of beams, the area of the bottom section and the depth are the most important elements in calculating its strength. From this mode of treating the subject, it was obvious that the same view was strictly applicable to a tube whatever might be the form, the ultimate strength depending mainly on the area given to the top and bottom sections.

This was the shape which the subject took in my mind between the 16th and 23d of March, when Sir John Rennie, Mr. Rendel, and Captain Vidal, visited the site of the proposed bridge over the Menai Straits, in compliance with instructions issued by the Lords Commissioners of the Admiralty, for the purpose of ascertaining how far the proposed cast-iron bridge of two arches was likely to interfere with the interests involved in the navigation. The Reports of these gentlemen will be found annexed, from which it will be seen that the cast-iron bridge was deemed ineligible, and that a clear passage throughout the whole span, as in the existing chain bridge, of at least 100 feet above high-water line, would be insisted upon.

My apprehensions respecting the fate of the design for

the cast-iron bridges were now realised, and it appeared evident that the tubular bridge was the only structure which combined the necessary strength and stability for a railway, with the conditions deemed essential for the protection of the navigation.

It became my duty, then, to announce to the Directors of the Chester and Holyhead Railway Company that I was prepared to carry out a bridge of this description. They did me the honour of giving me their confidence after I had generally explained my views; not, I believe, without some misgivings on their part.

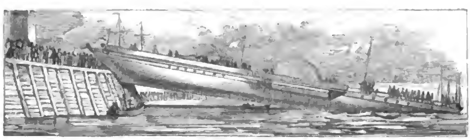
It soon became known to many of my friends what my intentions were, and to several I fully explained my views in detail, and entered into the calculations, especially with Mr. G. P. Bidder and others, in my office.

General Pasley (now General Sir Charles Pasley), then Inspector-general of Railways, called upon me, I believe, about the beginning of April. To him I shewed my sketches and explained my views. He concurred in the soundness of the idea, but most decidedly objected to the removal of the chains, urging as a reason, that no object could be answered by taking them down, if once put in their place, for the purpose of constructing the tube. To this argument I felt that there was no sufficient answer, especially if any contrivance could be devised for making them serviceable in giving strength to the tube, the possibility of which I did not doubt, although from the observations I had made on the Stockton Suspension Railway Bridge, I considered there was considerable difficulty and several objections to rendering a flexible chain available for strengthening a rigid platform. General Pasley, however, urged so strongly the propriety of allowing them to remain on prudential grounds, that I ceased to urge the intention as a part of the design, for it was evidently a step which might be allowed to depend

entirely on the results developed in the progress of the work. When I had explained to the Directors my views, Mr. John Laird, the well-known iron ship builder, then one of the Chester and Holyhead Board, expressed his confidence in the great strength which such a structure as I proposed would possess, and adduced some instances where the extraordinary strength of iron ships had been tested when stranded. The most remarkable case, however, of this kind, where the strength of the hull of an iron vessel had been strikingly evinced, was brought before me by Mr. Miller, the eminent marine engine builder, and having occurred under his own eye, he was enabled to afford me the minutest information.

The incident here alluded to took place in launching the Prince of Wales iron steam-vessel, at Blackwall, at the works of Messrs. Miller and Ravenhill, and was deemed so demonstrative of the excellence and surprising strength of iron ships, that an engraving was published by Mr. Miller, exhibiting the position and dimensions of the vessel, with a brief description of the accident, as under. *

* *The Prince of Wales iron steam-vessel.*



"The vessel is entirely of iron, and is intended for the Margate station; she is 180 feet long between the perpendiculars; in launching, the cleet on the bow gave way in consequence of the bolts breaking, and let the vessel down so that the bilge came in contact with the wharf; she was ultimately forced off by screw-jacks and two tug-vessels, cutting her way deeper into the concrete and planking of the wharf, until she assumed the position represented in the drawing; and at that period the distance measured from the face of the wharf to the point of contact of the vessel and the surface of the water was 110 feet. The whole of the deck in the centre of the vessel was

The circumstances here brought to light were so confirmatory of the calculations I had made on the strength of tubular structures, that it greatly relieved my anxiety, and converted my confidence into a certainty that I had not undertaken an impracticable task.

The period was now approaching when I should be called upon to give evidence before a Parliamentary Committee on the subject of the proposed bridge. My late revered father, having always taken a deep interest in the various proposals which had been considered for carrying a railway across the Menai Straits, requested me to explain fully to him the views which had led me to suggest the use of a tube, and also the nature of the calculations I had made in reference to it. It was during this personal conference that Mr. William Fairbairn accidentally called upon me; to whom I also explained the principles of the structure I had proposed. He at once acquiesced in their truth, and expressed confidence in the feasibility of my project, giving me at the same time some facts relative to the remarkable strength of iron steam-ships, and invited me to his works at Millwall to examine the construction of an iron steam-ship which was then in progress. Mr. Fairbairn's experience in this department of engineering being well known to me, and also his investigations in connexion with Mr. Hodgkinson on the subject of the strength of cast-iron, it occurred to me that he would be well qualified to assist me in the experimental inquiry which I had deter-

left unfastened for the reception of the machinery; when completely afloat it was found that the shear of the vessel was not broken, and that she had received no injury except that the bow was twisted in consequence of letting go the stern-rope, and thus exposing the vessel to the sweep of a strong ebb tide. On examination it was found that three of the angle-iron ribs, or frames, were broken, and one of the plates cracked, occasioning a considerable leak, which was accompanied by no other inconvenience than that of filling the bow compartment as far as the first bulk-head; and after hauling the vessel into dock, the necessary repairs were effected in four days."

mined upon making prior to finally deciding on the exact dimensions of the tubes or mode of procedure.

He readily agreed to assist me, and it was forthwith decided to consider and arrange a series of experiments.

Nothing, however, but preparatory steps were taken, when the Bill for the deviation of the line in the vicinity of Bangor was brought before a Committee of the House of Commons on the 5th of May, 1845.

The evidence which I gave before the Committee on the above day was received with much evident incredulity; so much so, that towards the end of that day's proceeding the Committee stated they would require further evidence, and especially that of the Inspector-general of Railways, before they could pass the Bill authorising the erection of such a bridge as that which I had proposed. The preamble of the Bill was passed, but a resolution come to which left the question of the bridge entirely open for further consideration. In this position of things it became evident, from my knowledge of the decided opinions held by the Inspector-general respecting the propriety of not dispensing with the chains, that I should not persist in the opinion that they were unnecessary; accordingly it will be observed, that whilst I expressed an unequivocal opinion as to the sufficiency of the tube alone, I was driven, from the circumstances that surrounded me, to leave the impression upon the minds of the Committee that at all events the chains might be left as auxiliaries to the tube if necessary.

The Bill passed the Committee, and in due course became law by receiving the royal assent, June 30th, 1845.

I now commenced an experimental investigation on tubular constructions. The performance of the experiments I entrusted to Mr. Fairbairn. Before any experiments had been performed, he suggested a modification of my view, similar to that which has been since proposed by Mr. Cowper, as a

mode of constructing rigid suspension-bridges, and described in a paper read before the Institution of Mechanical Engineers, October 27, 1847.

The notion is the converse of the first, or beam-platform. Mr. Fairbairn proposed to transfer the rigidity from the platform to the chain. This was effected by converting the chain into a large flat tube or tubes, with sufficient flexibility to assume a curved form, but with sufficient rigidity to resist much distortion of curvature, by an unequal or varying pressure, such as is usually communicated by the transit of a heavy weight along the platform of suspension-bridges.

The tubular construction in both ideas is resorted to for the purpose of obtaining the requisite stiffness; and the question is really, in which way are strength and stiffness attained most economically and efficiently. This view occupied my attention for some little time; but the difficulties of erection appeared to me insuperable, whereas, with the rigid platform, the ordinary chains offered great facilities for constructing and erecting the tubes. The rigid beam, instead of the rigid chain, was therefore persevered in as preferable, not only because it afforded greater facilities for erection, but on account of the rigidity of the curved tube being very problematical.

There can, however, be no question that a rigid suspension-bridge, with a tubular chain, in some cases, may be employed with success. Its construction, however, did not appear to offer such advantages as to justify my rejecting the rigid tube, which offered greater facilities for erection, as also the probability of dispensing with the chains altogether.

We had not proceeded far in this experimental investigation when Mr. Fairbairn suggested that Mr. Eaton Hodgkinson's aid should be solicited. To this proposal I instantly consented; for being familiar with the valuable contributions

of this gentleman to engineering science, more especially in the department which comprehends the very subject then engrossing my attention, I felt, considering the responsibility which I had publicly assumed, that I should be doing injustice to the Board of Directors, who had placed such confidence in me, if I did not avail myself of all the practical and scientific aid which might be within my reach. I did so, freely and unhesitatingly, from every quarter. The experiments conducted by these gentlemen are given in detail in a subsequent chapter. The facts elicited by these experiments were carefully discussed with them from time to time, and the best method of pursuing the investigation was determined upon. In the majority of the early experiments failure took place by the crushing of thin plates in the upper side of the tubes. This defect induced me at once to return to the original form in which the tube had occurred to me, viz., that of an ordinary flanged girder. To carry out this notion, keeping in view the tendency of the plates to buckle, a double top of corrugated iron was applied, and a corresponding increase made in the strength of the bottom, which arrangement, it will be seen, was attended with favourable results.

The top of the tube thus came to be considered simply as a series of parallel hollow pillars, to resist the compression to which it was subjected by transverse strain. Among the numerous modifications which presented themselves, a series of rectangular cells possessed so many practical advantages, as regards construction, that I did not hesitate to give this arrangement a preference; and although subsequently some advantages in the use of circular cells were clearly developed by the experiments, I did not consider them of sufficient importance to change my decision. There is no doubt, from subsequent experience, that the fears at this time entertained with respect to buckling were, to a considerable extent, ex-

aggerated, for the thickness of plates is an element of far greater importance in this resistance to buckling than was then imagined.

In March 1846, sufficient data were accumulated to enable me, for the first time, to decide somewhat definitely on the required dimensions, and I accordingly gave instructions to my assistant, Mr. Edwin Clark, to prepare a model of the tube, which was made with rectangular cells in the top and bottom, the sectional area of the top being then provisionally fixed at 600, and that of the bottom at 400, square inches. The various modifications which afterwards took place, as new facts were disclosed, will be found fully described hereafter.

The success which has attended our exertions in this laborious and anxious investigation demands of me this public acknowledgment.

To Mr. Fairbairn I am indebted for the zeal with which he entered upon the experimental investigation, for the confidence he displayed in the success of my design, for the sound practical information which he brought to bear upon the subject, and the assistance he rendered as we progressed.

To Mr. Hodgkinson for devising and carrying out a series of experiments which terminated in establishing the laws that regulate the strength of tubular structures in a manner so satisfactory that I was enabled to proceed with more confidence than I otherwise should have done.

To Mr. Edwin Clark, the resident engineer, for the important assistance he rendered me in strictly scrutinising the results of every experiment, whether made by Mr. Fairbairn or Mr. Hodgkinson, and for the separate and independent scientific analysis to which they were invariably subjected by him before I finally decided upon the form and dimensions of the structure or upon any mode of procedure.

I have now brought the history of the Conway and Britannia Bridges to a date, after which all that was done has either been communicated in official reports to the Board of Directors, or has been carefully registered by my assistant, Mr. Edwin Clark, who has given his undivided attention to the subject since the beginning of 1846, during which period he has collected a mass of information which cannot fail to be both interesting and important to the profession to which he belongs. I cannot close this statement respecting two works which have caused me years of increasing and intense anxiety without expressing my regret that one of the gentlemen to whom I have always been most anxious to award all the credit to which he is entitled, should have endeavoured to enhance his own claims by detracting from the credit fairly due to all those with whom he has been associated in this great work. But I sincerely trust that the facts and views put on record in the following pages will enable those who take an interest in the subject to do justice to all parties concerned.

CHAPTER II.

CONTINUATION OF THE HISTORY OF THE DESIGN.

No additional observations can further elucidate the very remarkable circumstances and restrictions under which the engineer proceeded with his designs for the Britannia Bridge. The natural difficulties to be overcome in crossing such a gulf were increased by the formidable opposition of conflicting personal interests and the requirements of the Act of Parliament at length obtained; the construction of an arch of colossal dimensions, and without centering, or some modification of the suspension-bridge, which should render it sufficiently rigid for railway traffic, appeared the only available alternatives affording any precedent, and both these projects being rejected, nothing remained but to devise some new method of construction applicable to such unusual contingencies. The successful opposition to the magnificent cast-iron bracket arch, described in Mr. Stephenson's observations, is certainly to be regretted. Some further details of this bold and novel device will be found in Plate XXXIII., in which the requisite dimensions of the tie-rods for resisting the horizontal thrust are calculated, and the proposed form of the voussoirs is also shewn, although these plans were never matured, and are to be considered only as the original conceptions of the design. The general outline of this beautiful structure is shewn on Plate XXXII., which is copied from the original drawings submitted in evidence on the proposal of this bridge. The arched viaduct at either end shewn by the architect would

have been supplanted by the counterpoise semi-arches described at page 17, which would have been fac-similes of the corresponding semi-arches in the plate; the magnitude of the piers, which were objected to on account of their size, is in good proportion with the general features of the structure; and, moreover, their dimensions were necessary, not only to insure stability in the structure during the progress of its construction, but also ultimately to furnish a counterpoise to passing traffic, while the inertia of such a mass on such a base placed it out of all danger of damage from wind.*

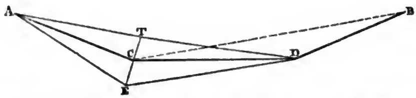
With respect to the use of the present suspension-bridge for the proposed traffic, it was found difficult to devise any means of sufficiently strengthening it that did not involve an almost entire reconstruction,† and great difficulty was similarly found in attempting to render any suspension-bridge sufficiently rigid for railway traffic by means of ordinary trussing. When the passing load is small compared with the weight of the chains and of the structure itself, there is indeed no difficulty, but the construction of a platform 450 feet long, sufficiently rigid for locomotive traffic, almost amounts to the construction of the tube itself. The ordinary reader would hardly be aware of the difficulty of rendering a suspended structure rigid when exposed to great local strain. Rigidity is excluded by the very principle of suspension, in which the element of strength is the very instability of the chain, which, converting every strain upon it into a direct tensile strain, at once allows the material to act with the greatest possible advantage. If this motion of the chain be prevented by any

* A very beautiful model of this bridge is in the possession of the Directors at their offices at Euston Square, as also an elaborate model of the natural features of the locality, by Mr. Salter of Hammersmith.

† The strain on the chains from its own weight, supposing them to be all acting uniformly, amounts at present to full five tons per square inch of section.

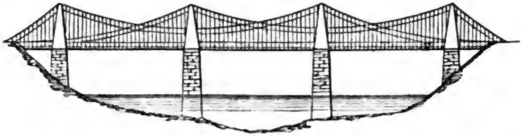
means within the structure itself, such as trellis-work, some transverse strain or thrust is at once produced which is incompatible with the principle of suspension; and a compound of the beam and the chain is the result.

A suspension-bridge to be rigid would consist of only two links, and they must in themselves be of sufficient rigidity as beams to prevent their flexure—a thing impossible in large spans.



In a bridge, A B, consisting of three links, a weight at C, tending to lower that point, would revolve on the radius A C round the centre A. This motion could be prevented by direct or indirect communication with the point B, that is, by another link C B, forming a suspension-bridge of only two links, and therefore impracticable on a large scale, as the link C B would itself become a flexible catenary; or, otherwise, we must give depth and immobility to the joint C to prevent its yielding, which involves a thrust at T and an extension at E, A C D becoming an ordinary beam.

The same would hold good with any number of links, and under the effect of any load passing over such a bridge. The strain caused by the load would be similar in kind to the strains on a girder, and therefore incompatible with the qualities of a chain.



Chains alternating so that the point of suspension of one should come over the centre of the other, or rods so arranged

as to keep each other rigid, while every point of the road-way might be considered as sustained by a suspension-bridge of only two links, would only partially remedy the evil, until the rods become so numerous and so interlace each other that the bridge would be a trellis suspension-bridge. If we again imagine the trellis-work to become more and more dense, the bars become plates, and we arrive at a chain of riveted plates, which in the tubular form was at one time suggested as a modification of the tube first proposed by Mr. Stephenson.

This trellis-work, in fact, exists in the sides of the present tube, for the close side of the tube may be considered as dense trellis-work; a trellis-beam bridge being really nothing more than the vertical rib of an ordinary girder, without the most important additions of top and bottom flanges. And, in like manner, does every possible combination of rods exist in the sides of the present structure.

Another difficulty occurs with a rigid platform on a suspension-bridge. The expansion and contraction, owing to changes of temperature, acting on so long a chain, increases or diminishes the versed sine of the catenary, while the horizontal roadway, merely lengthening and shortening from the same cause, must rise and fall at the centre of the span, which is impossible if it be rigid.

Now, the extension of a chain for a span of 400 feet, with a versed sine of 30 feet, as required at Conway, with a strain of 6 tons per square inch, would amount to 3 inches nearly, from its elasticity alone. The variation in length of the same chain from changes of temperature would amount to 3¾ inches; and this latter extension would increase the versed sine, or lower the centre of the horizontal roadway through a space of nearly 9 inches, while the extension of the roadway itself from the same cause, if made of iron, would be 3¾ inches nearly, taking place in a horizontal direction only; so that, supposing the rigid roadway

to be borne up by the chain in cold weather, in the summer the chain at the centre of the span would be dipping 9 inches below it, leaving it entirely unsupported.

Now, 300 tons placed as nearly as was practicable in the centre of the Conway Tube, only deflected it 3 inches, so that with a roadway as rigid as this tube the chain would at times afford literally no support, even after making allowance for the deflection of the roadway: for we have no right to reckon on the elasticity of the chain, without supposing that it is sometimes to support one weight and sometimes another, which presumes a roadway capable of supporting itself. The chain, in fact, will alternately be doing everything and nothing.

Mr. Stephenson had practically seen the difficulty of employing the ordinary suspension-bridge for railway purposes on the Stockton and Darlington Railway, where he was called in to erect a new bridge across the River Tees, in consequence of the failure of a bridge of this description which had been constructed there: this was a case in which an attempt was made to render the roadway rigid by ordinary trussing. It is remarkable, in this case, that after the roadway was strengthened and rendered rigid by piles driven into the bed of the river, the chains only affording partial support, their vibration literally destroyed the framework under the platform, and drew the piles out of the ground. These considerations led Mr. Stephenson to abandon the attempt of rendering an ordinary suspension-bridge rigid (*see* his evidence at p. 47), and to resort to an independent beam.

The peculiar advantages of a beam, under the circumstances referred to, are at once evident.

There are but two general principles of construction for crossing any horizontal space:—

Either the horizontal strain must be resisted by the abutments, as in an arch or suspension-bridge, the material being exposed merely to a direct crushing or tensile strain;—

Or, secondly, the horizontal strain may be resisted within the system itself, as in an ordinary girder or trellis-bridge, or in a trussed roof, in which the materials employed have both tension and compression to resist simultaneously.

The independence of all extraneous support except direct vertical pressure, which characterises this latter class of structures, was peculiarly available where it was requisite that a structure of such vast dimensions should be transported entire into its place, or constructed without scaffolding.

The bracket arch before described involved no horizontal thrust, and may be treated as a beam supported at the centre instead of the extremities.

With such an extension of the usual signification of the term girder or beam, the Britannia Bridge has been called a tubular beam.

A tube is here applied to a purpose so entirely novel, that some modification of our familiar interpretation of its meaning is also necessary to avoid erroneous impressions as to its application. The term tubular, in this instance, merely signifies "hollow," which is not in reality an essential characteristic of this species of construction.

Perhaps the more peculiar feature of novelty in the tube consists in its being a constructed beam. Hitherto beams had been principally confined to single castings, or to few separate castings, with wrought-iron tension-rods to relieve the lower flange of its tensile strain, as in trussed girders, on account of the inadequate strength of cast-iron to resist such a strain, and more especially on account of its brittleness or incapability of accommodating itself to change of form, without fracture. Castings for beams are limited in their size by the impracticability of running large quantities of metal without air-bubbles or flaws, by the fracture of large masses from unequal contraction during the process of cooling, by the crystallisation of the internal portions, and also by the

weight of the beams themselves becoming an important part of their breaking-weight, when constructed of a metal which requires so large a section to resist the tensile strain on the lower flange. The longest single beams have consequently seldom exceeded 60 feet, and are generally limited to considerably less dimensions. The longest compound girders constructed in parts with wrought-iron tension-rods do not exceed 104 feet, as in the bridge over the Arno, on the Florence and Leghorn line, by Mr. Robert Stephenson.*

The dimensions of an arch, or suspension-bridge, are almost unlimited, as regards construction, because in the former there is no dependence on bolts or any other means of uniting the voussoirs, which simply butt against each other, and depend on friction for their stability; and in the latter, the union of plates or bars by the pin at the joint is peculiarly simple and practical, and may be carried to any extent. In the construction of a beam of several independent parts, the effective union of these parts, subjected as they are to both tensile and compressive strain, offers considerable difficulties; and in 1845, as though intended for an experiment on this very subject, Mr. Stephenson constructed a cast-iron beam of 120 feet span, of 16 castings, breaking joint like brickwork, and bolted together with flanges on each casting, with provisions for wrought-iron trussing. Of this beam, Plate XLIV. is a representation. It was intended to be erected on the Wisbeach line for crossing the River Nene. The expense of fitting was, however, very considerable, and the weight required for the connecting flanges is a great drawback. The most important features of a beam, moreover, the top and

* It is rather remarkable that the application of cast-iron to the construction of large girders for railway bridges originated with Mr. George Stephenson, who first employed such girders on the Liverpool and Manchester line. The beam is, in fact, peculiarly applicable to railways on account of its simplicity and rigidity, and its history is inseparable from that of their progress.

bottom flanges, are not proportioned to the mass of the vertical rib, in which the main strength is made to consist, and the extent to which such a construction can be carried is, consequently, limited.

The drawing is inserted from its peculiar resemblance in principle to the wrought-iron tube, and as an example of the tendency of Mr. Stephenson's mind towards this principle of construction.

The difficulty of bolting these castings together, and the nice fitting requisite with such brittle material, would naturally suggest the superiority of wrought-iron plates, united by means of rivets, as employed so successfully in ship-building and boiler-making, while the wrought-iron platform designed for the North Eastern Railway, as early as 1841, is a still more direct attempt at employing this material in the construction of a bridge.

A beam of sufficient dimensions for a span of 460 feet was, however, quite beyond the limit of any precedent; and Mr. Stephenson was compelled to extend the theory of beams to an extent that left all previous practice unavailable, and to leap at once to dimensions far exceeding those of even the largest arches in existence. It was natural, that, in conceiving the probable character of these new structures, he should be guided by comparison with existing types. In fact, it was in combining mentally, side by side, two ordinary flanged girders, of sufficient dimensions for his purpose, that he was led to the original conception of the tube. Thus, in the course of his inquiries he had considered all means heretofore employed for crossing a great space; the contraction and expansion of large arches, the oscillations of chains, and the necessity of employing a structure that could be moved entire, led him to the beam; in the beam these difficulties were much lessened; the expansion and contraction had to be met only in a longitudinal direction; the cal-

culation of the necessary strength was simple and easy on received and well-established principles; it combined the strength of the chain with the rigidity of the arch; and was well adapted for independent construction. In selecting a material for his purpose, wrought-iron was remarkably applicable. Its tensile and compressive strength, and also its ductility, or power of accommodating itself without loss of strength to change of shape, are very considerable, and the means of uniting it by rivets simple and efficient. Mr. Stephenson appreciated these advantages, and for this material the tubular form was peculiarly available.

Such would, probably, be Mr. Stephenson's first views of the subject. With these principles matured by subsequent meditation and discussion, he gave the evidence quoted in the following pages. The complete manner in which he had then investigated the subject will be seen from the minute details of his evidence.

CHAPTER III.

MINUTES OF EVIDENCE AND REPORTS ON THE SUBJECT OF THE BRITANNIA BRIDGE.

HOUSE OF COMMONS.

Select Committee on Railway Bills: Group 2; Chester and Holyhead Railway Bill.

Lunæ, 5° *die Maii*, 1845.

THOMAS HENRY SUTTON SOTHERON, ESQ., in the Chair.

MESSRS. BURKE, PRITT, VENABLES, AND CO., *appeared as Agents in support of the Bill;* MR. AUSTIN, MR. TALBOT, AND MR. ROBINSON, *appeared as Counsel in support of the Bill.*

ROBERT STEPHENSON, ESQ. *called and examined by* MR. ROBINSON.

You are the engineer for the intended line of railway?—I am.

You are aware of the manner in which it is proposed to cross the Menai Straits?—I am.

Is there to be a bridge of 104 feet high, with the arches of 450 feet span?—There is.

Is that the form of crossing sanctioned by the Government?—Yes, it is: it is in conformity with a report that has been made to the Admiralty by Sir John Rennie, Mr. Rendel, and Captain Vidal.

Do you consider that a practicable and safe mode of crossing the Menai under the circumstances?—Yes, I do.

Do you consider it the best mode of crossing the Menai?—No; not for the railway—for the navigation I think it is: I prepared, originally, an ordinary arch of 360 feet span, similar to the Southwark Bridge one, but the Admiralty objected to that in consequence of the haunches or spandrels of the arch interfering with the masts of the vessels, and it was in consequence of that original design not being in conformity with their views that they sent down those Commissioners. I went with them, and we received evidence from the pilots of the Straits and others for two or three days; and Sir John Rennie and Mr. Rendel have since reported to the Admiralty that they considered the span of the bridge I proposed of 360 feet should be made a span of 450 feet and that a clear space of 105 feet should be given underneath the platform of the bridge.

You feel no doubt as to the practicability of so great a span as 450 feet being used for a railroad?—Not now; although I did at the time.

What has induced you to change your opinion as to the practicability of the span?—I thought we ought not to adopt any span exceeding 360 feet with a cast-iron arch; and I thought, also, that that span could only be exceeded by the adoption of the chain bridge, which I do not approve of for the passage of locomotive engines, on account of the undulation into which the platforms are usually thrown. I have had one instance of the kind brought specially under my attention, that is, the Stockton and Darlington Railway Suspension Bridge, near Stockton. I saw there a strong chain bridge which was erected for railway purposes, which utterly failed, in consequence of the undulation of the platform. I have thought of adopting another plan in connexion with suspension, which would render the platform quite rigid; and if the platform be made rigid, then I think the suspension principle may be applied, but until it is made rigid I had my doubts about it.

You adopted the scheme of an arch of this construction above the span of 360 feet?—Yes, I did; for I found that even with 360 feet span the expansion and contraction between summer and winter would raise that bridge about 8 or 9 inches in the middle.

At a span of 360 feet?—Yes; that made me hesitate a good deal; but I thought of arranging it in such a way that the contraction

might take place without disturbing the arch. In the Southwark Bridge the expansion and contraction go on there and raise and lower the arch considerably, and it is in some measure interfered with by the soundness of the masonry at the bottom. The span of that is 240 feet. Having determined on 360 feet, and finding the expansion would raise the road at least 8 inches in the middle, I considered that any increase of span with cast-iron would create a difficulty which I was not prepared to encounter.

Are you now satisfied that that difficulty may be overcome? —Yes, I am; by having a platform of wrought-iron stiffened, by giving it a peculiar form; and in that case I should have it merely as a beam laid across the Straits, and it would be loose at the ends, to expand and contract, so that the platform may simply be this iron beam lying across the Straits, with the ends resting on the masonry. It would be fixed on the middle pier, and the expansion would take place at each end, which I find would be about 4 inches at each end.

What would become of the supporting arch when the expansion and contraction go on?—I do not propose to do it in that way; I propose to do it on a beam, or in connexion with chains.

Is it not an arch on the plan of Southwark Bridge?—No. Perhaps I may at once explain to the Committee the idea I have adopted. I conceive a tube: supposing a wrought-iron tube to extend across the Straits; that tube to have, we will say, 25 feet diameter to hold a line of railway, and the line of railway would run inside of it. In addition to that, we should have to erect a *chain platform* for the purpose of the building. Then the question would arise, whether *the chains would be allowed to remain*, or whether they would be taken down. My own opinion is, that a tube of wrought-iron would possess sufficient strength and rigidity to support a railway train. I am instituting a series of experiments in conjunction with Mr. Fairbairn of Manchester; he is already in possession of experiments with respect to iron ships, which place the thing beyond doubt. He has ascertained that a vessel of 250 feet in length, supported at the ends, will not yield with all the machinery in the middle. There are several cases which I could quote of iron vessels having been stranded with the steam-engines in the centre, without injuring the construction of the vessel.

What would be the thickness of the side?—I should propose the

bottom side to be three-quarters of an inch; it would be strengthened by a side trussing.

In the nature of a tunnel?—Yes.

What is the diameter of the tube?—It would be about 25 feet.

For the whole train to run in?—Yes; it is nothing but putting an arch over a wooden bridge, which is very common, for the purpose of protecting them from the weather. I should not venture to build a cast-iron arch of that description of 450 feet span, on account o the expansion and contraction. I am sure it would not be sufficient but under the restrictions I have to labour under, it becomes necessary to do as I propose; we have no centering.

Is this mode of construction quite original?—It is.

Is it your own view?—Yes; meeting the contingencies which have been put upon me by Government engineers.

Then the whole length of the tube would be about 900 feet?—Yes, it will.

And it will be supported in the centre upon a pier?—Yes.

Do you propose two?—No, one, because one will add to the strength.

That will not allow the contraction and expansion to take place at the centre of the pier?—No, only at each end, because I keep the pier entirely free from disturbance by fixing the tube down to it.

How do you propose to get the tube there? You will in fact make the tube on the spot?—Yes.

How would you place it in the position you mean it to occupy?—I would construct a platform suspended by chains, just the same as they bind an iron vessel.

The nature of it would be this: it would consist of tubes suspended above the Straits, and with the two extremities unconnected with the masonry, so as to allow of expansion and contraction?—Precisely so; it is one tube in effect. Here is the middle resting upon the centre pier; this being fixed down, both ends expand and contract as the temperature of the air changes, without the masonry. With an arch, either the masonry must give way or the arch must rise. In this I had prepared myself for a rise of 8 inches. Any arrangements by which the elevation of the railway should exceed that I consider would not only be dangerous, but destructive to the structure itself.

In point of security, the only question is whether for the distance of 450 feet a tube of three-quarters of an inch thickness would be of strength enough to support the weight of a train as it passes: Do you think it will?—Yes.

You are satisfied upon that point in consequence of the experiments you have made?—Perfectly.

It is not possible to have any ribs to support it; is it?—There is no difficulty in having ribs, but I do not think it is necessary.

The reason for having a tube is on account of the strength of it?—Yes, on account of its shape; also you cannot disturb the shape of a curve as if it were a piece of flat iron plating. The question was, whether a suspension-bridge could be made available with 450 feet span. I had no other course but to make it fit for railway trains; there is no construction of timber that I can conceive would answer the purpose: on account of the great length it would not be perfectly rigid; it would be put together in a great number of parts, and those parts, each of them, would be liable to decay at the joints; therefore it occurred to me that a rigid platform might be obtained by substituting a tube in addition to the chains. Then, in going into the calculation of the strength of the tube, I found that I did not require the chains themselves, and therefore I have since proceeded upon the idea of the plating merely and simple tubes.

Have your calculations been submitted to many other engineers? I have made them in conjunction with Mr. Fairbairn of Manchester, whose experience is greater than any other man's in England.

There is no experience of a bridge being formed of a tube of this kind; is there?—No, there is no experience of it; nor was there of the iron vessel some time ago. There is now one building by Mr. Fairbairn, 220 feet in length, and he says that he will engage that when it is finished that it shall be put down in the stocks at each end, and she shall have a thousand tons of machinery in the middle of her, and it will not affect her.

What is the length?—220 feet: but that is not so strong as a tube; and therefore any experience that this would carry out the tube would fully bear.

I wish to ask you whether this is your own suggestion?—It is entirely.

Mr. Robinson.]—From the experiments you have made, and from

the inquiries you have also made, are you satisfied that that suggestion of yours is a practicable and safe one ?—I am not only satisfied that it is practicable, but I must confess that I cannot see my way at present to adopting anything else.

So that, if the Menai Strait is to be crossed at all, that is the only way you know of doing it ?—That is the only way I can conceive at present; there may be others, but I do not know of them.

Starting from the Britannia Rock ?—Yes. I make out the weight of each span to be about 450 tons; the weight of the cast-iron design which I had made would have been nearly 2000 tons.

Your other design would have had other supports at the sides ? —Yes; but I am comparing the weights of the two.

A great part of the weight of the first plan was composed of bars which supported the bridge ?—Yes; entirely.

In all cases of suspension-bridges the real difficulty has been occasioned by the alteration of the superstructure ?—As regards railway bridges the difficulty is in keeping the platform steady; because when the train went on to the Stockton and Darlington line, the rails rose up three feet in front of the engine; they were unable to use it.

HOUSE OF COMMONS.

Select Committee on Railway Bills: Group 2; Chester and Holyhead Railway Bill. (*No.* 1.)

THOMAS H. S. SOTHERON, ESQ., in the Chair.

Martis, 6° *die Maii,* 1845.

MR. RENDEL *was called in and examined by* MR. ROBINSON, *as follows:—*

You are a civil engineer?—I am.

You have had considerable experience in building bridges—iron suspension-bridges, and other bridges?—I have.

Under the direction of the Lords Commissioners of the Admiralty did you go down to inspect the Menai Straits for some proposed Railway over them in the month of March last?—I did.

Was that for the purpose of advising the Government as to the proper place for crossing the Menai Straits for this railway?—It was for the purpose of advising the Admiralty as to the suitability of a design that was proposed for crossing the Menai Straits at the Britannia Rocks by the intended railway to Holyhead.

Did the place at which they proposed to pass the Menai Straits appear to you an eligible site for the purpose?—That was the substance of my Report to the Admiralty upon that particular point,—that it was the most suitable site for crossing the Menai Straits.

And did you give your opinion to the Admiralty as to the height of the bridge, and as to the water-way between the two piers?—I did; my Report specified what should be the width of each arch, and the height of the roadway above high water, so as to avoid interruption to the navigation.

Now what was the height you recommended?—I have not my Report, but I think 105 feet above high water.

And the span?—I thought the circumstances required a span of 450 feet, so as to prevent injury to the navigation in the erection of a bridge.

And you thought there should be two such spans, and that the height should be 105 feet?—Yes; two such spans, each having a height of 105 feet.

Now, in your judgment as an engineer, and from your experience, do you think a bridge could be made in that form so as to carry the railway safely across the Straits?—I have no doubt whatever that a bridge may be constructed across the Menai Straits, with openings of the dimensions laid down in this drawing, with sufficient rigidity and strength to carry the trains across it.

The arch being, as you say, 450 feet?—Yes; I should wish to explain to the Committee, that in this Report I advised the Admiralty to leave the question of the principle of the design entirely with the Railway Company, that the only thing which the Admiralty could with propriety prescribe, was the minimum dimensions of the arches, with a view to the protection of the navigation.

And I understand you to say, from your judgment and experience in these matters, that you think those recommendations could be carried into effect practicably and with safety?—I have no doubt about it.

Cross-examined by Mr. Poole, *who stated that he appeared for the Petitioners, the Trustees of the Harbour of Carnarvon:—*

I believe you stated, Mr. Rendel, that the present bridge was perfectly innocuous, or nearly so, to the navigation?—That was the result of the examination of witnesses that was made on the spot.

The site of the intended bridge you considered the next best?—Yes.

You recommended that the piers to support that bridge should not exceed 50 feet square?—I did.

I believe the rate of the run of the tide at the Britannia Rock is considerably greater, and more violent and wild, than at the Menai Bridge, in consequence of its being obstructed by the Rock?—It is.

Do you consider a pier of 50 feet will be free from the objection of becalming vessels coming under the lee of it?—Yes.

I believe the Britannia Rocks are rugged, and not wall-sided?—Certainly; they are irregular.

Do you not think that it would be a necessary improvement when that pier is erected to the height of 100 feet, that some improvements should be made in the formation of that Rock for the purpose of protecting the navigation?—I think it would be very desirable that the Rock should be scarped.

Perhaps it would be difficult at this moment to point out the particular form of the details of this scarping?—I think it would be injudicious at the present moment to point out the mode of scarping. I think if that is determined on, that should be done after the effect of the pier on the winds and currents is ascertained.

Committee.] What is the square of which this pier is intended to be made?—50 feet.

What is the square of the Rock itself in round numbers at this moment?—130 feet wide, but I should think 200 feet in length.

In short, larger than the pier?—Much larger than the pier; the object being to place the pier in the centre of it, so that the keel of a vessel should strike the rock before the bowsprit could by possibility come in contact with the pier.

So that this pier will not encroach upon the water over the boundary of the rock, as it at present stands?—Certainly not; it will be far within the limits of the rock.

Mr. Robinson.] Do you not think the height of 100 feet would be sufficient for that purpose?—When I recommended the 105 feet it was from seeing on Mr. Stephenson's plan, which accompanied also my Report, that is, the skeleton of it there, that the arch proposed by him was at that elevation; I should say, that in practice it would be quite sufficient if the roadway of the new bridge was made as high as the roadway of the Menai Bridge; for there are no commercial establishments—not a wharf, or a probability of there being any, between the two places, and therefore no vessel that could pass the Menai Bridge would pass here. I think the height of the Menai Bridge is 100 feet at the towers, and I believe it is 103 feet in the centre.

I believe when you were there you made inquiries and an

examination with Capt. Vidal, as to the perfect safety of the Menai Bridge; there has been no accident?—No. There has been only one. The accident was that a vessel had missed her stays, and broke the bowsprit on the towers.

Committee.] Are you acquainted with the plan of a tubular bridge for crossing the Menai?—The idea has been named to me.

Have you the plan sufficiently before you to pronounce an opinion as to its security?—I think the principle is one capable of being carried out with perfect safety: whether it is the best under all circumstances, I think one should not be tied down to without further consideration.

Perhaps you were not acquainted with the details of the construction of the tube?—The plan itself, the main principle of a tube, is so simple that I think the construction would be very manifest. The principle, as I understand it, is that the objection to an ordinary formed roadway, is that it has a slight tendency to motion which would by repetition make all the framing liable to partial dislocation and to slight undulation. A tube would have the advantage of presenting no joints liable to motion, and would consequently retain for a longer time its rigidity than an ordinary trussed roadway.

In this plan of yours do you propose to confine the masonry entirely to the pier, or do you have any arch of mason-work?—In that plan the object I had in view in attaching that plan was to shew, as I said before, the least opening that the Admiralty ought to consent to. How the bridge should be constructed, the plan does not contemplate; it assumes a skeleton, and the engineer of the Railway Company has, according to the Admiralty permission, the power to fill up the skeleton in any way he chooses, but it prescribes the least dimensions that the bridge shall have.

The plan does not profess to be a plan of what the Company purpose to do, but only gives you the minimum line, that you, on the part of the Admiralty, would accept?—Yes, that is the plan.

Do you think it could be safely constructed without mason-work arches?—I think it may. I can only speak from the experience I have had of a horizontal roadway of 150 feet in width.

A bridge?—A bridge that is at Montrose: that bridge fell,—in point of fact, it fell twice; and, at the instance of the Treasury, I re-constructed it, and, in its re-construction, I adopted a trussing

principle, and since its re-construction it has not had the slightest perceptible motion. Since its erection they have in several instances had gales of wind, that used to put it in a state of undulation to the extent of four or five feet; but since its re-construction, it has not had the slightest motion in any gale of wind. I think about four or five weeks ago, I had letters from the local parties, who told me they had just such a hurricane as the last which threw it down, and that it had not the slightest perceptible motion during that hurricane.

How is it supported?—The road-way is trussed in a particular way to give it a particular rigidity,—a rigidity which would distribute any weight that would go over the wood-way over the whole bridge, whereas, in most of the suspension-bridges, they are so flexible, that a weight placed in any particular part depressed that part without affecting the other parts.

But the road-way is supported by a chain?—It is.

And therefore has really nothing to do with what is the question before the Committee; the question of the tube?—A tube may be supported by a chain.

I suppose a tube must either be supported by chains or by masonry arches?—Yes; undoubtedly.

Is the bridge of wood or iron that you spoke of?—It is wood. It is simply a truss of wood instead of a tube of iron; the chains, I presume, would be the same in either case.

Do you think, from your knowledge and experience of these matters, that it is possible to construct a bridge of this description with equal strength of iron as with stone and masonry?—Not of equal; an irresistent bridge would of course be stronger with reference to all ordinary acceptations of the word, but I can imagine a tube, a large cylinder of wrought-iron, thrown across an opening of 450 feet, supported by chains, as a bridge of excessive strength. I can imagine it so.

But you cannot imagine it so strong as if it were of masonry?—Oh! certainly not.

Would it not be difficult to construct a bridge of stone arches upon the data the Admiralty have given?—Yes; it would be next to impossible.

MR. STEPHENSON *was called in and examined by* MR. ROBINSON *as follows:—*

I will, first of all, ask you your opinion in general. Do you consider, from your knowledge and experience in these matters, that a bridge might safely and practically be built over the Menai of the dimensions recommended by the Admiralty; that is to say, the height 100 or 105 feet, with two spans of 450 feet?—Yes; on the plan I proposed, or on the plan Mr. Rendel has spoken of.

Now you think it can be done, what is your present view as to the best mode of doing it?—I cannot better explain it than by almost repeating what Mr. Rendel said; that the object is to have a platform which shall be perfectly rigid, and that rigidity may be obtained either by a frame-work of wood or by a frame-work of iron. I conceive that the erection of a wooden platform there, with a great mass of timber, would be highly objectionable, as I believe it would yield in time to heavy weights that would have to pass over it. In taking that view of the matter, I then considered the best mode of obtaining a rigid platform of iron; I considered well the means of trussing iron together in a similar way to wood, avoiding, of course, the yielding that wood would have from its texture, and I found there was no way so simple, so cheap, or so rigid, as throwing the iron-work into the form of a tube. As I said yesterday, I propose, in erecting that tube in its place, to erect a wooden platform, in the usual way, by chains, so that men may have a platform to work upon; and I said I had not made up my mind whether I would remove those chains or not; but after the idea of the tube had occurred, I went into a calculation as to the strength of the tube, merely considering it as a beam, to see how far I could depend upon the tube itself, and how much would come upon the chain; and then I found that with 100 or 200 tons in the centre the tube itself would deflect but in a very small degree indeed, but I had not made up my mind whether I should venture to throw over the tube and depend upon that alone, or whether I should leave the chains I employed to construct it. I think it probable they would be left as a precaution, but yet I find that the tube itself would be quite sufficient to support any ordinary railway train.

What would be the longitudinal expansion and contraction of such a tube at each end?—Four inches.

Not more than four inches from shore to shore?—It is eight inches from shore to shore: it is fixed in the middle.

But, taking the whole tube as one, from shore to shore?—It would not be more than eight; I took the lowest temperature of this country, 15° below freezing point, and I considered that exposed to the sun it might rise to 120°, so that would be a difference of, say 140°, of temperature.

And in what way do you propose to unite the plates?—In the same way as the iron is that is used in a ship, united in the same way.

It will be one mass of iron?—Yes; a smooth tube made of wrought-iron, the same as a ship.

A succession of plates united together?—Yes; with rivets.

No rods?—No rods.

Running the whole length?—No; there may be what is termed "angle-irons" put on, that is, a species of iron they use to stiffen vessels, and I might employ the same expedient here, although I do not think it absolutely essential.

Should you have no apprehension of any tendency to rending in any part, as happened with an iron steam vessel of war at the Cape of Good Hope, where the iron began to rend behind the paddle-wheels?—That would be from heavy seas.

You would have no apprehensions of that?—None whatever.

What is the diameter of it?—I propose 25 feet.

Will there be two lines of rails for it or one?—In the present designs which I made, this sketch of it was only one tube, but I found it would be better in building it to have three chains, one on each side and one in the middle, because two smaller tubes would be better than one.

Then you would leave two lines of rails, one in each tube?—Yes; just so.

In fact, two small tubes instead of one?—Just so.

What would be the diameter of each of these tubes?—I should make them elliptical, and then 25 feet in height, and just wide enough to hold one line of railway trains. There would be rather more facility in the construction of two tubes than in the construction of one large.

What do you say would be the thickness of the plate?—I made my calculations upon the lower plate being seven-eighths of an inch.

Would that thickness be at the sides, bottom, and top?—No; I should make it seven-eighths at the top and bottom; and at the sides, where there is not so much pressure, there I should probably make it half an inch.

Would it be of the same thickness in the middle as it is near the ends?—Yes.

What would be the distance below without support?—450 feet.

In each of them?—In each of them.

450 feet in each of them without support?—Yes; if the chains were left, it would not be so.

If you had two tubes, what would be their diameters, each of them?—About 25 feet high by 15 feet the other way, elliptical.

With two tubes it would be 25 feet?—25 feet, or rather more; but there is an advantage in throwing it into two, because you have the advantage of the elliptical form, and there is great facility in the construction. If the chains were left, then there would be one chain, as it were, between the two tubes, which would be a great advantage.

The question I wish to ask you is, whether the tubes would be separated or be united together?—They would be so near together, that perhaps there would be only space for a chain to pass down between them.

They would not be the same piece of iron?—Certainly not. I think it desirable it should not be so, because in a large tube I had a difficulty, and that led me to the two tubes. Suppose a railway train comes and occupies one line of rail, it would not be in the line of the centre of gravity of the tube, and therefore it would have a tendency to tilt the tube; but if you have two tubes, each independent of the other, it is always in a state of repose.

Besides that, it would considerably diminish the pressure in the centre?—Clearly so.

Which is the great difficulty?—I hope the Committee will not consider the idea as chimerical because it is new; it is only, as Mr. Rendel stated, substituting an iron tube for the purpose of getting a rigid platform; and the question resolves itself into that; that is the simple question,—how can you get a rigid platform? Is it better

to get it by trussing wood or iron, or by throwing it into this simple state, which admits of easy construction, and would be completely free from wear and tear?

You say you took the idea from the construction of an iron vessel?—Yes; in order to test the idea, I went to an iron vessel.

And what weight did you state was placed in the centre of the vessel?—The vessel was about 220 feet in length; and Mr. Fairbairn stated he was ready to prop that vessel up with a weight of 1200 tons in the middle; and he stated the machinery and boilers in working order would weigh about 1000 or 1200 tons.

You have not made up your mind as to the safety of dispensing with the chains?—No, I have not.

It would be impossible to do so until it is constructed, would it not?—I would rather leave that, because I would make the design so that the chains might either be taken away or left; and during the construction we should have ample opportunity of ascertaining whether we could safely take away the chains or not.

There would be no great advantage from taking away the chain?—No; only it would make it more costly if they remained; they would be applicable to other purposes, and they would cost from 30,000*l.* to 40,000*l.*

You have no doubt, Mr. Stephenson, that the principle applied to this great span will give ample security to the public?—Oh, I am quite sure of it.

And you said that, you thought that an iron tube of the thickness you have mentioned, viz. seven-eighths of an inch, above and below, and a little less on the two sides, will bear any weight that is likely to be put upon it in the shape of trains?—Yes.

You feel perfectly confident upon that point?—Yes, I feel perfectly confident; but, with a view to remove any doubt upon that point, I feel it necessary to make a series of experiments: not that it will convince me more than I am at present, but that it shall convey confidence to the Board of Directors under whom I am acting; not that I have any doubt in my own mind of it.

Have you considered what the pressure will be in a gale of wind upon the side?—Yes, I have; and I find that is very small indeed. It would not be felt upon that tube, because you have only to consider the parallel case of a chimney; and it is very unfrequently that

you have chimneys interfered with by wind; and when they are they are imperfectly constructed: and a chimney is much weaker than this.

And would there be no advantage in having longitudinal rods in them, to lace them and strengthen them?—It would be very much strengthened indeed, in the same way as vessels are strengthened from the kelson. We shall have to raise a platform in the inside, of course, because we cannot run the engines at the bottom of it. There will be a wrought-iron or wooden platform at the bottom, which would add great strength also to it. But it is as certain that a tube will bear the weight as that the Menai Chain Bridge is standing now, because it stands upon just as simple a calculation.

Do you agree with the opinion that has been just expressed to the Committee, that stone-work would be still stronger for a bridge of this size than any iron-work that can be put into it?—I do not think so.

You think you will be able to make as strong, and safe, and durable a bridge with iron as you can with masonry?—No; but I think, in the ordinary acceptation of the word strength, it can be made quite as strong: but a stone bridge has no vibration, that is all.

Would it not be much more durable?—I think it would; but, in the first instance, there is the question of the practicability of erecting a stone bridge.

Do you think that a stone bridge would be freer from vibration, and perhaps more durable, but that it would be impossible to put a stone bridge here on account of considerations connected with the navigation?—Clearly, I do not believe it would be possible by any arrangements that could be conceived.

Why is that?—Because there is no centering; you could not have a centre to build a bridge upon.

And the arch would interfere with the navigation?—Yes; the arch would interfere with the navigation.

Why would not this rock upon which you propose to place the pier afford you a centre?—The wooden centre must reach across the whole channel, 450 feet.

And that is too wide for the purpose?—It is not too great a width, but it would block up the navigation entirely, and it would block up

the Straits for four years. I do not think you would build it in four years.

In point of fact, the Admiralty would not sanction it?—Oh, no.

Would the expense be much greater?—Very much indeed; I would not undertake it. I do not like to say it would be impossible to build a stone bridge there, but I am very sure it would be next door to it.

What is the greatest span of any known stone arch?—250 feet, I think. Mr. Rendel says, only 200 feet.

You stated you thought it would not be possible to use wood?—I should certainly not recommend it; it would not be desirable to do so.

Did you ever see the bridge at Yarmouth that fell the other day?—Yes, I know it very well.

Was that well constructed?—No, it was a most rickety thing. In fact, its fall was pretty well foretold. It was most insufficient.

Could you point out, in a way intelligible to us, what was the difference of construction between that bridge and a bridge which you would call well constructed?—Why, it was a single chain thrown across, one chain on each side; and the whole structure was very light, and evidently intended for traffic of the lightest description, which it would have stood, I dare say; but in this case it got covered with people; and the evil had been increased very much by a footpath having been added on to the width of the bridge, which projected beyond one of the chains I believe about 6 feet wide. The platform of the bridge, therefore, was suspended by two chains, one from the edge of the platform here, the other chain from here; and this platform projected over, and the people got upon the footpath, and tilted the platform.

What I want to know is the difference of construction between that bridge which failed and others which stand, or how you would recommend them to be built. Is it not more in strength of works than in principle of construction?—I think not. I say distinctly I only use the chain, and should have the chains there as a precautionary measure. I would build my tube, which is essentially different from the other, sufficiently strong to bear the weight.

As far as I understand, the tube is to be supported on the Britannia Rock from one to the other shore?—Yes.

You trust as much to that support as to any chains?—Yes, clearly so.

Therefore it is upon the principle of a suspension-bridge?—No, certainly not.

But it is partly upon the principle of a suspension-bridge?—Well, I do not know. The idea at present is that it would not partake in any way its strength from the chains.

How far is this proposed bridge from the present bridge over the Menai Straits?—Above a mile.

Have you ever considered the possibility of carrying a railroad over the present Menai Bridge?—That was the first proposal.

Do you consider that at all feasible?—Oh, it is feasible.

It is?—Yes.

Do you consider the present Menai Bridge could be so altered and improved and strengthened as to be made able to support a railroad?—I think it might; but it would leave it merely a suspension-bridge, which I do not like.

Have you taken into consideration the difference of expense between so altering and improving the present Menai Bridge and the erection of an entirely new one, as is now proposed?—I do not think there would be much difference; and it would be a very imperfect job when done.

You mentioned yesterday there was a bridge in the north over which the railroad was carried, and that that suspension-bridge entirely failed?—Yes, entirely; and I have since built another bridge in lieu of it.

Is there to be one chain carried across the whole Straits, or are there to be two chains, meeting at the middle tower?—Yes, meeting at the middle tower.

I think you mentioned that Captain Vidal was with you?—Yes.

You went down to Bangor with him?—Yes, we spent three days together.

Have you seen the Report he has made to the Board of Trade? —Yes.

Did you concur with him in his observations in that Report?—Yes, as far as I am able to judge.

The principal gist of the Report relates to the navigation, does it not?—Yes.

Are we likely to find in this Report any opinion on the stability of your bridge?—I think not.

Now, after your bridge is constructed, do you think the navigation will be as free for vessels passing up and down, as unobstructed, as at the present moment?—I think quite so.

In fact, it will be according to the specification given by the Admiralty in every respect?—Yes, in strict conformity.

Cross-examined by MR. POOLE.

You used the words, "a gale of wind;" you said you thought a gale of wind would not much affect it. In a very light breeze do you think that a pillar of 50 feet, with the addition of 25 feet on the top of that, 50 feet square, 100 feet high, and 25 feet upon the top of that, will not produce some calm in the neighbourhood of the Britannia Rock?—It must produce some, but the question was well considered by the engineers who went down, and I concurred with them in their opinion, that it would not injuriously affect the navigation.

But it must produce some?—Yes, it would produce some.

Supposing a vessel in consequence of some calm was to drift upon the Britannia Rock, she would go upon a shelve?—Yes.

It is not steep, is it?—No.

If she struck there she would most likely hang, and not drift off?—She might or might not.

It is not like the bottom of Southwark Bridge?—Yes.

Might it be got rid of by cutting that part?—Yes it might.

It might be premature at present to point out the details of that cutting, but it might be done?—Oh, it might be done.

Mr. Robinson.] You said it was at first proposed to carry the railway over the present Menai Bridge?—It was.

But the Government objected to that last year?—They did.

In consequence of that the new line was projected, was it?—It was.

GENERAL PASLEY *was called in and examined by the Committee as follows:—*

The plan of the bridge which is projected by this company has been laid before you as the responsible adviser of the Board of

Trade?—The original plan proposed by Mr. Robert Stephenson was laid before me in February, and I had two conferences with him upon the subject, and we differed in opinion at that time. I objected to Mr. Robert Stephenson's plan, which was to have been 116 feet in the centre, about 50 feet at the spring of the arch, and 350 feet span in the two arches, and to add a pier upon the centre of 120 feet wide. I objected to that bridge, because I thought it would injure the navigation, and also because I thought that the execution of it would be so very difficult as to amount nearly to an impossibility. He proposed to make a cast-iron bridge of the dimensions I have mentioned, something similar to the Southwark cast-iron bridge.

Having objected to that first project, you have since seen the other project, which is now laid before us?—I have.

Will you favour us with your opinion upon that subject?—In objecting to that project I took the liberty of suggesting a general principle without entering into details, which was, that I thought the suspension-bridge might be erected over the Menai Straits, by means of which they might be enabled to obtain a platform, and when the platform was obtained that they might convert that suspension-bridge into what is called a latticed bridge, such as they have done in America, and such as Sir John M'Neill has lately made in the Royal Canal at Dublin; either a latticed or a trussed bridge, partly of timber and partly of iron, or entirely of wrought-iron; and when this latticed bridge, or trussed bridge—when this was put across the Straits, that the suspension-chains, or rods, by which it was put together might be removed, and that the trussed or latticed bridge would have sufficient stiffness within itself to bear railway trains with safety: but I would not recommend the suspension part to be removed; it could do no good to remove it: but that either of those independently would have strength enough to carry a railway train,—either the suspension or trussed bridge, which would require three longitudinal ranges of trussing, one in the centre and one at each side,—they would have sufficient stiffness to bear the weight of railway trains in safety. It would be prudent to leave the suspension-chains, which would also have strength enough, and by that means a bridge might be obtained of superabundant strength; and if in the

course of time any part became decayed, that part might be removed without injury to the stability of the bridge.

And you consider the plan now laid before us to be upon that principle you have pointed out?—No, it is quite a new principle. Mr. Robert Stephenson, in proposing a tube, when he proposed that to me, and shewed me his drawings, I immediately saw that it was practicable, and I believe it to be efficient, and very sound in principle. Whether it would be better than a latticed or a trussed bridge, I cannot pretend to say; but I think that tubing will be particularly strong and safe, and there are some examples of the strength of tubes that I know. When the Standard Yard was burnt in the fire in the Houses of Parliament, after that the Astronomical Society undertook to make a new Standard Yard for this country, and they found everything in the shape of a flat ruler bent more or less, and they made it in the form of a tube, and that is now the form of the Standard of the Astronomical Society; and another has been made for the House of Parliament, and it is found that a tube will not bend like a solid beam.

Would it be equally strong, whether it is a circular tube, as described by Mr. Stephenson, or two smaller elliptical ones?—I should think the two tubes would be preferable, for the reasons assigned by Mr. Stephenson, which I think are very obvious.

Do you think it would be safe to dispense with the suspension-chains?—I do not see any advantage in dispensing with the suspension-chains. The suspension-chains must necessarily be erected in the place of centering. They can have no centering in the Menai Straits; they are supported from below, without disturbing the navigation: therefore the suspension-bridge affords the only practicable means of supporting a stiff bridge whilst in its progress; and having once made a suspension-bridge for that purpose, I do not see any advantage in removing it—on the contrary.

That chain would be an additional support?—Yes; there would be three chains, two for the sides and one for the end.

On the whole, therefore, General Pasley, you think a bridge built on the plan proposed by Mr. Stephenson would give ample security for trains passing there?—I am perfectly convinced it would.

And you believe it to be a practicable plan?—Quite so.

But you do not advise the removal of the chains?—I do not; I see no advantage in it.

Do you think there would be any hazard in removing them?—I think it would be better to leave them.

It is difficult to answer the question until the bridge is actually built, is it not?—Yes.

If you leave the chains, must you not raise the piers higher?—No; because you must necessarily have chains and rods to obtain a platform for forming the tubes upon.

You would not raise the masonry of the pier at all higher?—No.

Just look at that plan. Would that pier have to be raised any higher?—Oh, yes; it would require a pyramid for the chain here, and one here, and another here.

Then, would that have any effect upon the navigation?—I should think not; because, as I have observed frequently when I have seen harbours crowded with great fleets of men-of-war forty years ago, the smallest vessels sailed through them without danger.

But the higher you can carry these masonry piers, the more likely you are to affect the navigation?—Not at all; because the farther part of the bridge is above the high-water level of the spring-tides.

But are not these piers likely to affect the navigation by taking off the wind?—I should think not. When a boat or small vessel passes a three-decker which is becalmed, it makes very little difference, and shoots along and causes no danger.

We have had it given in evidence to the Committee that it must produce some effect upon the navigation by taking off the wind; what I wish you to state is, whether you do not think it will produce a greater effect the higher you raise them?—No; I should think not.

Previously to a railway being opened it is usual to send you to ascertain the security of the railway; is it not?—Yes.

And therefore you probably will be sent down to ascertain the security of this bridge before the railway is opened to the public?—Yes.

And could those chains be removed without the sanction of the Government?—I do not know. I do not see any objection to their being there; I should recommend their not being removed.

You would not, as the Government engineer, have the power to prevent their being removed, except by special Act of Parliament? —I really do not know.

But you would have the power of reporting the bridge was insecure?—Yes; I should have the power of reporting whether the removal would be safe or not.

The question in reality is, whether, without any reference to the chains, this plan of the tube will give sufficient strength to support the trains?—When the tube is put together, it will be supported by the chains; when once placed, it will have strength to support itself; but when once there, I think it would be better not to remove them.

Do you think it possible by the help of the chains and the tube —by both—to build as strong a bridge as could be built of masonry? —I think a bridge of masonry there is impracticable, unless the navigation of the Straits were stopped altogether, because you must have a number of points centering from below, like the bridge at Chester (which is 250 feet span), which was built from a great number of centerings from below. And another thing, a bridge of 450 feet in dimensions is a thing not paralleled in history, and I have some doubts whether it would be secure.

Do you think additional strength would be given to this proposed bridge by introducing more masonry than is proposed by the present plan?—No, I do not think it will give additional security. The pier in the centre of 50 feet square, as was proposed, is quite strong enough; and I understand that the high-water of spring-tides must either be made to flow over the level of the rock (and in that case it should turn off further to impede the current), in that case the ends of the pier should be pointed, to avoid the current.

Could you not afford some support to the superficies of the bridge by introducing masonry work into the arches of the bridge and at the angles?—No, certainly not.

I believe, sir, you have said you observed no danger occurred to a small vessel when sailing through a fleet; do not you consider that less danger must necessarily occur when a vessel comes against a wall-sided substance than when it drifts against a rugged shelving-rock?—A man-of-war is the least dangerous of the two; but I have been in small boats and not experienced any danger.

Do you think the ends of the pier ought to be pointed?—I said

the ends of the pier above the level of the high-water of spring-tides ought to be pointed; but pointing might improve it. I suppose the present question is not to improve the navigation of the Straits, but to make a bridge over the Straits as they exist.

Then, if you consider it necessary to point the pier which is above high-water mark, do you not consider it necessary, *à fortiori*, to point the abutment below the water?—Not the whole of it; only that part that goes above high-water mark at spring-tides. I believe at neap-tides the current would not strike the pier at all.

It would strike the rock though?—It would strike the rock now.

And if a vessel were drifted upon the rock, it would be liable to receive greater damage than if it were carried against a wall-sided substance?—Yes.

SIR JOHN RENNIE *was called and examined by* MR. ROBINSON *as follows :—*

I believe you went down at the instance of the Government to view the site of this intended bridge over the Menai?—Yes, I did.

Do you agree in opinion that the site chosen is an eligible one?—I think it is.

Are you of opinion that the bridge may be safely built over the Straits, by the Britannia Rock, to carry a railway train?—I think a bridge may be built from the Britannia Rock with sufficient safety to carry a railway train.

Now are you acquainted with the mode by which Mr. Stephenson proposes to accomplish that?—I cannot say that I am. I was merely sent down by the Admiralty to ascertain what should be the dimensions of the opening, and the particular mode of constructing a bridge across the Menai Straits from the Britannia Rock, so as not to interfere with the navigation of vessels passing up and down. I made a Report, simply stating what I considered should be the openings, and the height of any part above high-water, the dimensions of the proposed pier at the Britannia Rock; but as to the mode of construction of the proposed bridge, I have no knowledge whatever.

Are you satisfied with what you recommended to be done? The openings you recommended, I believe, were the same as Mr. Rendel advised?—I believe I recommended openings of 450 feet in the

clear; a pier upon the highest part of the Britannia Rock. I made no sketch upon the subject; I merely stated in my Report that I thought any pier upon the Britannia Rock ought not to be more than 50 feet square, and the arch in the spring 100 feet above high-water spring-tides, so as to enable vessels navigating the openings to the bridge to have the same opening as they have now for working vessels.

The space, you say, between the piers was to be 450 feet?—Yes.

Now you have had so much experience in these matters, do you think a bridge could be safely and practically built so as to carry those views into effect?—Oh, I have no doubt about it.

Mr. Poole *read the following passage from* Sir John Rennie's *Report*.] "First, That the navigation of the Menai Straits, between the Britannia Rock and the present suspension-bridge, is extremely difficult, and even dangerous, on account of the great velocity of the tidal currents, the numerous eddies and rocks which are situated there, as well on account of the baffling winds which prevail; and that nothing but the greatest skill and experience can enable the pilots to navigate vessels through it.

"Secondly, That the navigation of the Straits is capable of being greatly improved, although even now it is much frequented by a numerous class of trading vessels; nevertheless, if improved, it might be rendered of much more importance."

Cross-examined by Mr. Poole.] I believe, Sir John, you looked at the Britannia Rock?—I did.

The sides of it are very shelving, rugged, and uneven?—They are.

Do you consider, supposing a vessel were to drift against one of those sides, would she not be in much greater danger as the sides are at present, than if they were scarped, and were sided or made perpendicular?—Certainly.

Re-examined by Mr. Robinson.] Are you of opinion that if a bridge were built in accordance with the suggestions you made to the Government, it would leave the navigation uninjured?—I think it would.

Do you mean to introduce any masonry into the arch?—I did not at all enter into the question of how the bridge was to be made. I did not consider that part of my province.

Then you have not considered the question whether that could be laid down with sufficient strength and solidity if made entirely of iron?—Generally speaking, a bridge may be made of iron of sufficient stability to carry a railway train.

And you think it might also be done with some portion of the arching constructed of mason-work?—A combination of the two?

Yes.—Why, that is rather difficult to give an off-hand answer. By adopting arch-ways to a certain extent, it might be done.

Would not that, generally speaking, if it could be done, be stronger on that account?—I am not at all clear that it would be. I look upon a cast-iron bridge, of a proper construction, to be as strong as one of mason-work.

You do not mean upon the chain principle?—Yes, I mean a chain principle.

You do not contemplate a chain-bridge?—No.

But you think, generally speaking, that the principle of arching, whether of stone or iron, is stronger and more durable than that of chains?—I think, for a railway-bridge, where it can be done, a fixed bridge on the insistent principle, whether of iron or stone, is better adapted for that particular object; but a chain-bridge may also be made sufficiently secure, I consider.

You consider the arches are stronger of the two?—Yes, the arches are better; they are subject to less vibration.

REPORTS TO THE ADMIRALTY.

SIR JOHN RENNIE'S REPORT.

London, 16*th April*, 1845.

SIR,

In compliance with the instructions communicated to us by the Lords Commissioners of the Admiralty in your letter of the 10th March, I proceeded to Bangor on the 23d, and during that and the two following days carefully examined the Menai Straits between the present suspension-bridge and the Britannia Rock, where the Chester and Holyhead Railway Company propose to erect a bridge across the Straits, in company with Captain Vidal, Mr. Robert Stephenson, and Mr. Rendel. I also received evidence from pilots and captains of vessels, and others well experienced in the navigation of the Straits, particularly that portion between Britannia Rock and the present suspension-bridge, and generally investigated everything connected with the subject during the equinoctial spring-tides, which were very favourable for the purpose.

I now beg leave to give my opinion as follows, as regards any bridge which may be constructed upon the Britannia Rock.

First,—That the navigation of the Menai Straits between the Britannia Rock and the present suspension-bridge is extremely difficult, and even dangerous, on account of the great velocity of the tidal currents, the numerous eddies and rocks which are situated there, as well as on account of the baffling winds which prevail, and that nothing but the greatest skill and experience can enable the pilots to navigate the vessels through it.

Secondly,—That the navigation of the Straits is capable of being improved, although even now it is much frequented by a numerous class of trading vessels; nevertheless, if improved it might be rendered of much more importance.

Thirdly,—No bridge, or work of any kind, should be erected upon the Britannia Rock, which might in any way interfere with the present navigation, or prevent it from being improved at any time hereafter, if funds can be found for that purpose.

Fourthly,—That no pier should be constructed larger than 50 feet square on the Britannia Rock, and that it should be erected in such a position on the highest part of the rock as to give the least possible hindrance to vessels navigating the Straits.

Fifthly,—That no pier, or abutment, should be erected on either side of the Straits projecting beyond the line of ordinary high-water mark, so that it might not interfere with the bowsprits of vessels navigating the Straits.

Sixthly,—That there should be a clear headway of about 100 feet above high-water line of ordinary spring-tides at every part under the proposed bridge where vessels now pass. I further beg leave to observe, that I also examined, in company with Captain Vidal and Mr. Rendel, that portion of the Holyhead and Chester Railway where it crosses the River Clwyd, and the measures proposed by the Railway Company to accommodate the navigation, and I beg leave to observe, that I see no objection to them under the circumstances.

I am, Sir,

Your most humble Servant,

John Rennie.

To the Secretary of the Admiralty.

CAPTAIN VIDAL'S REPORT.

Wellington Street, Woolwich, 7th April, 1845.

Sir,

In obedience to the orders of the Lords Commissioners of the Admiralty, conveyed to me in your letter of the 12th March, informing me of the determination of Parliament that the Holyhead Railway is to cross the Strait of Menai on a bridge of at least the same height above the water as the present suspension-bridge at Bangor; and that the exact site of the new bridge and its specific form having been left to the decision of their Lordships, they had

employed Sir John Rennie and Mr. Rendel to meet Mr. R. Stephenson, the engineer of the railway, on the spot, and to report their opinion on those two points as connected with the free navigation of the Strait; and desiring I would meet those gentlemen, who would be there on the 23d, and that I should assist them with any nautical advice in the above matters, making to you a report of my proceedings for their Lordships' information.

I have now the honour to report to you that I proceeded to the Menai Strait a few days previous to the appointed meeting, with a view to study its localities and peculiar modes of navigation, and to understand the difficulty of the subject in a nautical point of view. For this purpose I watched attentively for some days the manœuvres of the vessels beating both ways through the Strait, and conversed with several sailing-masters and pilots upon the subject. I made some measurements of distances, and a close section of the bed of the Strait over which it is proposed the bridge should pass, and had a tide-gauge erected on the Gorred Goch, the nearest islet to the Britannia Rock which remains uncovered at spring-tide. I had also a float prepared, with a view to any experiments which might be desired on the set and velocity of the tides.

I also directed my attention to the number and class of vessels navigating the Strait, and measured several varieties to ascertain their height of masting.

On Monday, 24th, I accompanied Sir John Rennie, Mr. Rendel, and Mr. Stephenson, to the Strait, a little before high-water of spring-tide, and landing on the islet of Gorred Goch, we observed the effect of the tide from the top of flood to low water, determining its fall and velocity, and observing its set and the eddies and disturbances occasioned by the irregularity of the channels and the numerous rocks which encumber the Strait, from the vicinity of the present bridge to the Britannia, over which it is proposed to carry the new one.

The 24th was devoted to these various objects:—The Britannia Rock and those adjacent to it, with both shores of the Strait in their locality, were all seen at low water, and many matters connected with the site of the proposed bridge and its construction were considered on the spot, with these objects before us.

On the 25th, by appointment, we met a deputation representing

the interest of Carnarvon opposed to the erection of the bridge, and in our presence several sailing-masters and pilots were examined as to their opinion of the injuries which a bridge of the proposed construction would occasion to the navigation of that part of the Strait commonly called the Swilleys, and which is understood as extending from the present suspension-bridge, near Bangor Ferry, to the Britannia Rock, a distance in the direction of the stream of tide of about 1700 yards.

Mr. Evans, Mayor of Carnarvon, and Mr. Poole, a solicitor of that town, conducted the inquiry, and the result may be stated concisely thus:—

The bridge proposed by Mr. Stephenson to be constructed over the Strait of Menai consists of three piers, each 55 feet above high water of spring-tide, supporting two iron arches of 360 feet span, the crowns or soffits of which are 50 feet above the piers, and 105 feet above high water. The centre pier, resting on the Britannia Rock, extends 130 feet in width across it, and the other two piers are to rise from low-water mark of spring-tide on either side of the Strait.

It is objected to this proposed bridge—

First,—That it will cause a diminution of head-way and water-way, very injurious to the navigation of the Strait.

Secondly,—That its piers being placed at low water will increase the eddies of the tides, already very difficult and dangerous.

Thirdly,—That it will occasion baffling eddy winds, which, keeping in view the rapidity of the tides and the violent eddies at the place in question and all through the Swilleys, will greatly increase the difficulty of working the vessels, and thereby add seriously to the existing dangers of the navigation.

Fourthly,—That it is apprehended the piers rising from low water of spring-tide will obstruct the free influx and reflux of the tides, and that, in consequence, a greater deposit of sandy particles, held in suspension in the water, and now carried out to the deeps, will take place in the form of sediment, and gradually increase the sand-banks in the Strait and at Carnarvon Bar.

Some explanations appear to me necessary to the right understanding of these objects, and I will endeavour to give them as concisely as I can.

The Strait of Menai in the vicinity of the Swilleys is bounded by a very rocky, broken, irregular coast-line, and its channel contains several islets and dangerous rocks—some generally uncovered, and others only visible at particular periods of the tides.

The tide on this occasion of our examination, which was a spring-tide, rose 19 feet 3 inches; and an experiment made at the time of its greatest velocity on the ebb, gave its rate five nautic miles per hour.

Such a current of water passing along an extremely irregular line of coast, and over such an uneven bottom, proceeds with a turbulence which disturbs the whole body of the stream, and occasions mischievous eddies, most dangerous and embarrassing to the navigation. A further difficulty is experienced in the high lands on either side the Strait, which give rise to uncertain, baffling winds, especially during the summer months, when it is stated that at the present Menai Bridge the wind is frequently easterly, while in the reach between Dinorure and the Britannia the wind at that time is from the westward. These circumstances render the lofty sails indispensable, as they are, at times, more serviceable than the lower sails; and I have ascertained that of late years the vessels built in the vicinity of the Strait are higher masted than they used to be, and indeed of larger construction generally, possibly on that account.

With beating winds the channel on the south side the Swilley and Cribinniau Rocks is not used, as being too narrow for working the vessels, and under the same circumstances the channel on the north side the Britannia is seldom taken on account of the strong eddies in it when the tide is moving to the westward; and when it is moving to the eastward, vessels working in it make invariably a longer reach by tacking to the south of the Britannia.

With the wind well free the channel along the Carnarvon shore is preferred, because the stream of tide passes more directly through it.

I now proceed with the reasons for the objections to the bridge, as those resulting from the inquiries made before us by the deputation from Carnarvon.

And 1st. That it will cause a diminution of head-way and water-way very injurious to the navigation of the Strait.

The diminution of head-way is occasioned by the form of the

bridge, its piers being only 55 feet above high water; and as a sloop of 37 tons has a mast of 73 feet in height, a numerous class of vessels will meet with interruption; and it has been explained why the lofty sails cannot be dispensed with. Again, we are informed, that frequently vessels to the number of fifteen, or more, may be seen beating down together under the influence of a favourable tide; and in the eddies and baffling winds with which they have often to contend, it becomes a matter of extreme difficulty to take exactly that part of the arch through which they may safely pass.

From the peculiarity of the tides in the Menai Strait, which run to the westward or make ebb in reality an hour and a half before high water, all vessels bound to the westward against a westerly wind endeavour to reach the Swilleys as nearly as possible at the turn of tide, by which means they effect their passage while the tide is rising, or by the time it is high water.

At this time of tide, under present circumstances, vessels could tack with their jib-booms over the spot where it is proposed to place the south pier of the bridge, and also over the Britannia Rock, where the great central pier is designed; and to weather the western end of the Britannia it is necessary they should stand over close to the south pier, which, if the present plan of the bridge be carried out, they could not do, both on account of the pier itself and the arch which rests upon it.

The first objection, therefore, appeared to me to be satisfactorily established, viz. that a diminution of head-way and water-way, injurious to the interests of the navigation, will be caused by the proposed position and height of the piers.

The second objection is, that the piers will increase the eddies of the tides, already very difficult and dangerous. The effect of the south and central piers placed at low water of spring-tide would no doubt create some little additional difficulty by their enlargement of the existing eddies, but their contraction of the navigable part of the channel is in my estimation the more serious evil.

The third objection to the proposed bridge is, that it will occasion baffling eddy winds, which, keeping in view the rapidity of the tides and the violent eddies at the place in question and all through the Swilleys, will greatly increase the difficulty of working the

explanation, but I have placed the south pier where it really must stand according to my measurements, if the pier on the Britannia and the span of the arch are to retain the dimensions originally proposed.

I have the honour to be, Sir,

Your very obedient Servant,

(Signed) ALEXANDER J. E. VIDAL, *Captain.*

MR. RENDEL'S REPORT.

8 *Great George Street, Westminster,*
14*th April*, 1845.

SIR,

In compliance with the directions of the Lords Commissioners of the Admiralty, contained in your letter of the 21st of February last, I met Sir John Rennie and Captain Vidal, R.N., at Bangor, on the 24th ult., and with them made an examination of the Menai Strait, particularly that portion of it where the bridge is proposed to be built by the Chester and Holyhead Railway Company. Mr. Robert Stephenson, the engineer for the railway, attended us, and fully explained his plan for the bridge, and his views on the subject generally. On the following day the several parties immediately interested in the navigation of the Straits met us, and adduced evidence which established the local importance of the navigation, its peculiarities, and the necessity for due caution in deciding a question so important to their interests. This evidence was of the most satisfactory character, from its practical nature, being given by numerous pilots belonging to the Straits, and by masters of vessels accustomed to their navigation.

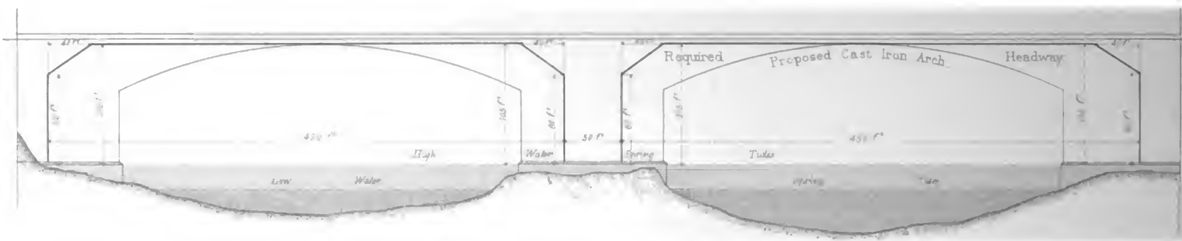
Presuming that their lordships desire from me the *results*, rather than the detailed particulars of this inquiry, I shall proceed to report the conclusion at which I have arrived after a very careful consideration of my own observations, and the evidence adduced on the inquiry.

I am of opinion that the site selected is the freest from objection of any which the Straits afford. The site of the present bridge is

To face P. 81

CHESTER AND HOLYHEAD RAILWAY

Section of Proposed Bridge at Britannia Rock



certainly a preferable one for the convenience of the navigation, and next to it I consider that selected by Mr. Stephenson for his railway bridge. The structure of the present bridge is also of a kind most favourable for meeting the peculiar difficulties of the navigation. It offers little or no impediment to the navigable water-way, and presents so small an obstruction to the winds that their currents are not materially interrupted. It appears from the evidence of the pilots that no case has occurred of damage or delay to shipping from the present bridge, excepting one instance of a very trifling nature, which was admitted by the pilot who had charge of the vessel to have been occasioned by neglect.

This satisfactory result of experience in the present bridge points out very strongly the propriety of adopting as nearly as possible a similar *form* for the one now proposed, leaving, however, the parties who will have the responsibility of the work to determine its *principle* of construction.

The diagram attached to this Report represents in black tint the form and dimensions of the bridge proposed by the Railway Company; in red lines and figures, that which I would recommend their lordships to require. By the proposed alteration, the pier on the Britannia Rock and the abutment on the Carnarvonshire shore will stand so far beyond the limit of navigable water as to prevent the slightest interference with the width of the channel, and from the former being reduced to not exceeding 50 feet square, and the latter placed close under the cliffs, their effect on the winds will be the least possible. In regard to headway, it will be observed that for the whole width of the navigable water it is to be 105 feet in the clear above high water of spring-tides. In its outline, therefore, this plan is similar to the present bridge, and like it would, I am of opinion, be unattended with injury to the navigation.

Captain Vidal has so fully reported to their lordships on the peculiarities of the Straits, and the importance of their navigation, as to leave nothing to be added by me on these points, excepting a confirmation of his statements and opinions.

In returning from the Menai Straits, Sir John Rennie, Captain Vidal, and myself, inspected the Voryd, and in compliance with the instructions contained in your letter of the 14th ult., examined that harbour and the river from thence to Rhydlan.

This second survey confirms the opinions stated in my Report to the Lords Commissioners of the Admiralty of the 24th April last year; and I am of opinion that if the Railway Company execute in a proper manner the works therein recommended, and which I understand they are prepared to do, then the inhabitants of Rhyl, Rhydlan, Abergele, and St. Asaph, will have all the accommodation provided which they can fairly expect from the Railway Company, and will be placed in the most advantageous position for extending the improvement of the harbour, as well as the river up to Rhydlan.

I have the honour to be, Sir,

Your obedient Servant,

JAMES M. RENDEL.

To Captain W. A. B. Hamilton, R.N., &c. &c.
Admiralty.

SECTION II.

THE PRELIMINARY EXPERIMENTS.

CHAPTER I.

EXPERIMENTS ON CYLINDRICAL AND ELLIPTICAL TUBES.

In order to remove any doubts as to the practicability of his proposal, and for the purpose of further maturing the design, Mr. Stephenson, with the permission of the Company, proceeded to institute a series of direct experiments on the transverse strength of tubes.

The experiments, which were designed and proceeded with under his personal superintendence, were not, at first, specific in their object. It was necessary to determine what kind of information was required, rather than to pursue any definite course, and to ascertain generally in what manner tubes might be expected to fail, and to what extent their strength might be modified by form. Although the square or rectangular section was first proposed, round or elliptical tubes appeared to offer considerable advantages. If suspended in chains, they had equal rigidity in every direction, and small tendency to change of form; they were simple of construction; and if the vertical portions of the circle or ellipse could be securely retained in shape, the top and bottom were well adapted to resist extension and compression; but the most important advantage was the small resistance they

offered to the violent winds, to which their great height and peculiarly unsheltered situation would expose them; and on these accounts they were suggested instead of the square tube. It is difficult to retrace the steps by which any design is perfected; and it is remarkable that the first conceptions are frequently returned to, and discovered to have been correct. In the present instance this was eminently the case, and in the course of experiment the round tube was superseded by the elliptical, which in its turn merged into the original square.

In submitting his views to the test of experiment, Mr. Stephenson called to his assistance, first, Mr. Fairbairn, and subsequently (viz. in August 1845), at Mr. Fairbairn's suggestion, his friend Mr. Eaton Hodgkinson, to assist him in the experiments which we shall detail. With these gentlemen his views were constantly reasoned on, numberless suggestions and difficulties were discussed, and the most efficient manner of conducting the requisite inquiries was decided on.

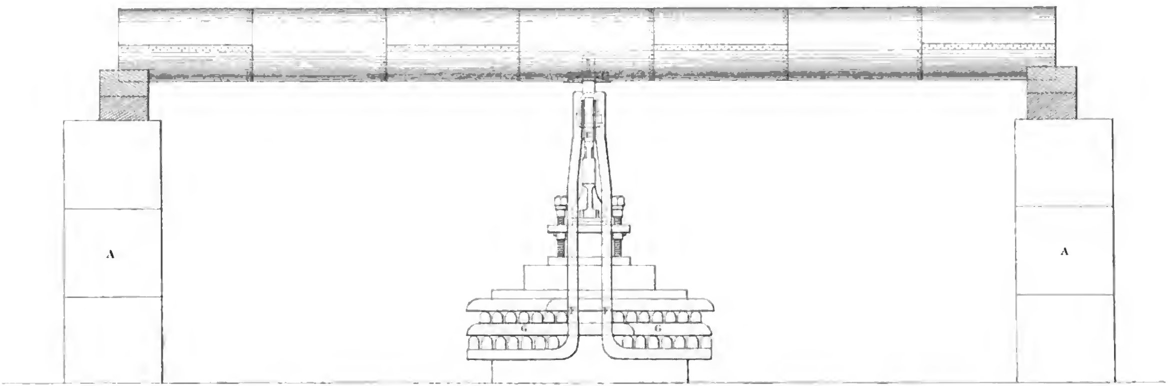
It was determined to begin with simple cylindrical tubes of sheet iron, the plates being curved and united by overlap joints with two rows of rivets, and the models resembling the ordinary funnel of a steam-vessel.* They were constructed and broken at Mr. Fairbairn's works at Millwall, London; these works, being specially adapted for ship-building and boiler-making, offered every facility for such an investigation, and were most conveniently accessible.

The material employed was rolled boiler plate of ordinary quality, punched and riveted as in common boiler-work; the

* Some of the models were afterwards used as chimneys for the rivet-furnaces at the works at the Straits (*See* Plate IV). The two square furnace chimneys in the foreground, introduced with picturesque effect by the artist, were models broken in these experiments and afterwards applied to this purpose.

APPARATUS USED FOR THE EXPERIMENTS WITH THE SHEET IRON TUBES & BEAMS.

Fig. 1



rivets were inserted red-hot, and closed partly by machine and partly by hand. The models varied in length from 15 to 31 feet, and in diameter from 12 to 24 inches, bearing at the centre from 1 to 6 tons.

In determining the size of the models, it must be remembered that Mr. Stephenson had already, on other grounds than the proportions necessary for strength, decided approximately on the dimensions of this bridge. Its height was to be sufficient for the locomotive, while a much greater height would have endangered the sides; a good proportion for the depth of large cast-iron beams had been found to be about one-fifteenth of their length, and therefore the same proportion had been provisionally fixed for the tube, while the breadth was governed by the space necessary for the passage of the trains. The dimensions, then, were 450 feet length, 30 feet height, 15 feet breadth. The thickness of the plates was then assumed to be from half-an-inch to five-eighths, and the total weight of each tube 600 tons. It will be found that these proportions are maintained in the models with such variations as would give data for testing theories of the effect of change of any particular dimension.

The apparatus with which these models were broken consisted of a loaded scale, or platform, suspended by a shackle, or suspension link, from the bottom of each tube, which was perforated for this purpose. In order to distribute the strain over the part from which this shackle was suspended, a saddle, or cushion of hard wood, about 8 inches square, was placed inside the tube, the suspension link, passing through the bottom of the tube; and also through this cushion of wood, was there securely keyed by means of a cotter with an iron washer. The tube was strengthened at this spot by means of a strong plate riveted round this perforation.

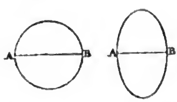
In order to ascertain from time to time the permanent

set, a powerful lever afforded every facility for removing the weight by means of a screw-jack.

All the experiments were thus made with direct weight, without the intervention of a lever. Pigs of iron, carefully weighed, were placed in this suspended scale, while supported by the screw-jack; the screw-jack being then slackened, the weight descended gently into action: the deflection was recorded, the weight again raised and increased, and the operation repeated until the model failed.

The thickness of the plates was carefully determined by shearing a strip from each plate and cutting this strip into several smaller pieces, which, being piled on each other, and pressed into close contact in a vice, were collectively measured, and the average thickness thus determined.

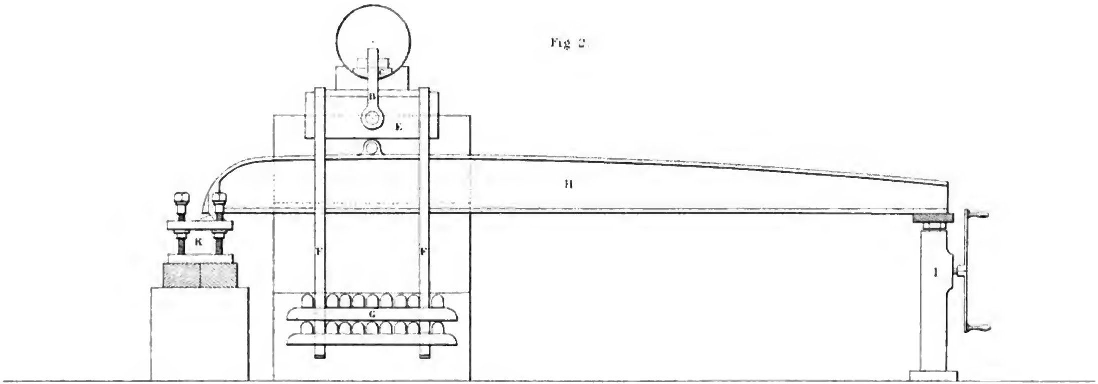
It will be more convenient to adopt a tabular form in the description of the experiments. The first column merely numbers the experiment for future reference, the dates are given in column 2. The total length of each model is given in column 3, which, on comparison with the clear distance between the supports in column 4, gives the extent of bearing on the supporting piers. The external diameter in column 5 will be found to vary in some of the experiments, in consequence of the tubes being drawn by the weight into an oval form. The decrease, therefore, applies to the transverse diameter A B. The thickness of the plates was determined as above mentioned. The



weight of the tubes was ascertained by direct means. The sectional area is merely the thickness multiplied by the circumference, or the surface in superficial square inches that would be exposed in cutting through the tube at the centre. The deflection and permanent set, as the weights were laid on, are recorded in columns 10 and 11, the ultimate deflection being calculated *pro ratá* for the last weight applied.

APPARATUS USED FOR THE EXPERIMENTS, WITH THE SHEET IRON TUBES & BEAMS.

Fig 2.



Day & Son, lith's to the Queen

The column of Remarks will sufficiently explain the method of failure. The models were supported at the ends on timber packing adapted to the shape of the tube. They were, moreover, kept in shape *at the extremities* by the insertion of blocks of wood fitting the tubes. The last weight recorded is the actual breaking-weight.

The illustrations represent the method of fracture, for which purpose a short length only of the centre of each tube is engraved, with the form of section.

TABLE I.

Preliminary Experiments on the Transverse Strength of Cylindrical Tubes of Riveted Boiler Plate.

No. of Experiment.	Date.	Length. Total.	Length. Between Supports.	Diameter.	Weight of Tube. Total.	Weight of Tube. Between Supports.	Sectional Area.	Weight Applied.	Deflection.	Permanent Sets.	Remarks.
		Feet.	Feet.	Inches.	Tons.	Tons.	Inches.	Tons.	Inches.	Inches.	
1	July 6, 1845.	18·1	17·	12·18	·0455	·0429	1·56	·357	·06		
								·857	·25	·02	Tube puckered and crushed at top, 13 inches from the centre, before the whole of the last weight was laid on.
								1·357	·39		
2	July 7	18·125	17·	12	·0478	·0446	1·4	·357	·2		
								·607	·32	03	
								·857	·41	·03	Failed, as No. 1, at top, after bearing the last weight about 1¼ minute.
								·96	·46	·03	
								1·018	·6	·05	
								1·057	·6	·1	
								1·071	·6	·1	
								1·156	·61	·1	
								1·207	·65		

Experiment 1:

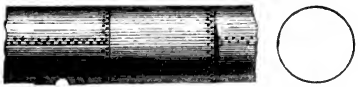
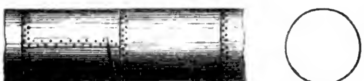
Thickness of Plate, ·0408 inch.

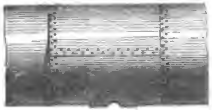
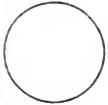
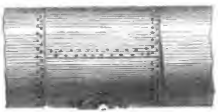
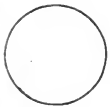
Experiment 2:

Thickness of Plate, ·0375 inch.

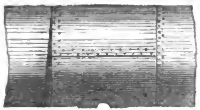
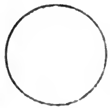
3	July 11	16·83	15·62	12·4	·175	·1625	5·05	·357	·07		
								·857	·17		
								1·357	·30	·025	
								1·607	·31	·025	
								2·107	·42	·075	
								2·607	·5	·1	
						Thickness of Plate, ·131 inch.		3·107	·65	·102	Before failure, the horizontal diameter was diminished by ·4 inch.
								3·607	·85	·175	
								4·107	·97	·25	
								4·607	1·12	·35	Failed at bottom, by breaking through shackle hole.
								4·857	1·2	·4	
								5·107	1·29		
4	July 12	25·	23·42	17·68	·1545	·1446	3·32	·357	·1		
								·857	·2		
								1·357	·3		Before failure, the horizontal diameter was diminished 1·5 inch.
						Thickness of Plate, ·063 inch.		1·857	·42	·05	
								2·357	·51	·075	
								2·607	·60	·09	
								2·732	·71	·1	Failed at bottom, through the rivet-hole, 25 inches from centre.
								2·857	·74		

Transverse Strength of Cylindrical Tubes.

No. of Experiment.	Date.	Length.		Diameter.	Weight of Tube.		Sectional Area.	Weight Applied.	Deflection.	Permanent Sets.	Remarks.
		Total.	Between Supports.		Total.	Between Supports.					
		Feet.	Feet.	Inches.	Tons.	Tons.	Inches.	Tons.	Inches.	Inches.	
5	July 12, 1845.	25·08	23·42	18·18	·3469	·3245	6·8	·357	·05		
								·857	·15		
								1·357	·2		The horizontal diameter before failure was diminished by ·42 inch.
								2·357	·38	·05	
								3·357	·55	·06	
							Thickness of Plate, ·119 inch.	4·357	·75	·09	
								5·357	·95	·13	
								5·607	1·	·15	
								5·857	1·05	·18	Failed at the bottom, 38·5 inches from centre, after bearing the weight half a minute.
								6·107	1·13	·2	
								6·357	1·19		
6	July 30	24·87	23·42	18·26	·1491	·1406	3·31	·357	·08	Not observed.	
								·857	·15		Before failure, horizontal diameter was diminished by 1·12 inch.
								1·357	·25		
							Thickness of Plate, ·058 inch.	1·857	·35		
								2·357	·46		
								2·607	·51		
								2·857	·56		Failed at bottom, 6 inches from the shackle, the top retaining its shape.

7	July 30	32·66	31·27	24·	·4495	·4299	7·13	·357	·04		
								·857	·1		
								1·357	·17		Horizontal diameter diminished before failure by ·84 inch.
								1·857	·25	·02	
								2·357	·33	·03	
						Thickness of Plate, ·095 inch.		2·857	·4	·04	
								3·357	·47	·05	Failed at the bottom, 13·5 inches from the centre.
								3·857	·55	·05	
								4·357	·63		
8	July 30	34·54	31·27	24·3	·6183	·5598	10·24	·357	·05		
								·857	·15		
								1·357	·2		
								2·357	·33	·04	
								3·357	·48	·07	
								4·357	·6	·1	Horizontal diameter diminished before failure by ·7 inch.
						Thickness of Plate, ·135 inch.		5·357	·77	·13	
								5·607	·81	·14	
								5·857	·85	·15	Failed at the bottom, 15 inches from the centre, just as the last weight was suspended.
								6·107	·9	·17	
								6·357	·95		

Transverse Strength of Cylindrical Tubes.

No. of Experiment.	Date.	Length.		Diameter.	Weight of Tube.		Sectional Area.	Weight Applied.	Deflection.	Permanent Sets.	Remarks.
		Total.	Between Supports.		Total.	Between Supports.					
		Feet.	Feet.	Inches.	Tons.	Tons.	Inches.	Tons.	Inches.	Inches.	
9	July 31, 1845.	32·68	27·	24·2	·4486	·4295	7·2	·357			
								·857	·05		
								1·357	·13		
								2·357	·3		
						Thickness of Plate, ·095 inch.		3·357	·47	·04	Horizontal diameter diminished before failure by ·71 inch.
								4·107	·59	·07	
								4·357	·64	·08	Failed at bottom, 15 inches from centre, just as the last weight was suspended.
								4·607	·69	·1	
								4·857	·74		
10	Oct. 8	14·66	14·	17·68	·0908	·0865	3·32	·428			
								·814	·06		This tube was one of the halves of that broken in Experiment 4.
								1·201	·1		
								1·98	·2		
								2·749	3		
								3·506	·38		
						Thickness ·063	of Plate, inch.	3·886	·5	·16	
								4·268	·58	·28	
								4·64	·62	·47	Failed at top, puckering up, the sides being greatly distorted and the horizontal diameter being diminished by 3·7 inches.
								4·833	·92	·6	
								4·936	·92	·92	The weight 4·936 was left suspended for some minutes.
								5·033	·958		

11	Oct. 8	15·75	15·	18·18	·212	·2065	6·8	·415		Not observed.	This tube was one of the halves of that broken in Experiment 5.
								1·119	·04		
								1·961	·06		
								2·735	·06		
								3·506	·07		
						Thickness ·119	of Plate, inch.	4·259	·07		Horizontal diameter before failure was diminished by ·53 inch.
								5·792	·08		
								6·554	·08		
								7·335	·08		
								8·085			Failed at bottom, 8 inches from centre.
								8·283			
12	Oct. 9	10·	8·5	12·18	·025	·0214	1·56	·428	·02	Not observed.	This tube was one of the halves of that broken in Experiment 1.
						Thickness ·0408	of Plate, inch.	1·201	·13		
								1·772	·19		Failed at top, 6 inches from centre.

The experiments on the circular tubes were followed by a series of similar experiments on oval or elliptical tubes. These were devised simultaneously with those on the circular and rectangular forms, the principal object in view being to ascertain, previous to more detailed investigation, in what manner these tubes would fail. The oval tubes were from 17 to 24 feet long, varying in depth from 12 to 22 inches, and in width from 7 to 14 inches, and broke with weights varying from 1 to 8 tons, suspended from the centre. The riveting, as with the last, was extremely defective; indeed, a better method of riveting was the first result of the experiments. The plates were united merely by an overlap with two rows of rivets. There were no stops or diaphragms to keep the tubes in shape, excepting at the extremities.

In comparing these experiments with others which have been subsequently made, it will be convenient to derive in each case a Constant for cylindrical tubes from the formula hereafter investigated,

$$W = \frac{a\,d}{l}\,c,$$

or,

$$c = \frac{W\,l}{a\,d}.$$

In determining the values of c in the following table, the breaking-weight has been increased by the addition of half the weight of the tube:—

Cylindrical Tubes.

Experiment.	W, or Breaking-Weight.	a, or Area of Section.	Value of c.
	Tons.	Inches.	Tons.
1	1·38	1·56	14·8
2	1·23	1·4	15·0
3	5·19	5·05	15·4
4	2·93	3·32	13·5
5	6·52	6·8	13·3
6	2·93	3·31	14·7
7	4·57	7·13	9·9
8	6·64	10·24	9·9
9	5·072	7·2	10·5

The mean value of c in these Experiments is 12·9.

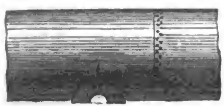
We shall also hereafter find it useful to compare the breaking-weight of a tube with the weight of the tube itself. We have for these cylindrical tubes:—

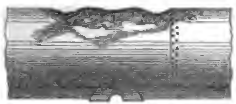
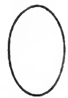
Comparative Weights and Strengths.

Experiment.	Length between Supports.	Weight of Tube between Supports.	Breaking-Weight.	Ratio of Weight to Breaking-Weight.
	Feet.	Tons.	Tons.	
1	17·	·0429	1·357	1 : 31·
2	17·	·0446	1·207	1 : 27·
3	15·62	·1625	5·107	1 : 31·4
4	23·42	·1446	2·857	1 : 19·7
5	23·42	·3245	6·357	1 : 19·6
6	23·42	·1406	2·857	1 : 20·3
7	31·27	·4299	4·357	1 : 10·1
8	31·27	·5598	6·357	1 : 11·3
9	27·	·4295	4·857	1 : 11·3

TABLE II.

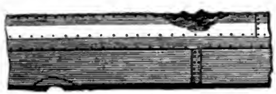
Preliminary Experiments on the Transverse Strength of Elliptical Tubes of Riveted Boiler-Plate.

No. of Experiment.	Date.	Length.		Diameter.		Weight.		Sectional Area.	Weight Applied.	Deflection.	Permanent Sets.	Remarks.
		Total.	Between Supports.	Major.	Minor.	Total.	Between Supports.					
		Feet.	Feet.	Inches.	Inches.	Tons.	Tons.	Inches.	Tons.	Inches.	Inches.	
13	Aug. 6, 1845.	17·916	17·	14·62	9·25	·0486	·0461	1·559	·357	·12		
									·857	·57	·09	
						Thickness of Plate, ·0416 inch.			·937	·62		Failed at top, just as the last weight was suspended.
14	Sept. 17	25·75	24·	21·66	13·5	·316	·2946	7·235	·379	·02		A block of wood 10 inches long was placed in the tube at centre to take the strain of the shackle.
									·779	·08		
									1·579	·17		
									2·379	·28	·02	
									3·179	·39	·04	
									3·979	·51	·06	
						Thickness of Plate, ·131 inch.			4·779	·64	·1	
									5·579	·8	·154	
									6·379	1·	·24	
									6·779	1·13	·3	Failed at bottom, after sustaining the weight a few seconds; the minor diameter having been diminished, before fracture, by 1·18 inch.
									7·179	1·27	·39	
									7·379	1·32	·44	
									7·623	1·36		

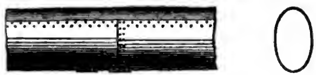
15	Sept. 18	26·25	24·	21·25	14·125	·1594	·1455	3·823	·379	·04	Barely perceptible.	Failed at top, before the whole of the last weight came on it. Breaking-weight probably 3·246 tons.
									·772	·09		
						Thickness of Plate, ·0688 inch.			1·152	·13		
									1·534	·17		
									1·915	·22		
									2·297	·28		
									2·672	·34		
									3·050	·42		
									3·444	·45		Minor diameter diminished before failure by 1·9 inch.
16	Sept. 18	19·66	18·5	12·	7·5	·1035	·0977	2·038	·379	·125		Tube with a cell or fin on top; which, however, soon became distorted, doubling up for some inches on each side of centre.
									·770	·2		
									1·161	·3		
									1·548	·4		
									1·930	·51		
						Thickness of Plate, ·071 inch.			2·311	·625	·04	
									2·499	·71	·08	
									2·685	·78	·1	
									2·875	·87	·14	
									3·065	·95		
									Not broken, but shewing much distress on top side.			

Transverse Strength of Elliptical Tubes.

No. of Experiment.	Date.	Length. Total.	Length. Between Supports.	Diameter. Major.	Diameter. Minor.	Weight. Total.	Weight. Between Supports.	Sectional Area.	Weight Applied.	Deflection.	Permanent Sets.	Remarks.
		Feet.	Feet.	Inches.	Inches.	Tons.	Tons.	Inches.	Tons.	Inches.	Inches.	
17	Sept. 18, 1845.	[As in Experiment 16.]							·415	·14		Tube of previous Experiment reversed in position.
									1·179	·61	·14	
									1·963	1·19	·48	
									2·153	1·32	·58	Failed at bottom, 22 inches from centre, shortly after putting on last weight.
									2·337	1·45	·67	
									2·52			
18	Sept. 19	19·083	18·5	13·	8·	·1192	·1152	3·509	·379	·08		
									·769	·162		Tube with a similar fin to that in Experiment 16.
									1·152	·241		
									1·548	·324		
									1·938	·413		
									2·688	·604	·025	
									3·266	·798	·050	
									3·416	·852	·061	Failed at top, doubling up 3 feet from centre.
									3·616	·974	·082	
									3·753	1·14	·26	
									3·934	1·19		



Thickness of Plate, ·0644 inch.

19	Sept. 19	18·2	17·5	15·	9·75	·1669	·1605	5·559	·379	·05		Failed, as the last weight was laid on, at bottom; the top side at the same time giving signs of failure.
									·954	·14		
									1·539	·22		
									2·115	·31	·04	
									3·271	·49	·057	
						Thickness of Plate, ·143 inch.			4·406	·69	·11	
									5·563	·96	·24	
									6·141	1·16	·34	
									6·525	1·36	·47	
									6·915			
									Breaking-weight perhaps more truly 6·696 tons, and ultimate deflection 1·39.			

It is evident that these first experiments are of little or no value as a foundation for any theory as to the strength of circular or elliptical tubes, of such dimensions as would have been requisite for a large structure. In four of the experiments out of twelve, with circular tubes, and in three out of five, with elliptical tubes, failure took place by compression or buckling of the top; and in all these cases the plates were *very thin*, viz. in two tubes $\frac{7}{100}$ of an inch, and in four of them only $\frac{4}{100}$ of an inch in thickness. In all the other experiments the bottom tore asunder, either through a line of rivets, or through the perforation made for the suspended shackle. The riveting was not at all adapted for a tensile strain, as the plates were simply overlapped, and united by two rows of rivets (fig. 1), instead of being con-

Fig. 1. 

Fig. 2. 

nected by a larger cover, as in fig. 2. The general result with all of them was a change of shape, by which the round tubes became oval, and the ellipse became more elongated, as the strain increased. This elongation or change of form had the effect of converting a large portion of the tubes into "side," to the detriment of the parts destined to resist extension and compression; and in consequence of this defect, and of other advantages promised by the rectangular form, it suggested itself as a means of obviating this change of shape, the experiments on these tubes were not continued.

This distortion may to some extent have been increased by the load being suspended wholly at the centre, but arose principally from the thinness of the plates.

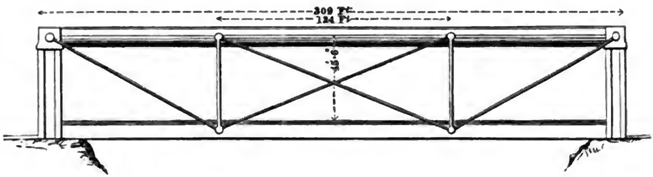
It is to be regretted that circular tubes, with thicker plates, were not experimented upon; as subsequent expe-

rience has shewn that no distortion would then have occurred, and valuable results would probably have been obtained. Permanence of form might, moreover, be entirely ensured by diaphragms or stops, at intervals, throughout the tube, or by stiffening-plates united by angle-iron, as in the Bridges. Such diaphragms have, indeed, been successfully adopted by Professor Airy in using wrought-iron tubes for the support of astronomical instruments, to which purpose they are peculiarly applicable, on account not only of their stiffness, but of their greater freedom from vibration or tremor than cast-iron supports. Diaphragms are used in the construction of the wrought-iron polar axes of the large equatorial telescope in the Observatory of Liverpool, which are formed of two semi-elliptical boiler-plate tubes, of exquisite workmanship.

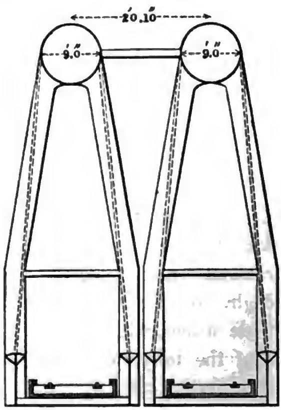
Circular wrought-iron tubes, of considerable thickness, and of magnificent dimensions, retained in shape by stops, are also being used by Mr. Brunel in the construction of a bridge over the Wye, at Chepstow, in South Wales. These tubes are, however, not strained transversely, except in supporting their own weight during the process of erection, and for this purpose it is intended to render them temporarily more rigid by cambering them to a slight extent by tie-rods along the bottom. They are 305 feet long, 9 feet diameter, and $\frac{5}{8}$ inch thick; and are employed as struts, or pillars, to resist the horizontal strain of the suspension links which support the wrought-iron girders of which the bridge is composed. By these means, without the usual tie-chains of a suspension-bridge, the lofty towers are relieved from all lateral strain.

The total span of this bridge is 300 feet, which is the length of the circular tube employed as a strut; a chain, consisting of three straight links, suspended from this strut,

divides the span into three equal portions of 100 feet each. The beam carrying the roadway is thus a continuous beam, 300 feet long, supported at each end and at two points in its length, as in the sketch below.



The circular tubes are supported on cast-iron standards, as in the section below (to an enlarged scale), where the direction of the chains and section of the lower wrought-iron girders are also shewn.



The following details were kindly furnished by Mr. Brunel:—

CHEPSTOW BRIDGE.—*(Approximate Weight.)*

	Tons.
298 feet run of tube and butt-plates	$127\frac{1}{2}$
Hoop of ditto over piers	$7\frac{3}{4}$
Side-plates, bottom ditto, &c. for attachment of main chains	15
Side plates for attachment of diagonal chains	$2\frac{1}{4}$
Stiffening diaphragms, 26 feet apart	$4\frac{1}{4}$
Rivet heads, &c.	$4\frac{3}{4}$
Total weight of one tube	$161\frac{1}{2}$
Main chains, eyes, pins, &c.	105
Diagonal chains, ditto	23
Vertical trusses	$18\frac{1}{2}$
Saddles, rollers, &c. at points of suspension	22
Main roadway, girders, transverse floor girders, &c.	130
Total weight of iron in one roadway	460

The tubes are of uniform section, consisting of 16 equal-sized plates, $\frac{3}{8}$ thick, and two side-plates $\frac{5}{8}$ thick. The plates are all 10 feet long, lapped together at sides, and butt-jointed at the ends with double butt-plates, and double riveted.

Circular tubes, 100 feet high, were also at one time proposed as supports for the platforms in constructing the abutment-tubes of the Britannia Bridge.

The round tube, as proposed for the bridge itself, if suspended in chains, and merely applied as a means of ensuring a rigid platform, would, if constructed with thick plates, properly united, have formed a most efficient structure, offering but little resistance to the wind, and having equal rigidity in every direction; while an elliptical tube of the depth necessary for the Britannia Bridge, and well retained in shape, possesses several important advantages as an independent beam. The curved plates of the top are well adapted for resisting compression, and for throwing off the wet, while the heavy riveting necessary for uniting the sides with the top and bottom in a rectangular tube is entirely obviated; although there are other more important practical advantages in favour of the rectangular form.

The tearing asunder of the bottom in all cases through the rivets, causes, as might be expected, considerable anomaly in the results of the experiments, which must be considered more as a test of the riveting than of the transverse strength of homogeneous sound wrought-iron cylinders. Accordingly, in applying theoretical formulæ to these experiments on round tubes, the tensile strain per square inch of section which the bottom sustained at the moment of fracture, is found to range from about 11 to 15 tons, as determined from the formula used by Mr. Hodgkinson:

$$f = \frac{w\,l\,a}{\pi\,(a^4 - a'^4)}$$

where $f =$ the strain per square inch of section; w, the breaking-weight; l, the length; a and a', the external and internal diameters respectively.

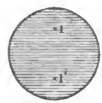
Now we know, *à priori*, that the average tensile strength per square inch of such boiler-plate is about 20 tons, which would have been nearer the value of f in those experiments in which the bottom failed, provided the plates had been fairly rent asunder without being weakened by rivets or change of form.

With respect to those experiments in which failure took place by the buckling of the top, f only represents in each experiment the resistance to *buckling* in a tube of that *particular* thickness of plate, and would not apply as a constant to tubes of other dimensions. This was really all the information directly obtainable from these experiments. No one knew, *à priori*, the resistance of plates to buckling, which was a new fact altogether, and one not involved in any of the formulæ hitherto employed. It, indeed, annihilated at once their practical utility; and, prominent as it became in subsequent experiments, it threatened temporarily even to frustrate the consummation of Mr. Stephenson's design. It was, therefore, at once, the most important object of investigation;

and being closely connected with the theory of the flexure of long pillars, the author of the only investigation of that subject could not fail to perceive the analogy, and his experiments were then directed (to use his own words), to ascertaining how far this value of f would be affected by changing the thickness of the metal, the other dimensions of the tube being the same.

The reader will easily arrive at the generally received simple rules employed in investigating the strength of tubes.

In solid cylinders employed as beams, supposing the length to remain constant—*i. e.*, the variation being only in diameter—since they are of symmetrical form, the strength will be dependent solely on the quantity of material in the section of fracture, and on the depth or leverage with which it is acting. Now, the section will be as the square of the diameter, in different cylinders; and with respect to the leverage or depth, if we suppose the section divided into horizontal layers, or laminæ, it is clear that, under transverse strain, the top layers will be compressed by the strain, and the lower layers extended, and that some intermediate layer will be in a neutral state. It is, moreover, evident that each horizontal layer is exerting a resistance to transverse strain, dependent directly on its distance from the neutral axis of the beam; and, secondly, dependent on the nature of the elasticity of the material, which may, or may not, exert a resistance proportional to the extension. But whether this is the case or not, and whether we can determine them or not, there will evidently be some points, I I′, so situated with respect to the neutral axis that we may assume the united effect of all the resistances to compression and extension to be accumulated at these points, about which they are in equilibrium.



Hence, then, if we double the dimensions of the section, *i. e.* the diameter of the circle, we shall, on the assumption of

similar action, under perfectly symmetrical conditions, in the first place, double the distance of the points I I′, or the leverage with which the resistance is exerted; and, secondly, we shall have four times as much sectional area, or material in action, with this double leverage; hence, altogether, we have twice four times, or eight times, the amount of resistance to overcome, or, in other words, the strength will be as the cube of the diameter. Thus, if one solid cylinder is three times the diameter of another, it will, as a beam, be twenty-seven times as strong; and with n times the diameter, n^3 times as strong, the length remaining constant. But an increase of length augments the strain from a given weight simply on the principle of an ordinary lever. A beam will be only half as strong by making it twice as long, because the leverage is twice as great. Hence the strength of different solid cylinders will be inversely proportional to their lengths and directly proportional to the cube of their diameters; or, in other words, directly proportional to their sectional area and depth, and inversely proportional to their length:

or, $$W = \frac{d^3}{l} c;$$

or, $$W = \frac{d^2\, d}{l} c;$$

where c is a constant depending on the material.

Thus, a round rod of cast-iron, 12 inches long and 2 inches diameter, broke with about 4·72 tons at the centre.

To find the strength of another rod, one inch long and one inch diameter, as a unit of comparison, we have by compound proportion,—

as $$\left.\begin{matrix} 1 : 12 \\ 3^3 : 1^3 \end{matrix}\right\} :: \overset{\text{Tons.}}{4{\cdot}72} : \overset{\text{Tons.}}{7{\cdot}08},$$

which is the value of the constant c in the above expressions.*

* The strength of a circular beam is to that of the circumscribed square as 1 : 1·7.

And with any other dimension, keeping the denominations inches and tons, we have merely to multiply the constant 7·08 by the cube of the diameter, and to divide by the length.

Thus, for a rod 20 feet long and 10 inches diameter, we have—

$$W = \frac{d^3}{l}\,7{\cdot}08 = \frac{1000}{240}\,7{\cdot}08 = 29{\cdot}5 \text{ tons.}$$

These general principles for the comparison of the strengths of different solid cylinders are equally applicable to elliptical, square, or rectangular beams, or T-shaped girders, or to any prismatic solids. In all cases, the strength of beams of similar section will be directly as the area of section and the depth, and inversely as the length;

or, $$W = \frac{A\,d}{l}c.$$

Where A represents the area of section, d the depth, l the length, and c is a constant to be determined for each particular form of beam, from experiment. With rectangular beams, the area of section being the depth multiplied by the breadth b, we have,—

$$W = \frac{d\,b\,d}{l}\,c = \frac{d^2\,b}{l}\,c,$$

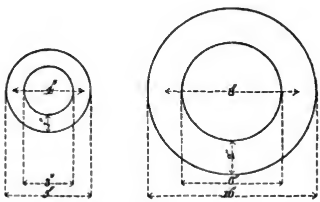
which is the usual form in use. These familiar empirical rules require explanation, because they are exclusively adopted by the practical mechanic, who has no leisure or inclination to avail himself of more exact investigation; and it is of the utmost importance that he should have a clear understanding of the foundation on which they are based. He will thus be the better able to discern those circumstances where they are inapplicable, and be able to modify them with some degree of security. It is indeed safer for him, in all cases, to rely on simple principles when their origin is thoroughly understood,

than blindly to attempt to use complicated formulæ based on reasoning which he is unable to comprehend, and involving intricate calculations which he is incapable of following. In all ordinary practical application he will thus go but little wrong, and even in the consideration of so unusual a construction as the Beams for the Britannia Bridge, a thorough appreciation of such elementary reasoning will lead to very sound results.

Similar principles have been usually applied in estimating the strength of tubes.*

One tube B is said to be *similar* to another A when all its dimensions, viz., its diameter, thickness, and length, are increased or decreased in the same proportion.

A B



The section B is of double the lineal dimensions of the section A. With respect to the strength of B as compared with A, supposing the length the same, if the thickness were equal in the two tubes, the area of the section B would be twice as great as that of the section A, on account of its being twice the diameter; the transverse strength of such a tube would, therefore, be twice as great on account of the increase of the quantity of material, if the depth were unaltered; but the depth, or leverage, being also doubled, the strength will be doubled again on this account. It would thus be four times

* We shall find hereafter that the transverse strength of two cylinders, one solid and one hollow, and of equal weight, will be as their diameters.

as strong as A. But the thickness is also doubled, and the strength is thus doubled a third time, so that, altogether, the strength of B is eight times as great as that of A; *i. e.*, the strength is proportionate to the cube of the diameter. It is also, as in a solid, inversely proportional to the length, and is thus proportionate to $\frac{d^3}{l}$ in similar cylindrical tubes.

Therefore, if the breaking-weight of A were 10 tons, the breaking-weight of B would be $\frac{2^3}{2}$, 10, or $2^2 \times 10 = 40$ tons; *i. e.* the breaking-weight of similar tubes is as the square of the lineal dimensions; for whatever number of times (n) we may make B greater than A, we should similarly have the strength as $\frac{n^3}{n}$, or as n^2.*

* That the strength of similar hollow wrought-iron cylinders, supported at the ends and loaded in the middle, is as the square of the lineal dimensions, may be thus demonstrated algebraically.

We have for the strength of a hollow cylinder, $W = \frac{\pi f}{a\,l}(a^4 - a'^4)$, a and a' being the external and internal radii, l the distance between supports, and W the breaking-weight, the other quantities being constant.

Whence the strength W′ of a tube n times the lineal dimensions will be

$$W' = \frac{\pi f}{n a\; n l}(n^4 a^4 - n^4 a'^4)$$

$$= n^2 \frac{\pi f}{a}(a^4 - a'^4) = n^2 W.$$

Also, in the rectangular tube,

Since $$W = \frac{2f(bd^3 - b'd'^3)}{3\,ld},$$

where d, d', are the external and internal depths; b and b', external and internal breadths; f dependent on the material and constant on the same material: the strength of a tube n times the size will be

$$W' = \frac{2f(nb\,n^3d^3 - nb'\,n^3d'^3)}{3\,n l\,n d}$$

$$= n^2\,\frac{2f(bd^3 - b'd'^3)}{3\,ld},$$

$\therefore$ $W' = n^2 W$, as before.

Hence the strength of similar tubes, as of prismatic solids, is proportional to their area of section and depth directly, and to their length inversely, for $\frac{d^3}{l} = \frac{d^2 d}{l}$, and this is proportional to $\frac{a d}{l}$. Thus in the preceding example, we have section of A $= (\overline{5+3} \times \overline{5-3})\ {\cdot}7854 = 12{\cdot}566$ square inches. The section of B is similarly $= 50{\cdot}264$ square inches.

$\therefore$ for the strength of B we have, by ordinary proportion,

$$\text{As } 12{\cdot}566 : 50{\cdot}264, \text{ or as } \left.\begin{array}{r} 1 : 4 \\ 1 : 2 \\ 2 : 1 \end{array}\right\} :: 10 : \frac{8 \times 10}{2} = 40 \text{ tons, as before.*}$$

The same reasoning applied to elliptical tubes would lead to the same conclusion, and thus the strength of all similar

* The sectional area of a round tube may be also calculated by multiplying the thickness by the mean diameter, and by 3·1416.

Thus, the area of A

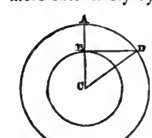
$$= 1 \text{ in.} \times 4 \times 3{\cdot}1416 = 12{\cdot}566, \text{ as before,}$$

or, by deducting the area of the inner cylinder from that of the outer, thus,

Area of outer cylinder	$= 5 \times 5 =$	25	circular inches.
Area of inner cylinder	$= 3 \times 3 =$	9	—
	Difference	16	—

which, multiplied by ·7854 = 12·566 square inches, as before.

It is extremely convenient to retain circular inches as the unit of measure in all circular engineering work, such as cylinders, hydraulic presses, circular columns or rods, pipes, pumps, valves, &c.; and it is to be regretted that this practice is confined to the practical mechanic and not introduced more extensively by theoretical writers.

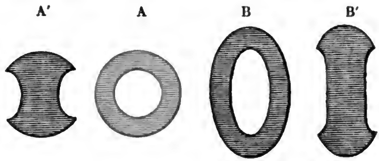


The sectional area of a round tube may be obtained geometrically as follows:—Draw the radius A C, then B D being drawn perpendicular to A C on B, will be intercepted by the outer circle at D, and a circle described with B D as radius will contain the same sectional area as the ring B D; because (B. i. 47. Euclid) $BD^2 = CD^2 - BC^2 = AC^2 - BC^2$.

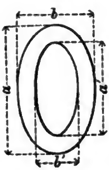
beams, whether solid or hollow, is directly as their sectional area and depth, and inversely as their length.*

We have only to bear in mind that the value of each horizontal layer in resisting transverse strain is directly proportional to its distance from the neutral axis, and independent of its lateral position.

Thus the tubes A and B must fracture in the same



* In finding the sectional area of an elliptical tube, we have the longer axis multiplied by the shorter and by $\cdot 7854$ = the sectional area of any ellipse.



i.e. $a\,b \times \cdot 7854$ = sectional area of outer ellipse;

similarly, $a'\,b' \times \cdot 7854$ = sectional area of inner ellipse;

$\therefore\ (a\,b - a'\,b')\cdot 7854$ = sectional area of the tube;

or we may approximate to the sectional area by multiplying the circumference by the thickness, the circumference being obtained by multiplying the mean of the two diameters by 3·1416, or by multiplying the sum of the two diameters by 1·57079.

As an example, we will take Exper. 19, where the conjugate and transverse external diameters are 15 inches and $9\frac{3}{4}$ inches respectively, the thickness being ·143; we have therefore 14·714 and 9·464 inches for the internal diameters.

And $\{(15 \times 9\cdot 75) - (14\cdot 714 \times 9\cdot 464)\}\ \cdot 7854 = 5\cdot 49$ sq. in. area;

or thus, $\dfrac{15 + 9\frac{3}{4}}{2} = 12\cdot 375$ = mean of the two diameters,

and $12\cdot 375 \times 3\cdot 1416 = 38\cdot 88$ in. = the circumference,

and $38\cdot 88 \times \cdot 153 = 5\cdot 55$ sq. in. area.

We should have approximated nearer by taking the mean between the external and internal diameters, instead of the internal diameter.

way as the solid beams A′ and B′ formed by transferring each horizontal layer of the rings into juxtaposition, so as to form solid beams. These beams are closely analogous to an ordinary T-girder; and by regarding tubes in this light we obtain a clear notion of their general properties and advantages, and some useful assistance in comparing them with each other; the whole of the principles we have explained arise at once out of this view of their composition.

These expressions are directly deducible from more general formulæ, which will be investigated in a subsequent chapter.

Convenient, however, as such simple views undoubtedly are, the anomaly of buckling, so prominent in the experiments described, precludes the possibility of employing any single empirical formula for tubes of all dimensions; for the strength of similar tubes is only proportional to the square of their linear dimensions, on the assumption that the resistance to tension and to crushing is proportional to the area to be extended or compressed; and a glance at the experiments recorded will shew how serious an error would be committed in computing the strength of any of these models from similar tubes of larger or smaller dimensions, and consequently of different thickness.

Considerable precaution is therefore necessary in the application of this reasoning to practical purposes.

Such an empirical rule is safely adapted, with some modification, to cast-iron girders, because the quantity of material in these beams is so great that, as generally constructed, they seldom or never fail by the compression of the top; and the quantity (a) in the ordinary formula for cast-iron girders,

$$W = \frac{a\,d}{l}\,26 \text{ tons},$$

is made to represent not the sectional area of the whole

girder, but only of the lower flange: consequently this is only applicable to beams in which the top is of sufficient strength to resist flexure as a column, and in which the vertical rib is small and supposed neutral.

Similarly with a wrought-iron tube, if the top were insured against flexure or buckling, it is evident that the strength of the bottom, and therefore of the tube, would be nearly proportional to the quantity of material extended; for this material can have no tendency to avoid the direction of the strain; and the tensile strength of wrought-iron must evidently be very nearly proportional to the area of section. It is possible that the surface of metal may be a little stronger than the internal portions; but this at the most can be but of small moment as regards the extended portion or bottom of the tube. If we, therefore, represent by (a) only the sectional area of *the extended material*, the formula,

$$W = \frac{a\,d}{l} f,$$

is practically safe; and the value of f will moreover be found proportionate to the tensile strength of the material.

But as regards fracture at the top, if a represent the sectional area of *the compressed material*, the other quantities remaining as before, this formula would only be applicable so long as the destruction of the top arises from actual compression in the direction of the strain. The top is a species of pillar, and follows to some extent the laws of pillars, in which the strength is not simply proportional to the sectional area.

When, however, the top is retained in shape by means of cells or corrugations, or other devices of construction, or when the plates are thick enough to resist flexure, then a similar expression may be admissible for the top; but the constant f will have different values, as it is applied either to the extended or compressed portions, and the formula will not apply generally to both the top and bottom.

The only manner in which such an expression should be practically employed is, consequently, to restrict its application entirely to the bottom or extended portion of the tube, always taking care, by special treatment, that the top be of sufficient strength to compel failure from extension; for in all well-constructed wrought-iron tubes of the same thickness on every side, as in the experiments, failure, we shall hereafter see, would always take place by the crushing of the top.

It will be hereafter shewn that the resistance to buckling or flexure, up to certain limits, is as the cube of the thickness of the plates; so that, if we make two similar thin models, the one being twice the dimensions of the other, the strength of the bottom of the larger model will be four times as great to resist tension as that of the smaller, but the strength of the top will be eight times as great to resist buckling. The strength of these models cannot, therefore, be simply as their area of section.

We have no means of deducing, from the experiments recorded, any constant of practical use for the strength of round tubes. The failure of the defective bottom through the rivets, and the crippling of the thin top in *every* experiment, give no positive results of any practical value. The value of f would, under other circumstances, have been about the same as that obtained from similarly constructed tubes of rectangular form. Practically the ultimate tensile strength of boiler plate may be taken at 20 tons, and its ultimate resistance to compression at 16 tons per square inch: which must not be understood as deduced from these experiments, but from subsequent experience.

From these experiments the mean values of c in the formula $W = \frac{a\,d}{l}\,c$ (*see* page 107) will be found to be as follows:

For the cylindrical tubes $c = 13{\cdot}03$.
For the elliptical tubes $c = 15{\cdot}3$.

In a subsequent chapter we shall give some additional experiments on circular and elliptical tubes of wrought and cast-iron, in which the preceding objections were partially removed, and the value of c was therefore found to be much greater.

We have many instances, in the vegetable kingdom, of the extreme rigidity and strength of circular tubes: the stems of the grass tribe generally are remarkable for their lightness and strength; the common wheat-straw and the river reed are familiar examples in our own climate; but in the tropics the gigantic stems of the bamboo and other grasses tower sixty feet above the jungle, and are extensively employed as beams for covering buildings, and even, in some cases, as the transverse bearers of light suspension-bridges. The angler's bamboo rod is the most perfect of tubular beams. Tapered off in proportion to the strain, its silicious coat (as in all the grasses) defies compression, while it is internally lined with woody fibre to resist extension in every direction; its strength, lightness, and stiffness, are thus equally marvellous; and we cannot fail to be struck with the provision of diaphragms, throughout the whole tribe, to preserve the circular form, which addition would certainly have much modified the results obtained from thin circular and elliptical tubes of wrought-iron.

CHAPTER II.

PRELIMINARY EXPERIMENTS ON RECTANGULAR TUBES.

The change of shape of the circular and elliptical tubes was considered so disadvantageous, that no further attention was given to the investigation of their properties. No experiments were made on thicker tubes, or on tubes better riveted, or with discs or rings to preserve their form; nor was any increase in the thickness of the top attempted, to resist the buckling. Their tendency to change of form led naturally to the rectangular section. Mr. Stephenson appears to have intended some of these modifications: he observes, "I hope some tubes of an elliptical form and with *thick plates* at the top and bottom will be tried, for in this way the disposal of the material will approach nearly to that in a common T-girder, which is doubtless the thing to be aimed at." No such tubes were, however, constructed, and the subject is still unexplored; but the rectangular form was more thoroughly investigated. More attention was paid to the riveting of the bottom plates, and to other details of construction, and even the first experiments gave favourable results, without the assistance of diaphragms. The rectangle peculiarly obviated change of form; sides, top, and bottom, were distinctly separated; and each was directly in the plane of the strain to which it was subjected, as in the ordinary flanged girder. The results obtained were consequently more uniform and intelligible; and this simplicity contributed much to the preference rapidly given to this original form in which the tube had been first conceived.

The rectangular tubes first constructed varied in length from 17 feet 6 inches to 24 feet, and in depth from $\frac{1}{15}$th to $\frac{1}{20}$th of the length, or from 8 inches to $18\frac{1}{4}$, the thickest plates used being about $\frac{1}{4}$ inch thick. They failed by weights placed at the centre, varying from a ton and a half to about

10 tons. These weights were suspended from a hole in the bottom of the tube, or from a bar passing transversely through the sides, and resting on a cushion on the bottom plates, the part pierced being strengthened by a plate riveted around the hole.

Some of the models were not of uniform thickness, the top and bottom differing considerably. The angles were not united by angle-iron, but formed by curving the side plates to form a flange for riveting to the top and bottom.

In conformity with the rule for cast-iron girders, the first experiments were made with the thickest plates at the bottom, and subsequently these same models were repaired and used with the thickest plates at the top, with much better results, as might naturally be expected with *thin tubes;* but the buckling of the top in thin models, as with the circular and elliptical tube, still interfered in all the results. Many ingenious suggestions to obviate this phenomenon were tested, as it was at this period imagined to furnish serious grounds for apprehension. Accordingly, in Exper. 16, with an elliptical tube, a fin was attached to the top to restrain this tendency; and subsequently, in Exper. 34, corrugated plates were substituted for plane plates with good effect. The cells in the top of the tubes used for the bridges may be considered as modifications of these devices, arising from similar apprehension. The real value of such precautions will be better appreciated hereafter in investigating the laws which govern the flexure of plates used as pillars; but considerable importance was attached to this subject during these experiments, and the results obtained with these corrugated tops, as compared with those obtained from plane *thin* plates, were very striking.

The experiments are here tabulated for facility of reference.

TABLE III.

Preliminary Experiments on the Transverse Strength of Rectangular Tubes of riveted Boiler-Plate.

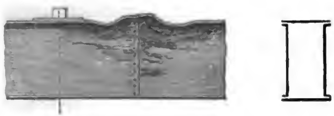
No. of Experiment.	Date.	Length.		Weight.		Sectional Area.			Weight applied.	Deflection.	Permanent Sets.	Remarks.
		Total.	Between Supports.	Total.	Between Supports.	Top.	Sides.	Bottom.				
		Feet.	Feet.	Tons.	Tons.	Inches.	Inches.	Inches.	Tons.	Inches.	Inches.	
20	July 31	18·5	17·5	·0902	·0853	1·074	1·432	1·074	·419	·17		Shackle hung across top of tube.
									·919	·55		
									1·419	·96	·1	Failed by compression, the top doubling up and the sides bulging out.
									1·679	1·1		
21	July 31	18·5	17·5	·1138	·1076	1·090	1·453	1·885	·441	·16		Shackle hung across top of tube, and a stiff plate, 30 inches long, laid under it, to distribute the weight on the top.
									·941	·45	.05	
									1·441	·8	·09	With 1·691 ton, failure by compression was rapidly approaching.
									1·691	·94		

No. 20:

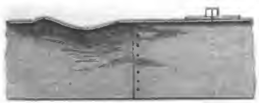
Depth 9·6 inches.
Breadth 9·6 —

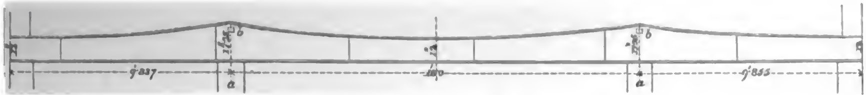
No. 21:

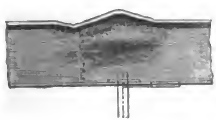
Depth 9·6 inches.
Breadth 9·6 —

22	July 31	18·5	17·5	·1138	·1076	1·885	1·453	1·090	·441	·17		The tube of Exper. 21 reversed in position.
									·941	·5	·07	Shackle hung across top of tube.
									1·441	·73	·14	
									1 691	·9	·18	
						Depth 9·6 inches. Breadth 9·6 —			1·941	1·05	·2	Failed by extension, starting the rivets in the bottom and adjoining side joints.
									2·191	1·21	·26	
									2·441	1·37	·32	
									2·691	1·54	·4	
									2·941	1·75	·5	
									3·191	1·88		
23	Aug. 1	18·5	17·5	·1415	·1339	1·951	2·168	3·394	·441	·15		Shackle hung across top of tube.
									·941	·3		
									1·441	·44	·05	Failed by compression, after bearing the weight about a minute.
									1·941	·6	·07	
						Depth 18·25 inches. Breadth 9·25 —			2·441	·7	·1	
									2·941	·9	·22	
									3·041	·93		

Transverse Strength of Rectangular Tubes.

No. of Experiment.	Date.	Length. Total.	Length. Between Supports.	Weight. Total.	Weight. Between Supports.	Sectional Area. Top.	Sectional Area. Sides.	Sectional Area. Bottom.	Weight applied.	Deflection.	Permanent Sets.	Remarks.
		Feet.	Feet.	Tons.	Tons.	Inches.	Inches.	Inches.	Tons.	Inches.	Inches.	
24	Aug. 1	18·5	17·5	·1415	·1339	3·394	2·168	1·951	·441	·08		The tube of Exp. 23 repaired and reversed in position.
									·941	·3		
									1·441	·42	·03	Shackle hung across top of tube.
									1·941	·55	·1	
									2·441	·7	·15	
									2·941	·8	·2	
						Depth 18·25 inches.			3·441	·92	·24	
						Breadth 9·25 —			3·941	1·1	·3	
									4·441	1·3	·31	
									4·691	1·35	·32	
									4·941	1·4	·4	
										1·6	·5	Increase in deflection from the weight being left on 20 hours.
									5·191	1·65	·53	
									5·441	1·73		Failed by compression.
25	Aug. 2	25·1	24·	·3518	·3364	2·051	3·902	3·615	·357	·07		Weight suspended from lower flange.
									·857	·2		
									1·357	·33		
									1·857	·5		
									2·357	·6		
						Depth 15 inches.			2·857	·7		
						Breadth 2·25 —			3·357	·83		
									3·857	·95	·07	
									4·357	1·2	·15	
									4·857	1·35	·2	Failed by extension, breaking through the plates of the bottom at a flaw in the iron.
									5·357	1·5	·25	
									5·857	1·613		



26	Aug. 3	37·71	17·	·2857	·1364	1·707	At Centre. 2·867	1·707	·357	·09		
						1·707	At Ramps. 3·831	1·707	·857	·2	·02	The ends of this tube were continued beyond the supports for 9·375 ft. and kept from rising.
									1·357	·32	·05	
									1·857	·45	·09	
									2·357	·59	·16	
						Depth at Centre 13· inches.			2·857	·71	·19	
	For elevation see above.					— at Ramps 17·25 —			3·357	·84	·22	
						Breadth throughout 7·5 —			3·857	·99	·27	With 4·857 the top side began to fail, puckering up 18 inches from shackle which was suspended to lower flange.
									4·357	1·18	·32	
									4·857	1 31		
27	Aug. 4	25·1	24·	·3518	·3364	2·051	3·902	3·615	2·357	·65	·08	The tube of Experiment 25, with a plate 25 inches thick and 14 inches long riveted over the fracture; shackle suspended to lower flange.
									3·357	·9	·18	
									4·357	1·2	·3	
									5 357	1·46	·21	
									5·857	1·6	·21	
						Depth 15 inches. Breadth 2·25 —			6·357	1·75	·21	
										2·	·6	Increase in deflections from weight being left on 17 hours.
									6·607	2·11	·62	
									6·857	2·17	·62	
									7·107	2·28	·68	
									7·357	2·36	·74	Failed by compression.
									7 607	2·58	·8	
									7·857	2·66		

Transverse Strength of Rectangular Tubes.

No. of Experiment.	Date.	Length. Total.	Length. Between Supports.	Weight. Total.	Weight. Between Supports.	Sectional Area. Top.	Sectional Area. Sides.	Sectional Area. Bottom.	Weight applied.	Deflection.	Permanent Sets.	Remarks.
		Feet.	Feet.	Tons.	Tons.	Inches.	Inches.	Inches.	Tons.	Inches.	Inches.	
28	Aug. 5	25·1	24·	·3518	·3364	3·615	3·902	2·051	4·357	1·4	·38	The tube of Experiments 25 and 27 repaired and turned upside down.
									4·857	1·65	·5	
									5·357	1·83	·5	
						Depth 15· inches.			5·857	2·03	·69	Failed by extension, the bottom flange tearing across at the line of rivets nearest to shackle.
						Breadth 2·25 —			6·357	2·3	·84	
									6 857	2·49	·97	
									7·107	2·59		
29	Aug. 10	3·71	18·	·2857	·1364	3·132	At Centre. 2·867	1·707	3·357	·68		The tube of Experiment 26 with strengthened top.
									3·857	·84	·04	
						1·707	At Ramps. 3·831	1·707	4·357	·99	·1	
									4·857	1·15	·16	
									5·357	1·31	·25	Failed as before, by the top puckering up 13 inches from shackle.
						Depth at Centre 13· inches.			5·607	1·49	·32	
						— at Ramps 17·25 —			5·857	1·64	·42	
						Breadth throughout 7·5 —			6·107	1·71		

30	Sept. 20	12·	11·	·0558	·0511	1·329	1·072	·897	·415	·06		Shackle hung across top of tube.
									·795	·11		
									1·174	·16		
									1·956	·26		
						Depth 8 inches. Breadth 1 —			2·725	·37	·01	
									3·123	·44	·035	
									3·509	·51	·05	
									3·882	·58	·075	
									4·261	·67	·118	
									4·453	·74	·150	With 4·453 tons the sides gave signs of twisting at the supports; the ends were, therefore, filled up with wood.
									4·636	·87	·234	
									4·833	1·06	·325	
									5·024			Failed by compression.
31	Sept. 20	12·	11·	·0558	·0511	·897	1·072	1·329	2·729	·51	·1	The tube of the preceding Experiment turned upside down. Shackle hung as before.
									2·924	·6	·13	
									3·115	·74	·23	
									3·190			Failed by compression.

Transverse Strength of Rectangular Tubes.

No. of Experiment.	Date.	Length.		Weight.		Sectional Area.			Weight applied.	Deflection.	Permanent Sets.	Remarks.
		Total.	Between Supports.	Total.	Between Supports.	Top.	Sides.	Bottom.				
		Feet.	Feet.	Tons.	Tons.	Inches.	Inches.	Inches.	Tons.	Inches.	Inches.	
32	Sept. 20	18·5	17·5	not	observed.	5·040	2·168	6·48	·428			The tube of Experiments 23 and 24, with strengthened top and bottom ; shackle hung as before.
									1·202	·09		
									1·976	·16		
									2·756	·25		
						Depth 18·25 inches.			3·508	·34		
						Breadth 9·25 —			4·265	·42		
									5·028	·6		
									5·405	·65	·14	Failed by compression, at joint nearest shackle, forcing out rivets.
									5·799	·72	·15	
									6·190			
33	Oct. 9	18·5	17·5	·1714	·1620	3·203	1·432	1·074	·428	·07		The tube of Experiment 20 with the top greatly strengthened. Shackle hung across top of tube.
									1·213	·25		
									1·897	·45	·07	
									2·370	·55	·1	
						Depth 9·6 inches.			2·759	·67	·11	
						Breadth 9·6 —				·73	·12	
									3·132	·84	·14	
									3·315	·94	·2	Failed by extension, the bottom tearing across at a joint 11 inches from centre.
									3·508	1·07	·29	
									3·693	1·13		

34	Oct. 14	19·66	19·	·2232	·2156	3·228	2·02	2·218	·441	·035		
									1·994	·19		Shackle hung across top of tube.
									2·774	·27		
									3·537	·34	·02	
									4 302	·424	·05	
									5·598	·523	·062	
									5·822	·64	·095	
									6·585	·735	·125	
									7·361	·87	·186	
									8·127	1·07	·276	With 8·127 tons deflection increased ·02 in 3 minutes.
									8·511	1·155		
									8·892	1·27	·4	
									9·269	1·425		Failed by the sides tearing away from the top.
									9·656	1·53	·59	
									10·030	1·59		

Depth 15·4 inches.
Breadth 7·75 —

Thickness of Plates.

Top .... ·23 inch.
Sides .... ·07 —
Bottom ...... ·18 —

If we select out of the foregoing experiments those in which there was more nearly a proper distribution of metal, and deduce from them a value of c from the relation $c = \frac{W\,l}{a\,d}$ for comparison with the values of c in circular and elliptical tubes, given at page 114, we shall have—

Experiment.	Breaking-Weight.	Sectional Area, or Value of a.	Value of the Constant c.
	Tons.		Tons.
22	3·24	4·04	9·5
27	8·03	8·00	19·3
30	5·05	2·90	28·6
33	3·73	3·20	11·7
34	10·13	7·05	21·3

Hence, the mean value of c is 18·07 tons, being somewhat greater than the value of c obtained from the other forms.

We shall give in a subsequent chapter some additional experiments on rectangular tubes, from which much higher constants have been derived.

Experiment 26 is a continuous beam, similar to the Britannia Bridge, the ends being kept down by blocks from above; but the puckering of the top vitiated the result of this experiment, from which no information was acquired.

In addition to the foregoing experiments on rectangular tubes, their close analogy to the ordinary T-girder, of which, indeed, they may be considered as a simple modification, led to some experiments on three solid wrought-iron T-girders, about ten feet long. They failed by bending over laterally without fracture of the flanges. On account of the ductility of wrought-iron, square bars, or beams, of this material invariably become useless or destroyed by bending under transverse strain, and not by tearing asunder, as in tubes, where the distance of the material from the neutral axis ensures great stiffness and prevents much flexure without fracture.

These experiments ended in the distortion of the beams, the top and bottom flanges not having sufficient breadth, as in the tubes, to prevent lateral deflection.

The dimensions and other details of these beams will be found in the following table. The results are the more interesting, as they agree so completely with results obtained from rectangular tubes of equal section and depth, and illustrate the little loss of strength that occurs with well-riveted joints, as compared with solid material.

TABLE IV.

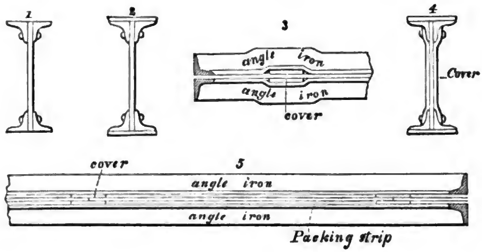
Experiments on the Transverse Strength of Solid Wrought-iron T-Girders.

No. of Experi-ment.	Date.	Length.		Total Depth.	Weight.		Sectional Area.		Weight Applied.	Deflection.	Permanent Sets.	Remarks.
		Total.	Between Supports.		Total.	Between Supports.	Top.	Bottom.				
		Feet.	Feet.	Inches.	Tons.	Tons.	Inches.	Inches.	Tons.	Inches.	Inches.	
35	Oct. 10	11·58	11·	7·	·1013	·0964	2·5	1·52	·395	·04		
									1·152	·12		
									1·927	·2		
									2·700	·26		
									3 457	·35		Failed by bending sideways.
									4·238	·46		
									5 024	·6	·09	
									5·783			
36	Oct. 10	10·66	10·	8·	·1103	·1035	2·75	1·892	·395			
									1·175	·04		
									1·945	·12		
									2·722	·15		
									3·494	·19		
									4·279	·21		
									5·034	·26		
									5·794	·3	·03	
									6·559	·35	·03	
									7·309	·45	09	With 8·087 there was a permanent lateral set of ·75 inch, which very much increased on the addition of more weight.
									8·087	·68	·26	
									8·465			

37	Oct 10	10·58	10·	8·9	·1232	·1165	2·75	1·80	·395	·02		With final weight there was a lateral deflection of 2·65 inches.
									1·163	.05		
									2·725	·11		
									4·268	·165		
									5·800	·22		
									6·575	·25		
									8·080	·29		
									8·857	·37		
									9·622	·475		
									9·994	·59		
									10·228			

This simple form of wrought-iron beam possesses great strength, and is extensively used in ship-building, and for the transverse bearers of bridges, and other purposes where the span is limited.

It is practically constructed for such purposes by riveting angle-iron to the top and bottom of the vertical plates forming the beam, fig. 1, which is sometimes further strengthened by the addition of horizontal plates, fig. 2.

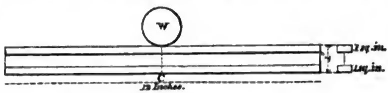


When of considerable length, the vertical plates are united by covers on each side the vertical joints; the angle-iron, in such cases, may be bent or *cranked* over these covers, fig. 3, or the covers over the angle-iron, fig. 4, but much neater and sounder work is made by packing between the covers, and thus leaving the angle-iron straight throughout the length, fig. 5, care being taken to unite the joints in the angle-irons of the lower flange by a proper cover, technically called by the workmen at the bridge "covering angle-iron," which is specially rolled for that purpose. This union of the angle-iron is far more trustworthy than welding, though evidently not so neat in appearance. The covering angle-iron, as used at the Britannia Bridge, is represented, full size, in Plate XXII.; other valuable details on this subject will be found in Plate XXI.

If the plates are thin, or the beam deep, it is of great importance that the vertical rib be kept from buckling by means

of vertical pillars of angle-iron or T-iron on one or both sides of the beam; the T-iron, in such case, being cranked over the flange of the angle-iron. (See T-iron gussets, Plate XXI.) This precaution is especially necessary at the bearings of such beams, the immediate strain at the extremity being directly a crushing strain, tending to buckle the vertical plate; the same holds good if weight is to be supported at any particular point in the length. The massive wrought-iron beams used to carry the presses for raising the tubes, and shewn on Plate XVI., are beams of this principle, with the modifications requisite for such unusual and peculiar requirements. Indeed, the tube itself may be regarded as a pair of these beams on a gigantic scale, united side by side, and the practical detail of their construction will be available generally in any application of this principle of girder.

With respect to the strength of such girders, the same principles apply to them as to rectangular tubes. We may approximate closely to the strength, rejecting the interference from buckling, or lateral distortion, on very simple considerations. Let us suppose a wrought-iron T-girder, twelve inches long, and one inch deep, the sectional area of the top and bottom flanges being one square inch. Now,



wrought-iron will fail by compression sooner than extension; the top flange will, therefore, probably be the cause of failure, and supposing the vertical plate connecting the two flanges to be very thin, and giving no other assistance than that of connecting these flanges, and supposing, moreover, wrought-iron to be crippled by a compression of twelve tons per square inch,* we may arrive at the breaking-weight of this beam as follows:—

* The large model FAILED with a compression of fifteen tons per sq. inch.

Whatever weight, W, is placed at the centre of this beam, half this weight, or $\frac{W}{2}$ is supported at A. The reaction at A is consequently $\frac{W}{2}$, and as regards the compression occasioned by this reaction at the point W, we may look upon A C W as a bent lever, the point C being considered the fulcrum, and the arm A C being six times as long as the arm C W; the reaction $\frac{W}{2}$ is, therefore, increased sixfold in compressing the central square inch of iron in the upper flange; *i. e.*, the strain at the centre is six times $\frac{W}{2}$, or 3 W, from any weight W, placed at the centre of the beam; a corresponding reaction being occasioned by the support at B. Now, since this square inch will only bear twelve tons before it is completely crippled, the beam will be destroyed when 3 W = 12, or when W = 4 tons, which is therefore the breaking-weight of this beam, one foot long, one inch deep, and with one square inch of section in the top and bottom flange. But the strength of these beams is proportional (on the assumption we have made) to the depth and sectional area, and inversely proportional to the length; *i. e.*, for any similar beam,—

$$W = \frac{d\,a}{l} 4 \text{ tons},$$

d being the depth in inches, *a* the sectional area of the upper flange in square inches, and *l* the length in feet. If we use the length also in inches, then,—

$$W = \frac{d\,a}{l} 48 \text{ tons}.$$

Applying this formula to Experiment 35, we have,—

$$\text{Breaking-weight} = \frac{2\tfrac{1}{2} \times 7 \times 48}{121} = 6 \text{ tons} = 13{,}440 \text{ lbs.}$$

The actual breaking-weight was 12,955 lbs. Similarly with Experiments 36 and 37,—

$$W = \frac{2\tfrac{1}{4} \times 8 \times 68}{120} = 8{\cdot}8 \text{ tons} = 19{,}712 \text{ lbs.}$$

The actual breaking-weights were 18,262 lbs. and 23,046 lbs.; the mean being 20,654 lbs.

We thus see how closely we should approximate to the strength of these beams with such simple reasoning. It is obvious that if twelve tons per square inch were also the ultimate tensile strength of wrought-iron, such beams should have the same quantity of material in both their upper and lower flanges. This is, however, not the case; the ultimate tensile strength is considerably greater,—viz., nearly twenty tons,—while the ultimate compressive strength is sixteen. The top and bottom flanges should thus be in the proportion of sixteen to twenty, or four to five; *i. e.*, the bottom flange should be four-fifths of the top flange for both to fail simultaneously; which is just the reverse of cast-iron, where the lower flange is required to be six times greater than the upper or similar grounds. With wrought-iron work, where riveted plates are used, the bottom alone is weakened by the rivets passing through the plates, the top remaining uninjured as regards compression, and on this account (neglecting the rivets in the calculation) little difference is practically made in the area of the top and bottom flanges of such girders.

With some trifling exceptions, we have now detailed the whole of the preliminary experiments on the transverse strength of wrought-iron girders, *i. e.*, the whole of the experiments first devised by Mr. Stephenson without other object than to test generally the properties of such structures, to discover in what manner they might be expected to fail, and to ascertain practically their applicability to purposes of construction.

In describing them, and deducing some practical results which have suggested themselves, we have had the advantage of much subsequent information, and the reader will thus be far more advanced in the subject after perusing these remarks

than the experimenter himself at this early period could possibly have been, engaged as he then was, not so much in actual progress on his journey, as in inquiring what direction he should pursue, like a traveller poring over some extensive map to determine what districts he should visit, and what subjects should principally command his attention.

The most prominent facts elicited by these experiments appeared, doubtless, at this period to be the information they gave as to the buckling of the top of the tubes and the demonstration of the superiority of the rectangular as compared with the circular and elliptical forms. They furnished valuable practical hints on the best methods of construction, pointed out the road for future investigation, and supplied some data for deductive reasoning; they were, moreover, satisfactory confirmations of all that Mr. Stephenson had expected as regarded strength and rigidity.

Little information was attained with respect to the sides. The distortion which occurred in Experiment 29 drew attention to their importance, and demonstrated the necessity of further inquiry; but the remarkable manner in which they generally retained their shape gave good grounds for belief that no great difficulty need be apprehended in this portion of the structure. But here again, as with the top, we have evidence of the great caution necessary in arguing from models to large structures; for whereas with the top the fears that first arose were afterwards proved to be groundless, so with the sides, the confidence their behaviour in these experiments at first inspired was subsequently discovered to be unfounded. And while little difficulty was experienced in the construction of the enlarged top, the greatest caution was necessary in properly proportioning and stiffening the sides; which, indeed, will be found the limit to any much greater extension of the magnitude of such structures.

Although it is thus evident that what had been done was

insufficient for enabling Mr. Stephenson to determine the definite proportions of his proposed bridges, sufficient information was acquired to warrant him in speaking with confidence on the probable result of his design. The general half-yearly meeting of February 1846 was fast approaching, and the directors and shareholders looked forward with much anxiety to his usual report, which was to embody the results of his investigations. On so important a subject Mr. Stephenson directed that Mr. Fairbairn and Mr. Hodgkinson should each prepare independent reports, addressed to him; these reports, together with his own, were laid by Mr. Stephenson before the shareholders.

MR. STEPHENSON'S REPORT.

To the Directors of the Chester and Holyhead Railway.

24 *Great George Street, Westminster,*
9th February, 1846.

Gentlemen,

In reporting to you the progress which has been made in the works, I beg to refer you to the statements of Mr. Ross and Mr. Forster, made from time to time, as regards those under contract. In addition, I need only state that last week I examined them personally, and found the whole progressing in the most satisfactory manner. I will, therefore, proceed at once to lay before you the results of the experimental investigation which, with your sanction, I commenced some months ago in reference to the construction of the bridge over the Menai Straits.

The object of this investigation, as you are aware, was to test the truth of the views I entertained respecting the employment of a large wrought-iron tube, instead of cast-iron

arches, as was originally proposed, but which we were compelled to abandon in consequence of the Admiralty refusing to allow the erection of such a structure, from the belief that it would injuriously interfere with the navigation of the Straits.

In conducting this experimental investigation, I saw the importance of avoiding the influence of any preconceived views of my own, or at least to check them, by calling in the aid of other parties thoroughly conversant with such researches. For this purpose, I have availed myself of the assistance of Mr. Fairbairn and Mr. Hodgkinson ; the former so well known for his thorough practical knowledge in such matters, and the latter distinguished as the first scientific authority on the strength of iron beams.

These gentlemen have pursued the subject with deep interest; and although they have not yet been able to bring the facts into a final and definite shape, they have each complied with my request that they would communicate their views upon the results which have already been arrived at. I therefore append to this Report their observations just as I received them. They will, I am confident, prove satisfactory to you.

I have throughout the experiments carefully studied the results as they developed themselves, and I am satisfied that the views I ventured to express twelve months ago were in the main correct; and that the adoption of a wrought-iron tube is the most efficient, as well as the most economical, description of structure that can be devised for a railway bridge across the Menai Straits.

In the course of the experiments, it is true, some unexpected and anomalous results presented themselves; but none of them tended, in my mind, to shew that the tubular form was not the very best for obtaining a rigid roadway for a railroad over a span of 450 feet, which is the absolute requirement for a bridge over the Menai Straits.

The first series of experiments was made with plain circular tubes, the second with elliptical, and the third with rectangular. In the whole of these this remarkable and unexpected fact was brought to light, viz. that in such tubes the power of wrought-iron to resist compression was much less than its power to resist tension, being exactly the reverse of that which holds with cast-iron: for example, in cast-iron beams for sustaining weight, the proper form is to dispose of the greater portion of the material at the bottom side of the beam, whereas with wrought-iron, these experiments demonstrate, beyond any doubt, that the greater portion of the material should be distributed on the upper side of the beam. We have arrived, therefore, at a fact having a most important bearing upon the construction of the tube; viz. that rigidity and strength are best obtained by throwing the greatest thickness of material into the upper side.

Another instructive lesson which the experiments have disclosed is, that the rectangular tube is by far the strongest, and that the circular and elliptical should be discarded altogether.

This result is extremely fortunate, as it greatly facilitates the mechanical arrangements for not merely the construction, but the permanent maintenance of the bridge.

We may now, therefore, consider that two essential points have been finally determined,—the form of the tube, and the distribution of the material.

The only important question remaining to be determined, is the absolute ultimate strength of a tube of any given dimensions. This is, of course, approximately solved by the experiments already completed; but Mr. Hodgkinson very properly states that others, with tubes of more varied dimensions, should be continued, in order to clear up some anomalies which still exist.

The formula as at present brought out by Mr. Hodgkin-

son, gives the strength of a rectangular tube of the dimensions I proposed, viz. 450 feet long, 15 feet wide, by 30 feet high (assuming the plates to be one inch thick), equal to 1100 tons applied in the centre, including the weight of the tube itself; but, deducting the latter, equal to 747 tons in the centre, or double this supposing the weight to be uniformly distributed over the whole 450 feet.

This amount of strength, although sufficient to carry any weight that can in practice be placed upon the bridge, is not sufficiently in excess for practical purposes. It is on this ground, therefore, I have requested Mr. Hodgkinson to devise a few more experiments in the shape best calculated to free the formula from all ambiguity. In the meantime, however, as I consider the main question settled, I am proceeding with the designs and working plans for the whole of the masonry, which I expect to have the pleasure of submitting to you in a fortnight from this time.

You will observe in Mr. Fairbairn's remarks, that he contemplates the feasibility of stripping the tube entirely of all the chains that may be required in the erection of the bridge; whereas, on the other hand, Mr. Hodgkinson thinks the chains will be an essential, or at all events a useful auxiliary, to give the tube the requisite strength and rigidity. This, however, will be determined by the proposed additional experiments, and does not interfere with the construction of the masonry, which is designed so as to admit of the tube, with or without the chains.

The application of chains as an auxiliary has occupied much of my attention, and I am satisfied that the ordinary mode of applying them to suspension-bridges is wholly inadmissible in the present instance; if, therefore, it be hereafter found necessary or desirable to employ them in conjunction with the tube, another mode of applying them must be devised, as it is absolutely essential to attach them in such a

manner as to preclude the possibility of the smallest oscillation.

In the accomplishment of this I see no difficulty whatever; and the designs have been arranged accordingly, in order to avoid any further delay.

The injurious consequences attending the ordinary mode of employing chains in suspension-bridges, were brought under my observation in a very striking manner on the Stockton and Darlington Railway, where I was called upon to erect a new bridge for carrying the railway across the River Tees, in lieu of an ordinary suspension-bridge, which had proved an entire failure.

Immediately on opening the suspension-bridge for railway traffic, the undulations into which the roadway was thrown, by the inevitable unequal distribution of the weight of the train upon it, were such as to threaten the instant downfall of the whole structure.

These dangerous undulations were most materially aggravated by the chain itself, for this obvious reason, that the platform, or roadway, which was constructed with ordinary trussing for the purpose of rendering it comparatively rigid, was suspended to the chain, which was perfectly flexible, all the parts of the latter being in equilibrium. The structure was, therefore, composed of two parts, the stability of the one being totally incompatible with that of the other: for example, the moment an unequal distribution of weight upon the roadway took place by the passage of a train, the curve of the chain altered, one portion descending at the point immediately above the greatest weight, and, consequently, causing some other portion to ascend in a corresponding degree, which necessarily raised the platform with it, and augmented the undulation.

So seriously was this defect found to operate, that immediate steps were taken to support the platform underneath

by ordinary trussing; — in short, by the erection of a complete wooden bridge, which took off a large proportion of the strain upon the chains. If the chains had been wholly removed, the substructure would have been more effective; but as they were allowed to remain, with the view of assisting, they still partake of these changes in the form of the curve, consequent upon the unequal distribution of the weight, and eventually destroyed all the connexions of the wooden frame-work underneath the platform, and even loosened and suspended many of the piles upon which the framework rested, and to which it was attached.

The study of these and other circumstances connected with the Stockton Bridge, leads me to reject all idea of deriving aid from chains employed in the ordinary manner.

I have, therefore, turned my attention to other modes of employing them in conjunction with the wrought-iron tube (as suggested by Mr. Hodgkinson), if such should be found necessary upon further investigation.

As I have already stated, in this I perceive no difficulty whatever; indeed, there is no other construction which has occurred to me, which presents such facilities as the rectangular tube for such a combination.

Having, I trust, clearly explained my views in reference to this important work, I have only to add, that in two months I expect every arrangement will be completed for commencing the masonry, which shall be conducted with the utmost activity and vigour.

I can scarcely venture to say, until after these arrangements are finally completed, at what period we may calculate upon the completion of this bridge; but I cannot recommend you to calculate upon the whole being accomplished in less than two years and a half.

I am, Gentlemen,
Your obedient Servant,
ROBERT STEPHENSON.

MR. FAIRBAIRN'S REPORT.

Abstract or short Summary of Results from Experiments relative to the proposed Bridge across the Menai Straits, addressed to ROBERT STEPHENSON, Esq. *By* W. FAIRBAIRN.

After a series of experiments undertaken at your request, for ascertaining the strongest form of a Sheet-iron Tubular Bridge across the Menai Straits, I have been induced, in order to meet the requirements for such a structure, and to ensure safety in the construction, to call in the aid and assistance of my friend Mr. Hodgkinson.

The flexible nature of the material, and the difficulties which presented themselves in retaining the lighter description of tubes in shape, gave exceedingly anomalous results; and having no formula on which dependence could be placed for the reduction of the experiments, I deemed it necessary, in a subject of such importance, to secure the co-operation of the first authority, in order to give confidence to the Chester and Holyhead Railway Company, with whom you are connected, and the public generally.

It will be observed, that the first class of experiments are upon cylindrical tubes; the second upon those of the elliptical form; and the last upon the rectangular kind. Tubes of each sort have been carefully tested, and the results recorded in the order in which they were made; and, moreover, each specimen had direct reference to the intended bridge, both as regards the length and thickness, as also the depth and width.

In the first class of experiments, which are those of the cylindrical form, the results are as follow:—

CYLINDRICAL TUBES.

No. of Experiments.	Distance between the Supports. ft. in.	Diameter in inches.	Thickness of Plate in inches.	Ultimate Deflection in inches.	Breaking Weight in lbs.	Remarks.
1	17 0	12·18	·0408	·39	3,040	Crushed top.
2	17 0	12·00	·0370	·65	2,704	Ditto.
3	15 7½	12·40	·1310	1·29	11,440	Torn asunder at the bottom.
4	23 5	18·26	·0582	·56	6,400	Ditto.
5	23 5	17·68	·0631	·74	6,400	Ditto.
6	23 5	18·18	·1190	1·19	14,240	Ditto.
7	31 3¼	24·00	·0954	·63	9,760	Ditto.
8	31 3¼	24·30	1·3501	·95	14,240	Ditto.
9	31 3¼	24·20	·0954	·74	10,880	Ditto.

With the exception of the first two, nearly the whole of the tubes were ruptured by tearing asunder at the bottom through the line of the rivets.

Finding the cylindrical form comparatively weak, the next experiments were upon tubes of the rectangular shape, which gave much better results. For the present it may, however, be more convenient to take the elliptical kind, as being the nearest approximation, as regards both form and strength, to the cylinders recorded above.

ELLIPTICAL TUBES.

No. of Experiments.	Distance between the Supports. ft. in.	Diameters transverse and conjugate in inches.	Thickness of Plate in inches.	Ultimate Deflection in inches.	Breaking Weight in lbs.	Remarks.
19	17 0	14·62 9·25	·0416	·62	2,100	Crushed on top.
20	24 0	21·66 13·50	1·1320	1·36	17,076	Broke by extension.
21	24 0	21 25 14·12	·0688	·45	7,270	By compression.
22	18 6	12·00 7·50	·0775	·95	6,867	By compression. This tube had a fin on the top side.
24	17 6	15·00 9·75	·1430	1·39	15,000	Both sides were ruptured.

It will be observed that the whole of these experiments indicated weakness on the top side of the tube, which, in almost every case, was greatly distorted by the force of compression acting in that direction. It is probable that those of the cylindrical form would have yielded in like manner, had the riveting at the joints been equally perfect on the lower side of the tube. This was not, however, the case, and hence arise the causes of rupture at that part.

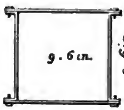
The next experiments, and probably the more important, were those of the rectangular kind; they indicate a considerably increased strength when compared with the cylindrical and elliptical forms: and, considering the many advantages which they possess over every other yet experimented upon, I am inclined to think them not only the strongest but the best adapted (either as regard lightness or security) for the proposed bridge.

RECTANGULAR TUBES.

No. of Experiments.	Distance between Supports.	Depth in Inches.	Width in inches.	Thickness of plate in inches.		Ultimate Deflection in inches.	Breaking Weight in lbs.	Remarks.
	ft. in.			Top.	Bottom.			
14	17 6	9·6	9·6	·075	·075	1·10	3,738	Broke by Compression.
14	17 6	9·6	9.6	·272	·075	1·13	8,273	(Reversed.) Extension.
15	17 6	9·6	9·6	·075	·142	0·94	3,788	Compression.
15	17 6	9·6	9·6	·142	·075	1·88	7,148	Extension.
16	17 6	18·25	9·25	·059	·149	0·93	6,812	Compression.
16	17 6	18·25	9·25	·149	·059	1·73	12,188	Ditto.
17	24 0	15·00	2·25	·160	·160	2·66	17,600	Ditto.
18	18 0	13·25	7·50	·142	·142	1·71	13,680	Ditto.
23	18 6	13·00	8·00	·066	·066	1·19	8,812	Compression. Circular bottom, fin at top.
29	19 0	15·40	7·75	·230	·180	1·59	22,469	Sides distorted. Corrugated top.

On consulting the above table, it will be found that the

results as respects strength are of a higher order than those obtained from the cylindrical and elliptical tubes ; and particularly those constructed with stronger plates on the top side, which, in almost every experiment where the thin side was uppermost, gave signs of weakness in that part. Some curious and interesting phenomena presented themselves in these experiments,—many of them are anomalous to our preconceived notions of the strength of materials, and totally different to anything yet exhibited in any previous research. It has invariably been observed, that in almost every experiment the tubes gave evidence of weakness in their powers of resistance on the top side to the forces tending to crush them. This was strongly exemplified in Experiments 14, 15, 16, &c., marked on the drawings and the table. With tubes of a rectangular shape, having the top side about double the thickness of the bottom, and the sides only half the thickness of the bottom, or one-fourth the thickness of the top, nearly double the strength was obtained. In Experiment 14 (marked



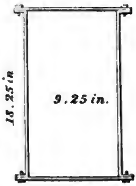
in the margin of the above table), a tube of the rectangular form, as per annexed sketch, $9\frac{1}{2}$ inches square, with top and bottom plates of equal thickness, the breaking-weight was 3738 lbs. Riveting a stronger plate on the top side, the strength was increased to 8273 lbs. The difference being 4535 lbs., considerably more than double the strength sustained by the tube when the top and bottom sides were equal.

The experiments given in No. 15 are of the same character, where the top plate is as near as possible double the thickness of the bottom. In these experiments, the tube was first crippled by doubling up the thin plate on the top side, which was done with a weight of 3788 lbs.

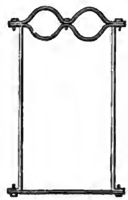
It was then reversed with the thick side upwards, and by

this change the breaking-weight was increased to 7148 lbs., making a difference of 3360 lbs., or an increase of nearly double the strength by the simple operation of reversing the tube and turning it upside down.

The same degree of importance is attached to a similar form, when the depth in the middle is double the width of the tube. From the experiments in No. 16, we deduce the same results in a tube of the annexed sectional form, where the depth is $18\frac{1}{4}$, and the depth $9\frac{1}{4}$ inches.



Loading this tube with 6812 lbs. (the thin plate being uppermost), it follows precisely the same law as before, and becomes wrinkled, with a hummock rising on the top side so as to render it no longer safe to sustain the load. Take, however, the same tube, and reverse it with the thick plate upwards, and you not only straighten the part previously injured, but you increase the resisting powers from 6812 lbs. to 12,188 lbs. Let us now examine the tube in the 29th experiment, where the top is composed of corrugated iron, as per sketch, forming two tubular cavities extending longitudinally along its upper side. This, it will be observed, presents the best form for resisting the "puckering," or crushing force, which, on almost every occasion, was present in the previous experiments. Having loaded the tube with increasing weights, it ultimately gave way by tearing the sides from the top and bottom plates, at nearly one and the same instant after the last weight, 22,469 lbs., was laid on. The greatly increased strength indicated by this form of tube is highly satisfactory; and, provided these facts be duly appreciated in the construction of the bridge, they will, I have no



doubt, lead to the balance of the two resisting forces of tension and compression.

The results here obtained are so essential to this inquiry, and to our knowledge of the strength of materials in general, that I have deemed it essential, in this abridged statement, to direct attention to facts of immense value in the proper and judicious application, as well as distribution, of the material in the proposed structure. Strength and lightness are *desiderata* of great importance, and the circumstances above stated are well worthy the attention of the mathematician and engineer.

For the present we shall have to consider not only the due and perfect proportion of the top and bottom sides of the tube, but also the stiffening of the sides with those parts, in order to effect the required rigidity for retaining the whole in shape. These are considerations which require attention; and till further experiments are made, and probably some of them upon a larger scale, it would be hazardous to pronounce anything definite as to the proportion of the parts, and the equalisation of the forces tending to the derangement of the structure.

So far as our knowledge extends—and judging from the experiments already completed—I would venture to state that a tubular bridge can be constructed of such powers and dimensions as will meet, with perfect security, the requirements of railway traffic across the Straits. The utmost care must, however, be observed in the construction, and probably a much greater quantity of material may be required than was originally contemplated before the structure can be considered safe.

In this opinion Mr. Hodgkinson and myself seem to agree: and although suspension chains may be useful in the construction in the first instance, they would nevertheless be

highly improper to depend upon as the principal support of the bridge. Under every circumstance I am of opinion that the tubes should be made sufficiently strong to sustain not only their own weight, but in addition to that load, 2000 tons equally distributed over the surface of the platform,—a load ten times greater than they will ever be called upon to support.

In fact, it should be a huge sheet-iron hollow girder, of sufficient strength and stiffness to sustain those weights; and provided the parts are well proportioned, and the plates properly riveted, you may strip off the chains, and leave it as a useful monument of the enterprise and energy of the age in which it was constructed.

In the pursuit of the experiments on the rectangular as well as other description of tubes, I have been most ably assisted by my excellent friend Mr. Hodgkinson; his scientific and mathematical attainments render him well qualified for such researches; and I feel myself indebted to him for the kind advice and valuable assistance which he has rendered in these and other investigations. I am also deeply indebted to yourself and the Directors for the confidence you have placed in my efforts, and for the encouragement I have uniformly received during the progressive developements of this inquiry.

But, in fact, the subject is of such importance, and the responsibilities attached to it are so great, as to demand every effort to demonstrate, calculate, and advise what in this case is best to be done. Both of us have, therefore, laboured incessantly at the task; and I am indebted to my friend for the reduction of the experiments, which I would not attempt to weaken by a single observation.

MR. HODGKINSON'S REPORT.

SUMMARY OF RESULTS,

Offered, in conjunction with one by William Fairbairn, *Esq., M. Inst. C. E. to* Robert Stephenson, *Esq., M. Inst. C. E., &c. &c., for the Directors of the Chester and Holyhead Railway, on the subject of a proposed Bridge across the Menai, near to Bangor. By* Eaton Hodgkinson, *F.R.S.*

Having in the month of August last year been requested to render assistance, principally in a scientific point of view, with respect to the experiments to ascertain the practicability of erecting a Tubular Bridge across the Menai Straits of sufficient strength for railway trains to pass through it with safety, I attended twice in London for that purpose; and as the experiments made there were on tubes of various forms of section, including several elliptical and circular ones, I investigated formulæ for reducing the strength of the leading ones. It appeared evident to me, however, that any conclusions deduced from received principles, with respect to the strength of thin tubes, could only be approximations; for these tubes usually give way by the top or compressed side becoming wrinkled, and unable to offer resistance, long before the parts subjected to tension are strained to the utmost they would bear. To ascertain how far this defect, which had not been contemplated in the theory, would effect the truth of computations on the strength of the tubes proposed to be used in the bridge,—and also to shew whether the principles generally received could be applied with certainty in reasoning as to the strength of the bridge from that of models comparatively very small,—for these two purposes I urged the neces-

sity of a number of fundamental experiments, which, besides supplying the wants above-mentioned, might enable me to obtain additional information to that from Mr. Fairbairn's experiments, with respect to the proportions that the different parts of the section of such a bridge ought to have, as well as what form it should be of, in order to bear the most.

Feeling that there might be objections against allowing me to follow the course I proposed, however necessary it might appear to myself, I suggested a much more limited series of experiments than now appear to me to be necessary; and as the time consumed in getting the plates rolled and the tubes prepared caused the experiments to be delayed till the beginning of the year, the time given me has been too limited to obtain all the facts which the few experiments proposed would have afforded.

I will now give the results, so far as they have been obtained and seem worthy of reliance, subject to correction from future experiments; beginning with the reduction of Mr. Fairbairn's experiments on the strength of tubes of wrought-iron made of plates riveted together.

Cylindrical Tubes.

The strength of a cylindrical tube, supported at the ends, and loaded in the middle, is expressed by the formula

$$w = \frac{\pi f}{a\,l}(a^4 - a'^4).$$

Where l is the distance between the supports: a, a', the external and internal radii; w, the breaking weight; f, the strain upon a unity of section, as a square inch, at the top and bottom of the tube, in consequence of the weight w; $\pi = 3{\cdot}14159$.

From this formula we obtain

$$f = \frac{w\,l\,a}{\pi\,(a^4 - a'^4)}.$$

As it will be convenient to know the strain f per square inch, which the metal at the top and bottom of the tube is bearing when rupture takes place, this value will be obtained from each of Mr. Fairbairn's experiments; the value w being made to include, besides the weight laid on at the time of fracture, the pressure from the weight of the tube between the supports, this last being equal to half that weight. Computing the results we have, from

		lbs.	
Experiment	1,	$f=$ 33456	Mean, 29887 lbs. = 13·34 tons.
,,	2,	$f=$ 33426	
,,	3,	$f=$ 35462	
,,	4,	$f=$ 32415	
,,	5,	$f=$ 30078	
,,	6,	$f=$ 33869	
,,	7,	$f=$ 22528	
,,	8,	$f=$ 22655	
,,	9,	$f=$ 25095	

Fracture in all cases took place either by the tube failing at the top, or tearing across at the rivet-holes; this happened on the average, as appears from above, when the metal was strained 13⅓ tons per square inch, or little more than half its full tensile strength.

Elliptical Tubes.

The value of f in an elliptical tube broken as before (the transverse axis being vertical), is expressed by the formula

$$f=\frac{w\,l\,a}{\pi\,(b\,a^3-b'\,a'^3)}.$$

Where a, a', are the semi-transverse external and internal diameters; b, b', the semi-conjugate external and internal diameters; and the rest as before, w including in all cases the pressure fom the weight of the beam.

Computing the results from Mr. Fairbairn's experiments, we have from

	lbs.		
Experiment 20,	$f = 36938$		
„ 21,	$f = 29144$	Mean, 37089 lbs. = 16·55 tons.	
„ 24,	$f = 45185$		

Rectangular Tubes.

If in a rectangular tube, employed as a beam, the thickness of the top and bottom be equal, and the sides are of any thickness at pleasure, then we have

$$f = \frac{3\,w\,l\,d}{2\,(b\,d^3 - b'\,d'^3)},$$

in which d, d', are the external and internal depths respectively; b, b', the external and internal breadths; and the rest as before.

Mr. Fairbairn's experiment No. 14 gives by reduction,

$$f = 18495 \text{ lbs.} = 8{\cdot}2566 \text{ tons.}$$

This is, however, much below the value which some of my own experiments give, as will be seen further on.

The value of f, which represents the strain upon the top or bottom of the tube when it gives way, is the quantity per square inch which the material will bear either before it becomes crushed at the top side or torn asunder at the bottom. But it has been mentioned before, that thin sheets of iron take a corrugated form with a much less pressure than would be required to tear them asunder; and, therefore, the value of f, as obtained from the preceding experiments, is generally the resistance of the material to crushing, and would have been so in every instance if the plates on the bottom side (subjected to tension) had not been rendered weaker by riveting.

The experiments made by myself were directed principally to two objects:—

I. To ascertain how far this value of f would be affected by changing the thickness of the metal, the other dimensions of the tube being the same.

II. To obtain the strength of tubes precisely similar to other tubes fixed on, — but proportionately less than the former in all their dimensions, as length, breadth, depth, and thickness, — in order to enable us to reason, as to strength from one size to another, with more certainty than hitherto, as mentioned before. Another object, not far pursued, was to seek for the proper proportion of metal in the top and bottom of the tube. Much more is required in this direction.

In the three series of experiments made the tubes were *rectangular*, and the dimensions and other values are given below.

Length of Tube.	Weight of Tube.	Distance between Supports.	Depth of Tube.	Breadth of Tube.	Thickness of Plates of Tube.	Last observed Deflection.	Corresponding Weight.	Breaking Weight.	Value of f, for Crushing Strain.
ft. in.	cwt. qrs.	ft. in.	Inches nearly	Inches nearly	Inches.	Inches.	Tons.	Tons.	Tons.
31 6	44 3	30 0	24	16	·525	3·03	56·3	57·5	19·17
31 6	24 1	30 0	24	16	·272	1·53	20·3	22·75	14·47
31 6	10 1	30 0	24	16	·124	1·20	5·04	5·53	7·74
	lbs. oz.						lbs.	lbs.	
8 2	78 13	7 6	6	4	·132	·66	9,416	9,976	23·17
8 2	38 11	7 6	6	4	·065	·32	2,696	3,156	15·31
8 2	...	7 6	6	4	...	...	...	...	...
4 $2\frac{1}{8}$	10 12	3 9	3	2	·061	·435	2,464	2,464	24·56
4 $3\frac{1}{4}$	4 15	3 9	3	2	·03	·13	560	672	13·42

The tube placed first in each series is intended to be proportional in every leading dimension, as distance between supports, breadth, depth, and thickness of metal, and any variations are allowed for in the computation. Thus the three first tubes of each series are intended to be similar; and in the same manner of the other tubes, &c.

Looking at the breaking-weights of the tubes varying only in thickness, we find a great falling off in the strength of the thinner ones; and the values of f shew that in these — the thickness of the plates being ·525, ·272, ·124 inch — the resistance, per square inch, will be 19·17, 14·47, and 7·74

tons respectively. The breaking-weights here employed do not include the pressure from the weight of the beam.

The value of f is usually constant in questions on the strength of bodies of the same nature, and represents the tensile strength of the material, but it appears from these experiments that it is variable in tubes, and represents their power to resist crippling. It depends upon the thickness of the matter in the tubes, when the depth or diameter is the same; or upon the thickness divided by the depth when that varies. The determination of the value of f, which can only be obtained by experiment, forms the chief obstacle to obtaining a formula for the strength of tubes of every form. When f is known the rest appears to depend upon received principles, and the computation of the strength may be made as in the "Application de la Mécanique" of Navier, part 1st, article iv.; or as in Papers of my own in the "Memoirs of the Literary and Philosophical Society of Manchester," vols. iv. and v., second series. I have, however, made for the present purpose further investigations on this subject, but defer giving them till additional information is obtained on the different points alluded to in this Report; and this may account for other omissions.

In the last table of experiments the tubes were devised to lessen or to avoid the anomalies which riveting introduces, in order to render the properties sought for more obvious. Hence, the results are somewhat higher than those which would be obtained by riveting as generally applied.

The tube, 31 feet 6 inches long, 24 cwt. 1 qr. weight, and ·272 inch in thickness of plates, was broken by crushing at the top with 22·75 tons. This tube was afterwards rendered straight, and had its weak top replaced by one of a given thickness, which I had obtained from computation; and the result was, that by a small addition of metal, applied in its proper proportion to the weakest part, the tube was increased

in strength from 22·75 tons to 32·53 tons; and the top and the bottom gave way together.

If it be determined to erect a bridge of tubes, I would beg to recommend that suspension-chains be employed as an auxiliary, otherwise great thickness of metal would be required to produce adequate stiffness and strength.

CHAPTER III.

EXPERIMENTS ON THE LARGE MODEL.

July 1846 to May 1847.

It is frequently difficult to retrace the successive steps by which any new principle has been matured, for every fact elicited appears at the time so important, and acquires such undue prominence, that the principle itself may be forgotten in the course of its developement. Such, however, was not the case in this instance, and it is highly interesting, now that the haven is reached, and the subject from more intimate acquaintance simplified, to mark the gradual progress, and preserve these embryo conceptions, of such novel structures. We cannot but participate with Mr. Stephenson in his unshaken confidence in the justice of the principles with which he had started in his inquiry, now confirmed by the test of experiment; nor can we fail to admire the zeal with which Mr. Fairbairn anticipates the result of the important investigation in which he was engaged, while his sanguine conclusions contrast forcibly with the abstract and minute deductions, the sceptical doubts and fears, of the exact Mathematician.

The publication of these Reports was an important epoch in the history of the tube. Public attention was now for the first time drawn to the subject, and the Directors of the Company were relieved from some anxiety by the more definite details submitted to them. As the other works throughout the line were now fast progressing, they became anxious

that operations at the bridges should be no longer delayed, and Mr. Stephenson was strongly urged to complete his plans; on the other hand, he was now incessantly harassed by the prodigious amount of other business arising out of the fatal glut of railway-bills that characterised this session, and was naturally anxious for some leisure hours to mature his design, and pursue still further his experimental inquiries.

The necessity for further experiments was obvious. Everybody had some doubts and fears to be overcome as soon as these details became known; dismal warnings came in on all hands, suggesting every imaginable apprehension, and Mr. Stephenson appeared at times disheartened when he withdrew, as was his daily custom, to give instructions on the subject, and to deliberate on the weighty difficulties that had to be encountered in his undertaking. Very few are aware of the painful anxiety that falls to the lot of the engineer in circumstances of such deep responsibility; he can be satisfied with no uncertainty or doubt,—and what other foundations were possible?

In preparing future experiments, Mr. Hodgkinson recommended the construction of a model-tube of such proportions, that while the sides were just thick enough to preserve the form of the tube, the top and bottom should be so adapted to each other as to be ready to fail simultaneously; he suggested that this top should then be replaced by circular tubes of wrought-iron of equal strength to resist crushing; and in order to investigate the necessary strength of these tubes, and the consequent saving in weight that was expected, he proposed to make a series of experiments upon the power of wrought-iron to resist a crushing force. Mr. Stephenson did not hesitate, with the approval of the Board, to sanction these inquiries, with some modifications, and Mr. Hodgkinson was desired to proceed with them, while Mr. Fairbairn was instructed to prepare the following model.

The experiments already described were decided on in May 1845, and finished in the course of the year; and in the month of December, sufficient information was obtained to induce Mr. Stephenson to construct, in conformity with the results, a model of very large dimensions, viz. one-sixth the lineal dimensions of the intended bridge, and corresponding, as nearly as possible, in every respect with what it was then supposed might be the best ultimate form to adopt. The proportions of this model were thoroughly discussed, and permission being obtained from the Board for that purpose, its construction was commenced in April 1846, at Millwall, near London, that it might progress and be tested under Mr. Stephenson's more immediate inspection. It was completed in July, and the experiments were immediately commenced.

The dimensions of the model were determined in reference to the requirements for the Britannia Bridge: every dimension being one-sixth of the eventual magnitude then thought necessary:

The Britannia Tube being 450 feet long, the clear span of the model between the bearings was $\frac{450}{6} = 75$ feet.

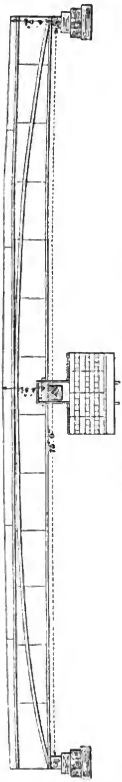
Similarly the depth at the centre was made $\frac{27}{6} =$ 4 ft. 6 in.

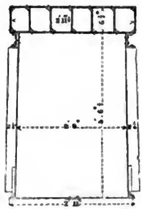
And the width .... $\frac{16}{6} =$ 2 feet 8 inches.

In settling the thickness of the plates the same proportion was maintained, *i. e.* six times the thickness of the plates of the model gives the thickness then proposed for the Britannia Tube. In enlarging the model to the actual size required, we have not only to make the plates six times as wide, but also six times as thick, hence it follows that the comparative sectional areas at any part of the model, and of the tube itself, would be as the square of 6; while the

comparative weights would be as the cube of 6; *i. e.* the weight of the tube would be 216 times as great as that of the model.

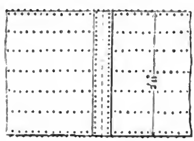
The elevation and transverse section at the centre of the model were as follows:—



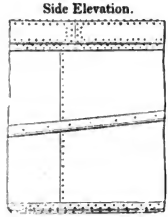


Its entire length was 77 feet 8 inches; between bearings, 75 feet; the top being cambered 6 inches at the centre, the end sections are 6 inches less in depth than the transverse one given above. The construction of this tube was particularly simple. The top consisted of six rectangular cells, $6\frac{1}{2}$ inches by 6 inches, the plates overlapping each other 1 inch. The joints, after the first fracture, were united by a cover.

Plan of the Top.



Side Elevation.



The cells were formed with a single angle-iron at each corner, the sectional area of which was

·175 inches (fig. 1), but the angle-iron uniting the sides with the top and bottom was rather stronger, having a sectional

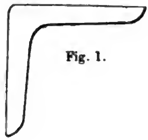


Fig. 1.

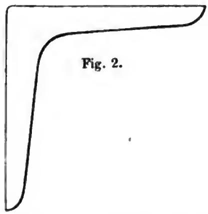
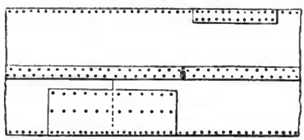


Fig. 2.

area of ·325 inches, as in fig. 2. The plates forming the sides were united by an overlap joint of 2 inches, and a single row of rivets; and wherever a break took place in the continuity of the angle-iron, a plate was riveted over the joint. The bottom was formed of two plates riveted to a strip 3 inches broad, running the whole length of the tube;

Plan of the Bottom.



and the transverse joints were covered by overlap plates 3 feet long, 1 foot 6 inches broad; they were unnecessarily thick, being nearly the same thickness as the bottom itself; whereas half this thickness would have been sufficient, there being two covers. In order to arrive at the correct thickness of the plates, eight or ten pieces were sheared from each plate, carefully flattened and laid together; the thickness of the whole pile was then accurately measured, and the individual thickness of each plate arrived at by dividing this

measurement by the number of plates in the pile. In this manner the following thicknesses were determined:—

	Thickness of each Plate.
A Plates of the Top	·179 inch.
B Plates of the Top	·151 ,,
× Plates of the Top	·141 ,,
S Plates or Sides	·1 ,,
D Plates, or Bottom Plates	·179 ,,
Longitudinal strip 3 in. broad, uniting the bottom plates	·330 ,,
Covers over the transverse joints in the bottom	·15 ,,
Thick plates in the sides extending 6 feet from each end	·212 ,,
Angle-iron in the top cells 1 in. × 1 in.	·175 ,,
Do. uniting the sides with the top and bottom $1\frac{1}{2} \times 1\frac{1}{2}$	·325 ,,
Arched flange on the side equal to a plate $4\frac{1}{4}$ in. broad, 80 ft. long	·3 ,,
Pillars at the ends 3′ 6″ high, 6″ broad	·4 ,,

The transverse sectional areas of the top, bottom, and sides, which were of constant thickness throughout, were also carefully determined as follows:—

Sectional Area of the Top.

	Ft.	In.	Inches.	Square Inches.
2 Angle-irons		3	× ·325	1·95
14 small do.		2	× ·175	4·90
A Top plate	2	11	× ·151	5·285
B Top plate	2	11	× ·153	5·355
X 7 Vertical plates		6	× ·141	5·922
Overlaps		4	× ·153	·612
Total				24·024

Sectional Area of the Bottom.

	Ft.	In.	Inches.	Square Inches.
2 Angle-irons		3	× ·325	1·95
Strap along the joint		3	× ·330	·99
Plate	2	11	× ·179	5·86
Total				8·8

Sectional Area of the Sides.

	Ft.	In.	Inches.	Square Inches.
2 Sides mean	3	9	× ·1 =	9·0
Total sectional area				41·824

The weight was calculated from these dimensions as follows:—

		lbs.
Weight between the supports, supposing no joints		10556
20 covering plates over the joints at bottom	$3' \times 1'\ 6'' \times \cdot 15 =$	540
Arched flanges on the sides..................		80
Overlap of side plates		60
Ditto of top plates		50
Rivets		500
Total weight between the supports		5 tons, 5 cwt. 23 lbs.

As it was of considerable importance to obtain the accurate weight, the tube, after the first experiment, was actually weighed in steelyards, and was found to weigh 5 tons, 15 cwt.

	Tons	cwt.	qrs.	
From........	5	15	0	
Deduct	0	12	2	weight beyond the supports.
Weight between the supports	5	2	2	

which agrees, within 3 cwt., with the weight estimated above.

In so important an experiment it was determined to break the tube by actual weight suspended from its centre without the medium of a lever; and for this purpose, a strong wrought-iron bar was passed through the sides of the beam, resting on a stiff plate on the bottom, the bar projecting on either side.

From these projections of the bar a strong timber platform was suspended by iron rods, and on this platform the weight, consisting of pigs of iron and boiler plates, was piled.

The weights were laid on about one ton at a time, and the deflection observed; the permanent set was also recorded, when the weight was removed.

Unfortunately, the deflections can be but little relied on in the first experiments, for the tube rested at each end on a timber pier which compressed considerably under the weight, and was materially affected by rain or sunshine, while the deflections were read at the centre from an independent

straight edge supported on uprights on either side of the tube, which was subject to similar variations.

The ground, moreover, under the timber piers, was swampy and bad.

In future experiments these objections were removed.

Weights applied.	Total Weight.	Deflection.
Tons.	Tons.	Inches.
·909	·909	·175
2·252	3·161	·275
2·256	5·417	·485
2·267	7·684	·635
2·178	9·862	·925
1·930	11·792	1·140
1·863	13·675	1·335
1·921	15·596	1·56
1·930	17·526	1·78
1·983	19·509	2·00
1·892	21·401	2·20
1·926	23·327	2·4
1·931	25·258	2·7
1·934	27·192	3·
2·077	29·269	3·4
1·017	30·286	3·575
2·239	32·525	3·726
2·001	34·526	4·375
1·999	35·525	

The ultimate deflection observed was 4·4 inches, and the permanent set, on the removal of 30·286 tons, was observed to be ·792 inches.

Fracture took place by the rending asunder of the bottom plates about 2 feet from the centre or suspended shackle. The bottom was a continuous chain 8·8 square inches in section, or 7·5 inches, deducting the rivet-holes; and this chain was so fairly and completely rent asunder, that it was evident that the chain had experienced a horizontal strain of sufficient magnitude to sever this area of iron.

It will be better to record all the experiments made with this tube *seriatim*, and ultimately to call attention to the

deductions to be drawn from the final result, although such deductions were necessarily progressive, and each alteration of the model was in accordance with the views derived from the previous experiments.

The cause of the fracture of the bottom in this experiment is evident. It had been intended, that, to prevent the breaking of the top, the sectional area of the top should be considerably greater than that of the bottom; and in this tube they were in the ratio of 24 to 9·2, or 2·6 to 1 nearly. This great difference was not, however, warranted even by the previous thin models.

Experiment II.

July 31st, 1846.

Wrought-iron work has one important advantage, that there is no difficulty in removing any particular plates and substituting fresh ones, by merely cutting out the rivets, without damaging the rest of the work; and precautions were taken at the time of fracture to prevent the tube from falling low enough to damage the top to any serious extent.

The result of the previous experiment led to an increase of the bottom of the tube, in the sectional area, from 8·819 inches to 12·8 inches, by the addition of two strips on the underside at the middle. These strips were each about 6½ inches broad, five-sixteenths of an inch thick, and 40 feet long, weighing 267 lbs. each. Extra covering plates, weighing about 28 lbs., were also necessary to repair the fracture.

The general proportions were, therefore, now as follows:—

Sectional area of top, as before, 24·024; of sides, 9·6; of bottom, 12·8 square inches, for 40 feet of the centre, but 8·8 square inches, throughout the remainder, as before; the weight between the supports was 5 tons, 7 cwt. 2 qrs.

This tube was broken by a weight of 43·3 tons, suspended as before from the centre.

The deflections were very regular so far as the imperfect means of determining them may be relied on; being at the centre one-tenth of an inch for each ton of additional weight.

Experiment 2.

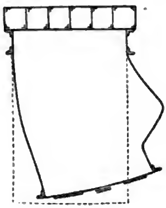
Weights Applied.	Total Weight.	Deflections.
Tons.	Tons.	Inches.
·915	·915	0·30
6·865	7 780	0·53
3·200	10·980	0·90
2·340	13·320	1·12
2·298	15·618	1·32
2·309	17·927	1·94
2·313	20·246	2·00
3·644	23·884	2·26
2·342	26·226	2·48
2·308	28·534	2·65
2·332	30·866	2·85
2·334	33·200	3·05
3·650	36·850	3·4
		3·5
1·108	37·958	3·6
1·109	39·067	3·66
2·794	41·861	4·06
·739	42·600	4·15
·750	43·35	

The ultimate deflection was 4·25 inches, but the permanent set could not be determined.

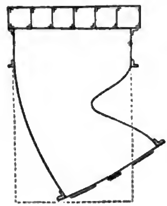
The tube failed by distortion of the sides, which had probably been a little damaged on the first experiment. The tendency of the top and bottom to approach each other was not resisted by any pillars on the sides, except at the extremities, over the bearings. The sides consequently collapsed, and the ultimate effect of the collapse, which extended along the central portions where there were no pillars, was to distort the extremity sideways, the ends being prevented from so

doing by the thickness of the plates and the pillars introduced there.

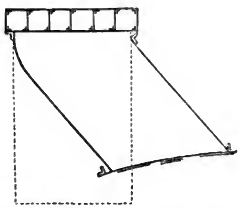
At A B.

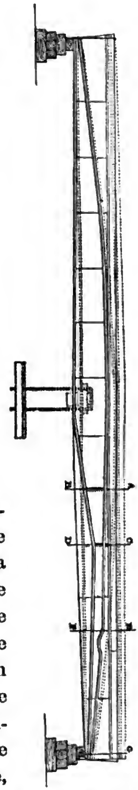


At C D.



At G H.





A diagonal wave of puckering clearly exposed the line of severest strain; this became evident, even in the first experiment with a weight of 29 tons, and at the time of fracture was so considerable, that it appeared probable that it would prove the cause of failure; at the time of failure in the first experiment its depth was estimated at nearly 3 inches. In the second experiment the undulation became evident with 25 tons, more especially near the centre, and increased up to the time of failure, when the side suddenly collapsed, and, simultaneously with the collapse, the tube rolled over to the opposite side, the

top and bottom remaining quite perfect, but the damaged side being torn asunder at two of the joints in the side plates. The other side was not permanently damaged, except from the secondary effect of rolling over, which injured the junction with the top and bottom.

A further and interesting proof of the position of the strain in the sides was afforded by the condition of the paint, which, being less elastic than the metal, did not follow the extending particles of iron, but peeled away in long flakes along the line of maximum tension, and parallel to its direction.*

Experiment III.

September 1846.

With respect to the requisite proportion between the top and the bottom of the tube, no information was acquired from the last experiment, but with respect to the sides, the result was of the greatest value. It was evident that they were exposed to unfair strain from the change of shape consequent on the tendency of the top and bottom to come together, the plates being strong enough, if they could but be kept in shape; and it was therefore determined, in this experiment, to modify the construction of the sides, leaving the top and bottom nearly as before. This was done by the addition of pillars of angle-iron throughout, of the whole height of the sides, riveted to them, having the effect of stiffening them, and at the same time of keeping the top and bottom in place. They were placed inside the tube, 2 feet

* Some pigment less elastic than paint, and with less adhesive properties, might probably be advantageously used for recording the motions of the particles of any material under strain.

apart; there were 33 on each side, and their section was somewhat greater than in fig. 2, page 159. They were prototypes of the T-iron pillars used in the large tubes, and their importance had not become prominent in smaller models.

The tube was, at the same time, thoroughly repaired, and the sides and top straightened, the damaged joints in the side being made good by plates riveted over them; and in order to give greater stability, a braced frame was inserted in each end.

The weight between the supports was now 5 tons, 9 cwt. 1 qr., other dimensions remaining as in the last experiment.

Weights Applied.	Total Weight.	Deflections.
Tons.	Tons.	Inches.
10·206	10·206	·875
10·984	21·19	1·875
10·093	31·283	2·875
3·327	34·61	3·25

This weight of 34·61 tons was left on for 19 hours, with no symptom of failure, the deflection at the end of that time having increased to 3·5 inches. Sixteen tons were then removed; and thus the tube was left till the 9th of September, when they were again laid on, and the experiment continued as follows:

	34·61	3·75
2·074	36·684	3·875
2·072	38·756	4·
2·053	40·809	4·16
2·09	42·899	4·31
1·036	43·935	4·44
1·025	44·960	4·5
1·037	45·997	4·62
1·036	47·033	4·69
1·037	48·070	4·8
1·035	49·105	4·94

In three minutes this deflection increased to 5·0 inches, the paint slightly peeling off from the bottom of the tube.

Tons.	Tons.	Inches.
.975	50·080	5·1
1·030	51·110	5·27
1·024	52·134	5·38
1·031	53·165	5·5
1·035	54·200	

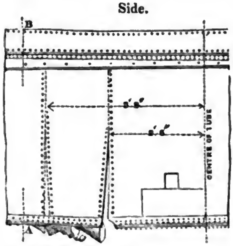
Buckling in the sides near the top was now becoming visible, when the shackle supporting the weights gave way, and the experiment was discontinued. When the tube was relieved of the weight, its elasticity was found to have been so little injured, that it very nearly resumed the set of 1·96 inches, which it had acquired in the previous experiments.

On September 9th the experiment was continued, by the last weight, 54·2 tons, being replaced; but the deflections caused by the following weights could not be accurately ascertained.

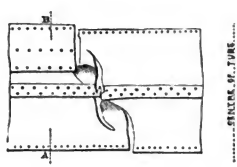
Weights applied.	Total Weight.	Remarks.
Tons.	Tons.	
	54·2	
1·018	55·218	Buckling observed to be increasing on the side; top slightly distorted; tube complaining a great deal at all parts.
1·077	56·295	

With this weight the tube failed, by the tearing asunder of the bottom, the fracture running up a line of rivets on each side.

Side.



Bottom.



Several rivets in the top plates at the middle of the tube were sheared off by the compression, and the holes had slid over each other nearly three-tenths of an inch, the plates being overlapped, and not butt-jointed, as they should have been.

The original curvature of the tube (6 inches) had been reduced by the previous breaking and repairs to a little less than 5 inches; consequently the top of the tube was below the horizontal line some time before the failure.

Experiment IV.

October 13th, 1846.

The result of the last experiment illustrated the importance of the pillars in the sides, as, with an addition of only 2 cwt. to the weight of the tube, the top and bottom remained precisely the same as before, while the breaking-weight was increased from 43 tons to nearly 56·5 tons, or more than ten times its own weight. This thin model, therefore, was capable of carrying 113 tons equally distributed over it, and was of itself sufficient for railway traffic, as the weight of a line of locomotives upon it would only be 75 tons.

As fracture had again occurred at the bottom, this portion only was strengthened, which was done through a length of 40 feet 8 inches of the middle, by removing the strips added throughout that space in Experiment II., and substituting stronger strips 9 inches broad and half-an-inch thick.

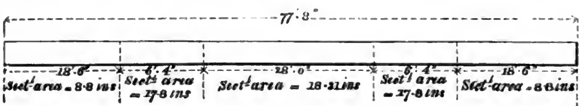
The central bottom strip also was replaced for a length of 28 feet at the centre, by a stronger strip 4 inches broad and three-eighths of an inch thick.

The top was so much damaged, however, that it was necessary to renew the upper plates for 30 feet at the centre; and butt-joints, with a covering-strip $3\frac{1}{2}$ inches broad, were

substituted for the lap-joints; but the new plates and covers were of precisely the same thickness as those removed. The lower and vertical plates of the top cells were retained and carefully straightened. The old angle-iron was replaced by similar new angle-iron. The damaged plates on the sides were replaced by new plates similar in every respect.

At the same time, a more accurate method of determining the deflection was adopted; which, it was thought, would be unaffected by any failure in the supports, &c. A fine wire, attached to one end of the tube, passing over a pulley at the other end, was retained in a constant state of tension by a weight; and the deflections were read on a scale attached to the centre of the tube. A comparison of this improved method with that before used shewed errors of more than an inch in the latter.

The sectional area of the top was still 24·024. The sectional area of the bottom was as under:—



The weight between the supports was 5 tons, 14 cwt.

This weight might evidently have been somewhat reduced without weakening the tube as regards bearing a weight at the centre, for the sectional area of the top remained constant throughout; whereas it is evident it might be reduced towards the ends in the same proportion as the bottom, and the bottom itself might still further have been reduced at the extremities.

The following was the result of the experiment:—

Weights applied.	Total Weight.	Deflections.	Remarks.
Tons.	Tons.	Inches.	
·909	·909		
26·612	27·521		
1·894	29·415	1·58	
1·886	31·301	1·73	
1·896	33·197	1·83	Left for an hour with no increase of deflection.
2·001	35·198	1·92	
2·261	37·459	2·08	
2·130	39·589	2·19	
2·079	41·668	2·33	
2·586	44·254	2·50	
2·085	46·339	2·63	
2·038	48·377	2·78	
2·108	50·485	2·97	Crippled a little at top of side, near the centre.
2·039	52·524	3·1	
2·114	54·638	3·28	Deflections slightly increased in five minutes.
2·075	56·713	3·48	Cracking, buckle near the top increasing.
·952	57·665	3·57	
1·046	58·711	3·68	
1·051	59·762	3·78	
1·015	60·777	3·88	
1·448	62·225	4·09	

It being now evening, the experiment was discontinued, and the tube partially relieved of the weight by the screw-jacks. On resuming it next morning, the deflection was found to have decreased by about ·1 inch, which was doubtless due to the difference in temperature between evening and morning—a cause which regularly produces similar effects on the large tubes.

·976	63·201	4·0	
·959	64·160	4·05	Undulations in the sides becoming very evident.
·98	65·140	4·12	
·984	66·124	4·18	Deflection increased without additional weight to 4·3 immediately before fracture.

With this weight of 66·124 tons the tube again tore

asunder at the bottom near the centre, and fell about 7 inches on to balks placed there for that purpose.

The breaking-weight in this experiment, as compared with that in Nos. I. and III. ought to be proportional to the increased area of the bottom. Compared with Experiment III.:

	Inches.		Inches.		Tons.		Tons.
As	12·8	:	18·3	: :	56·5	:	80·8

Compared with Experiment I.:

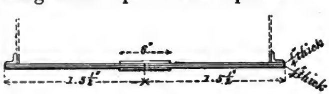
As	8·8	:	18·3	: :	35·	:	72·8

The mean of the two being 76·8 tons; whereas the actual breaking-weight was but 66 tons. However, on examination, the strips added to the bottom were found to be iron of very inferior quality.

Experiment V.

December 8th, 1846.

Failure having again taken place at the bottom, it was evident the proper proportion between it and the top was not yet obtained; and it was determined again to add to its sectional area at the centre portion of the tube. This was effected by using double quarter-inch plates.

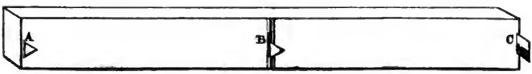


	Ft. In.		Ft. In.		
Two of these plates were	38 4	long, by	1 5½	broad.	
The two others	40 6	long, by	1 5½	broad.	
The two centre covering strips were	40 6	long, by	6	broad,	¼ in. thick.

We have therefore

Section of top as before	24	square inches.
Sides as before	9·6	,,
Sectional area of the bottom	22·45	,,
Weight between the supports	6 tons, 3 cwt.	

The method just described of taking deflections with a stretched wire was found to be imperfect, as the wire was much disturbed by wind; and a very effective plan was now adopted similar in principle to that since employed with the large tubes for the bridge.



A board with the upper half white and the lower black was secured to the tube at C. A bracket with a horizontal top was fixed at about the same level at A; and a similar white bracket moved vertically in a groove, with a scale, at the centre B. Observations were taken by looking from A to C. As the tube deflected, the centre bracket descended, exposing the black part of C. The space through which the centre bracket had to be raised, to just hide the black portion, was the required deflection.

The experiment made with this tube was a triple one:

First, Weights were placed on successively, amounting at last to 58 tons, and gradually removed to obtain the permanent sets.

Secondly, The tube was laid on its side, to test it laterally, and loaded with 12 tons, the experiment being then discontinued, before any injury could have happened to the tube.

Thirdly, It was replaced in its original position, and broken with 68½ tons.

Throughout this experiment considerable pains were taken to ascertain correctly the deflection and permanent set. The deflection of the tube on its side from its own weight was directly obtained by comparison with observations on the tube in its natural position.

I. The weights applied and removed were as follows :—

Weights applied.	Total Weight.	Deflections.	Remarks.
Tons.	Tons.	Inches.	
·909	·909		
22·271	23·18	1·02	
10·84	34·02	1·75	
4·904	38·924	1·98	
3·608	42·532	2·20	
4·150	46·682	2·45	Side bulging slightly.
4·266	50·948	2·70	
4·500	55·448	3·08	
2·144	57·592	3·20	

This last weight remained on for eighteen hours, during which time the deflection increased to 3·35, with no symptom of failure, after which it was taken off, and the deflections were observed as follows :—

Weights removed.	Total Weight suspended.	Deflections.
Tons.	Tons.	Inches.
	57·592	3·35
6·768	50·824	3·15
4·278	46·546	3·00
4·399	42·147	2·83
4·169	37·978	2·61
4·655	33·323	2·43
6·056	27·267	2·08
5·080	22·187	1·83
4·965	17·222	1·58
5·785	11·437	1·22
5·543	5·894	0·92
1·458	4·436	0·45
4·436	0·0	0·43

In ten minutes the deflection became, and permanently remained at, 0·15 inch. In these deflections no account is taken of the effects of local changes of temperature on the tube, which in the large tube have been found to produce considerable curvature. In other respects these deflections

are most carefully recorded. The temperature, moreover, was very constant at the time.

December 9, 1846.

II. The tube on its side.

The effect of wind on the tube at so great a height had always led to some apprehension. It was, moreover, found that a single person, by isochronous pressures against the tube, could produce very considerable vibrations. It was important to determine, therefore, the lateral strength of the model without carrying on the experiment far enough to damage it.

In its new position, the top and bottom were only one-tenth of an inch thick, while what had formerly been the top and bottom acted as sides, so that it was now merely two strong vertical ribs, with very small upper and lower flanges, not calculated to resist much strain. In such a beam, other dimensions remaining constant, the strength would be as the square of the depth, whereas with the same tube placed in its normal position the strength would be simply as the depth.

The following was the result of the experiment:—

Weights applied.	Total Weight.	Deflections.	Remarks.
Tons.	Tons.	Inches.	
0·	0·	·85	From its own weight.
·969	·969	·90	
1·973	2·942	1·13	
1·121	4·063	1·50	
1·136	5·199	1·70	
·996	6·195	1·87	
·967	7·162	2·00	Buckling along the top.
·962	8·124	2·28	
·958	9·082	2·50	
·946	10·028	2·66	Top side in waves; buckling very much.
·973	11·001	3·14	Taking off weight gave permanent set ·09.
·954	11·955	3·21	Increased in 10 minutes to 3·35.

This weight of twelve tons was not increased, through fear of damaging the tube: the experiment being discontinued, it recovered its original shape within ·1 inch.

III. The model being replaced in its natural position, was loaded and broken as follows:—

Weights applied.	Total Weight.	Deflections.
Tons.	Tons.	Inches.
1·	1·	
13·112	14·112	·7
11·667	25·779	1·32
11·323	37·102	1·92
9·845	46·947	2·39
5·305	52·252	2·78

In this state it was left till December 14th, with no increase of deflection. The weather was at the time very cold, the thermometer being 22°. The directions of the lines of tension in the sides of the tube were very evident, and they were marked with chalk for comparison in future experiments.

December 14.

	52·252	2·78
3·84	56·092	2·95
3·986	60·078	3·18

This weight was left suspended till the following day, when the deflection was found to have increased to 3·2; but between that day and the 23rd, when the experiment was resumed and completed, there was no further increase.

December 23.

	60·078	3·20
1·922	62·000	3·38
2·140	64·14	3·48
2·38	66·52	3·7
·932	67·452	3·81
1·196	68·648	

With this weight the tube broke, after sustaining it about three minutes.

The deflections throughout were well recorded, and average ·1 inch for every two tons.

With the additional area that had been given to the section of the bottom, it was expected this experiment would end in the top being crushed, and there was no evidence that this would not have been the case if the strengthening of the bottom had been continued far enough on each side of the centre; for though the tube failed, by tearing across the bottom, it was at a distance of 21 feet 4 inches from the centre, where the sectional area was, as in Experiment I., only 8·8 inches. Now, neglecting the weight of the beam and the camber in the depth, the strain at P, where fracture took place, is to the strain at C, where the weight was suspended,—

C P A

∧ 37′ 6″ × 21′ 4″ × 16′ 2″ ∧

as	A P	:	A C
or, as	16·2	:	37·6
or, as	1	:	2·32

In order that the beam might be equally strong at these two points, the section at P should therefore be equal to $\frac{1}{2·32}$ of the section at centre.

But the section at P was to the section at C as 8·8 : 22·45
or, as 1 : 2·55

It was therefore only $\frac{1}{2·55}$ of the section at the centre, and was consequently the weakest point, while, moreover, there is some increase of strain at P from the decrease in depth there.

Experiment VI.

The sixth and last experiment was made on the 15th of April, 1847. It is the most important of all the experiments, although it was made too late to be of much service in assisting in the design for the tubes, but the result was an interesting confirmation of the principles there acted upon.

In repairing the bottom of the tube for this experiment, it was not thought necessary to interfere with the section at the centre, which remained, as before, 22·45 square inches; but the additions were now carried farther on either side of the centre, by strips riveted to the old plates, which were also replaced at the fractures.

The principal object in view was still the determination of the proper proportion between the top and bottom. It was evident from the previous experiments that the top was nearly strained to its ultimate capability; it was accordingly still left as in the first experiment.

The sides, moreover, which, with the exception of the addition of pillars, remained as at first, gave evidences of very severe stress; and it may fairly be concluded that in the present model, which was the result of all the alterations made since the first experiment, we had a 75-feet tubular beam, with all its parts in good proportion for maximum strength, for the support of a load at its centre.

The weight between the supports was now 6 tons 5 cwt. 2 qrs. The total weight was 6 tons 18 cwt.

As regards the central sectional areas, we have—

		Sq. Inches.
Top	2 angle-irons	1·95
	14 small do.	4·90
	Top plates	16·56
	Longitudinal strips......	·61
	Angle-iron arch	2·54
		26·56

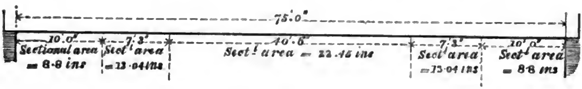
Sectional area — Sides at centre.

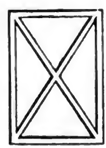
4 feet deep, $\frac{1}{10}$ inch thick = 9·6 sq. inches.

Ditto — Bottom at centre.

		Sq. Inches.
Two layers of plates, 2 ft. 11 in. broad, $\frac{1}{4}$ inch thick	=	17·5
Two strips, 6 inches wide, $\frac{1}{4}$ inch thick	=	3·0
Angle-irons		1·95
		22·45

The sectional area at other parts was as follows :—





There were frames over the bearings at each end, to preserve the shape, as in the margin, and angle-iron pillars, two feet apart, throughout the tube, attached to the sides, and an arch of angle-iron on each side as in the figure, page 158.

The angle-iron arch formed a portion of the tube from the commencement. It was originally added to the tube through a misunderstanding, and evidently affects none of the experiments, excepting the present one, where, being crushed together with the top, it is included in the sectional area.

	Tons	cwts.	qrs.
Total weight......................	6	18	2
Weight between the supports	6	5	2
Breaking-weight	86	4	0

Deflections at first about one-tenth of an inch for two tons, as in the last experiment. The deflections in the three last experiments, where they were better observed, were found to be very nearly proportional to the weight, even up to the point of failure.

Weights applied.	Total Weight.	Deflections.	Remarks.
Tons.	Tons.		
8·931	8·931	0·55	
7·041	15·971	0·78	
5·849	21·82	1·12	
5·980	27·800	1·48	
6·813	34·613	1·78	
6·592	41·205	2·12	
4·933	46·138	2·38	
5·049	51·187	2·70	
7·835	59·022	3·05	
2·165	61·187	3·23	
2·983	64·17	3·40	
2·100	66·27	3·58	
2·046	68·316	3·70	
2·098	70·414	3·78	Sides buckling near the middle.
1·857	72·271	3·88	
1·274	73·545	3·98	Sides at the end buckling a little.
1·283	74·828	4·10	Bottom, near the end, paint peeling off at the junction of the thick plates with the old bottom.
1·572	76·40	4·23	
1·24	77·64	4·33	Top uninjured.
1·414	79·054	4·47	Top beginning to shew undulation at the centre.
1·032	80·086	4·55	Sides buckling.
1·958	82·044	4·62	
1·204	83·248	4·72	
1·203	84·451	4·81	
1·661	86·112	4·88	

The weight here touched the ground; it was partly removed, and the ground dug out below the frame to give it play. After which, April 16th,—

	84·451	4·88	
1·661	86·112		

With this weight of 86·112 tons failure took place by the top crushing near the centre, at a part that had been previously injured but thoroughly repaired. The weight was not all suspended, but about eight tons were placed at the top. On examination the tube was found to be greatly strained all over; the undulations in the sides, which had at first

appeared near the centre, and were always greatest there, became, before failure, very evident from end to end; they formed angles of about 45° with the line of the bottom, and being longer than the diagonals of the parallelograms formed by the angle-iron pillars, they consequently ran across them.

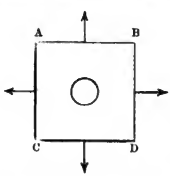
In conclusion, we may here recapitulate a few important phenomena in connexion with these experiments. With respect to the strength of the bottom of a tube, it is evident it has to be treated as a chain of plates riveted together; the number of rivets and the thickness of the covers over the bottom joints should be designed with this object. In the model tube the covers were 3 feet long and 18 inches broad, and were unnecessarily heavy, being of the same thickness as the plates themselves. They were placed on each side the joint, the covers and the plates being equally weakened by the rivets. It is evident half this thickness would have been sufficient.

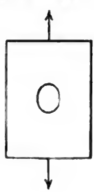
The continuous connexion of the side plates was secured by an overlap of about two inches; and wherever a break took place in the bottom angle-irons, a plate was riveted over the joint. The bottom was thus a continuous chain, and was fairly rent asunder, the line of fracture running through the rivets as the weakest section, but never shearing them. It was feared that, on account of the great width of the bottom, the strain which originates in the sides might not extend to the central portion. But in one of the experiments fracture began at the centre. In all cases, crackling noises preceded failure, which was otherwise rather sudden.

The top only failed once, viz. in the last experiment, though every time that failure took place at the bottom, from the tube descending through twelve or eighteen inches, the top was damaged to some extent, particularly in Experiment III. The ratio of the area of the top to that of the

bottom was in the last experiment as 26·5 : 22·45; but the bottom would still have sustained more weight. To have failed simultaneously, the ratio should have been as 27·5 : 22·45, or as 11 : 9 nearly. The top may be said to have been fairly crushed, although the result of the crushing was a buckling up of the plates and angle-iron. The failure was not so sudden as in failures of the bottom.

The side plates alone were unaltered throughout all the experiments. It is evident that whatever compression and extension took place in the top and bottom, these strains were solely induced by the action of the sides, through which, as a whole, the same strain must have been conveyed. The effect of this strain in the first experiment was to draw the sides into waves; now the sides, after bearing thirty-two tons in the first experiment, failed in the second with forty-three tons, by an increase of this same buckling, which allowing the top and bottom to approach nearer each other, diminished the depth and the strength of the tube. To illustrate this buckling, let A B C D be a square plate of any elastic material, Indian rubber, for instance, and drawn on every side in the direction of the arrows, a circle, inscribed at the centre, will still retain its form and merely become enlarged. Taking away the top and bottom tensions, the plate will become elongated and the circle an oval. Again, each particle, as it is more remote from the line of tension, will be less extended than its neighbour, and in endeavouring to restore the destroyed equilibrium in its elasticity, each particle will be drawn to a position nearer the line of tension, thus a wave or roll of pucker will be formed on





either side, as may be illustrated by stretching a piece of cloth.

The presence of these waves was, therefore, an evidence of tension and the index of its direction.

It was evident, from these experiments, that the tension throughout the bottom and the compression throughout the top stood in the relation of action and reaction to each other, the diagonal strain in the sides being the medium of communication. The necessity of vertical pillars thus became apparent, the sides themselves being too thin to resist the tendency of the top and bottom to approach each other from this diagonal strain. The great strain passing through the sides at the time of fracture in the last experiment was strikingly manifested by the undulations in the plates; and indeed, so equally distributed did the strains at this time appear about every part of the centre of the model, that it was impossible to surmise where failure would begin, till the tube, crushing at the top with a crackling noise, became more and more distorted, and at length failed suddenly, depositing its suspended burden on the ground.

CHAPTER IV.

DEDUCTIONS FROM THE PRELIMINARY EXPERIMENTS.

THE magnificent model described in the last chapter failed at length from the crushing of the top, after carrying a greater weight than even a double line of locomotives throughout its entire length. Nothing could be more satisfactory than this result; an addition of material of only one ton to a beam weighing originally only $5\frac{1}{4}$ tons, having increased the breaking-weight from $35\frac{1}{2}$ tons to upwards of 86!

A more striking example of the importance of the proper distribution of material can scarcely be imagined, and the modifications that led to such a result have been consequently minutely detailed.

The following analysis will be convenient for reference:—

Length of the model, 78 feet 8 inches; length between supports, 75 feet; extreme depth, 54 inches; breadth, 32 inches; sides, $\frac{1}{10}$ inch thick.

Date.	Weight between Supports.			Sect. Area of Top.	Sect. Area of Bottom.	Last obsd. Deflection.	Breaking-weight at Centre.	Remarks and Cause of Failure.
	Tons	cwt.	qr.	Sq. Inches.	Sq. Inches.	Inches.	Tons.	
1846. July 11	5	2	2	24·024	8·8	4·37	35·5 or 38 including ½ its own weight.	Tube uniform throughout. Bottom tore asunder.
Aug. 1	5	7	2	24·024	12·8	4·11	43·3 or 46	Bottom strengthened 40 feet at centre. Sides distorted.
Sept. 10	5	9	1	24·024	12·8	5·68	56·3 or 59	Sides stiffened with angle-irons. Bottom tore asunder.
Oct. 16	5	14	0	24·024	18·3	4·25	66·1 or 69	Bottom again strengthened for 40 feet. Tore asunder.
Dec. 11	6	3	0	24·024	22·4	3·81	68·5 or 71	Bottom increased in strength. Tore asunder 21 feet from the centre.
1847. April 16	6	5	0	26·5 including the arch of T iron which was crushed	22·4	4·88	86·1 or 89·24	The strengthening of the bottom continued farther from the centre. Top crushed.

In calculating the sectional areas of the top and bottom in the above table no deduction has been made for the weakening of the plates by the holes for the rivets,—and those experiments only are included in which the tube was actually broken. The deflections recorded are quite untrustworthy in the three first experiments.

We have seen (page 113), that in the formula

$$W = \frac{a\,d}{l}\,C,$$

for the strength of similar tubes, it is much more convenient to represent by a the sectional area of either the top or bottom alone, and to derive constants for each. We have in the large model most valuable data for deducing constants of great practical use on these assumptions.

These may evidently be at once arrived at by transposing C in the above equation, which gives

$$C = \frac{W\,l}{d\,a}.$$

But so important is it that the origin of constants in such daily use by the workman should be most clearly understood, that, for the benefit of some readers who object to x's and y's, it may be useful to deduce, in conversational language, from the last experiment on the large model,—

First, The strain per square inch when the top failed.

Secondly, The strain per square inch when the bottom failed.

Thirdly, An arithmetical rule for the strength of all similar or analogous rectangular tubes, or wrought-iron T girders.

To recapitulate the elements of the model, we have, in the last experiment,—

Length between supports, 75 feet.

Extreme depth, 54 inches, or 51 from the bottom to the centre of the cells.

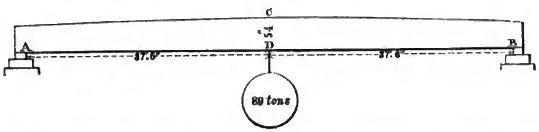
Sectional area of top, 26·5 square inches.
Sectional area of bottom, 22·45 square inches.
Weight of the tube between supports, 6·25 tons.
Breaking-weight (centre), 89·1 tons.

It is important to observe, that the iron used in the bottom was of very excellent quality.

In the first place, we must add to the breaking-weight of the tube half its own weight to obtain the actual strain at the centre; for the effect of a weight equally distributed over a beam, or of a beam's own weight, is at the centre the same as though half the weight were placed there.

The breaking-weight was, therefore, 86·1 tons + 3·127 = 89·2 tons, which we shall call 89 tons.

To find the strain per square inch at the top and bottom.



A B represents the model with 89 tons suspended at the centre. The weight on each of the supports A and B was consequently 44·5 tons. The reaction at A was consequently 44·5 tons at the end of the bent lever A D C, one arm of which, A D, is 37·5 feet long, or 450 inches; and the other, D C, 54 inches, or, rather, taking the depth to the centre of the top cells, 51 inches. The strain at C and D is increased in proportion to this leverage, or

As 51 in. : 450 in. : : 44·5 tons : 392·6 tons;

which is the actual tension at D and the actual compression at C, this tension and compression being resisted by a precisely similar strain arising from the reaction of 44·5 tons pressure at B.

Now there were 26·5 square inches in the top;

Therefore, $\frac{392 \cdot 6}{26 \cdot 5}$, or 14·8 tons, was the compression per square inch at the top, which crushed the material and destroyed the tube.

Moreover, there were 22·45 square inches in the bottom;

Therefore, $\frac{392 \cdot 6}{22 \cdot 45}$, or 17·5 tons, was the tensile strain at the bottom.

The bottom, however, did not fail in the experiment we are considering; we must, therefore, seek in the former experiments for the solution of our second question.

It is always well in such inquiries to neutralise the anomalies which invariably accompany all experiments, by taking the mean of several results. We were not enabled to do so with regard to the top of this model, but we may deduce a mean value of the tensile strain sustained by the bottom from the first, third, and fourth experiments, which all failed from the tearing asunder of the lower plates.

By the process adopted above we find,*

The tensile strain at fracture	in Experiment I.	= 19
	in Experiment III.	= 20·3
	in Experiment IV.	= 16·6
		55·9

* It will be convenient to deduce a simple expression for the strain at the centre of such a beam.

If W be the weight at the centre, the reaction at A and B is half W; the leverage, as before, is $\frac{450}{51}$ inches = 8·82.

The strain at the centre is consequently half W × 8·82 = 4·41 W; which, divided by the number of square inches (a), will give the strain per square inch, or,

$$\text{strain per square inch} = \frac{4 \cdot 41 \text{ W}}{a}$$

and, dividing this sum by 3, we have, for the mean tensile strength, strain = 18·6 tons per square inch of section.

Now, in the sixth experiment, where the top failed, the model was only sustaining 17·5; therefore the bottom was not on the point of failing, but would have supported about one-seventeenth more weight.

These two results are very valuable in all investigations of this nature, viz.:

That a cellular top similar to that in this model will fail with 14·8 tons per square inch of compression, and the bottom with 18·6 tons of tension.

We shall find hereafter the ultimate resistance of wrought-iron to compression and extension to be 16 tons and 20 tons respectively.

We may thus conclude, first, that no construction of top with equal section could bear much more compression than the cells in this model.

Thus, in Experiment I. strain per square inch at the bottom

$$= 4{\cdot}41\frac{38}{8{\cdot}8} = 19 \text{ tons.}$$

Or, as a general form, since in any tube the quantity 4·41 is represented by $\frac{l}{4\,d}$, where l = the length and d the depth, we have for the strain per square inch in the top or bottom of any analogous rectangular tube,

$$S = \frac{l\,W}{4\,a\,d}. \quad \text{Or, in words,}$$

Multiply the length by the weight at the centre, and divide this product by four times the depth, multiplied by the sectional area of the top or bottom, for the strain per square inch.

Again, we have from the above $\quad 4\,S = \frac{l\,W}{a\,d},$

and since $\quad W = \frac{a\,d}{l}C;$

therefore we have also $\quad C = \frac{l\,W}{a\,d},$

and consequently $\quad C = 4\,S.$

And, secondly, that the strength of the bottom falls somewhat short of the ultimate strength of the material, doubtless from the weakening of the plates by the rivets, the fracture always running through the rivet-holes.*

We shall complete the subject of the tensile strain in these experiments by ascertaining the amount of strain that produced the tearing asunder of the bottom in Experiment V., which did not fail at the centre, but at a distance of 21 feet

* The tensile strength per square inch of the bottom of the Millwall Tube, in Experiment I., was thus determined by Mr. Hodgkinson on the following data:—Depth to centre of top cells, 51 inches; semi-length, 450; sectional area of bottom, 5·348, deducting the rivet-holes, &c.

f being the strength per square inch of the metal, tensile strength of bottom plates $= 5{\cdot}348\,f$

$\therefore 51 \times 5{\cdot}348\,f = 272{\cdot}7\,f =$ moment of forces of bottom plates:— the sum of the thicknesses of the sides being $2 \times {\cdot}07 = {\cdot}14$ inches, and their depth, 51 inches. The moment of the strength of the sides being as f times the breadth multiplied by the square of the depth divided by 6, when the fulcrum or top offers equal resistance with the bottom; or by 3, when the top is supposed incompressible; *i. e.* as $\frac{f b d^2}{6}$

is $$\frac{f \times {\cdot}14 \times 51^2}{6} = 60{\cdot}69\,f;$$

or, $$\frac{f \times {\cdot}14 \times 51^2}{3} = 121{\cdot}38\,f$$

Taking the latter case, the fulcrum being on the edge, we get for the sum of the moments of the bottom and sides,

$$(272{\cdot}7 \times 121{\cdot}3)\,f = 394\,f.$$

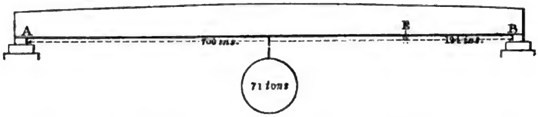
This moment being equal to the rectangle of half the length by half the breaking-weight,

$$= 450 \times 17{\cdot}75 = 7987{\cdot}5$$
$$\therefore 394\,f = 7987{\cdot}5$$
$$\therefore \quad f = \frac{7987{\cdot}5}{394} = 20{\cdot}27 \text{ tons, instead of } 18{\cdot}6 \text{ tons, as above.}$$

This result differs from the one obtained above, on account of the different data, the section of the bottom being taken at 5·348 instead of 8·8, and the sides being included in the sum of the moments.

The omission, however, of the sides as an element of strength gives increased security in practice; and in determining practical constants for similar tubes, the deduction of the rivet-holes will similarly affect all cases.

4 inches, or 256 inches from it; where the sectional area was only 8·8 square inches.



There being a camber or increase of depth of 6 inches at the centre, and the depth at the centre being, as before, 51 inches, the depth at E will be 47·6.

$$\text{The reaction at A is } \frac{71}{2} = 35{\cdot}5 \text{ tons.}$$

Therefore, for the strain at E,

$$\text{As } 47{\cdot}6 : 194 :: 35{\cdot}5 : 144{\cdot}7 \text{ tons.}$$

And the section being 8·8 square inches,

$$\text{The strain per square inch at E was } \frac{144{\cdot}7}{8{\cdot}8} = 16{\cdot}4 \text{ tons,}$$

or nearly the same strain as at the centre in Experiment IV.

For the strain at the centre at this time we have—

$$\text{As } 51 : 450 :: 35{\cdot}5 : 313{\cdot}2.$$

$$\text{And } \frac{313{\cdot}2}{22{\cdot}4} = 14 \text{ tons at C.}$$

The beam might, therefore, have been expected to break at E, but to have borne a greater weight than it did: now the sectional area was not gradually reduced, but suddenly changed at E; it failed, therefore, sooner on that account, in the same manner that a 2-inch bar, filed in one particular spot down to 1 inch square, will bear less at that spot than a bar of the same length 1 inch square throughout. The flexure is limited to the weakened spot. This is a case in which a beam is weakened by the addition of more material.

Thirdly, As to the arithmetical process of ascertaining the strength of any beam similar or analogous to this model, an ordinary rule-of-three statement is all that is required.

If we have any given tube of which we wish to know the breaking-weight at the centre, we have these compound proportions :—

As the amount of material to be compressed or extended in the model	:	the amount of material to be compressed or extended in the given tube	::	strength of model	:	strength of given tube.
As the leverage inducing the compression or extension in the given tube	:	the leverage inducing the compression or extension in the model				

Now if the given tube be exactly similar to the model, *i. e.* exactly twice, or thrice, or any number of times, the same depth, width, length, and thickness everywhere, the second ratio will be a ratio of equality; for since the leverage is the ratio of the depth to half the length, this ratio will remain unaltered if both the length and the depth are increased in the same proportion, as would be the case in similar tubes. Therefore, the strength of similar tubes is simply as the amount of material in their section.

Now if one tube is any number of times (say n times) both deeper and wider than another, its sides will be n times as high, and the top and bottom n times as wide. Its sectional area will therefore be n times as great. If we now increase the thickness also n times, the total sectional area will become n times n times as great, or n^2 times as great; *i. e.* the sectional area, and consequently the strength, of similar tubes will be as the square of their lineal dimensions.

Pursuing the analogy of similar tubes, it is evident that the weight of a bar or of a tube is proportionate to its sectional area, the length being constant; therefore, the weight of a tube n times as large in lineal dimensions as another will be

n^2 times as great per foot run. Now, a tube similar to another, but of n times the dimensions, will be also n times as long. Its weight will therefore be n times n^2 times as great, or n^3 times as great; *i. e.* the weight of similar tubes is as the cube of their lineal dimensions.*

It is seldom that we have to deal with beams exactly *similar* to the models for the intended bridges; and it is only with regard to such beams that these deductions are strictly true; but we may apply the following proportions for all *analogous* beams:

As the section of material in the top or bottom of the model	:	the section of material in the top or bottom of the given tube	∷	strength or breaking weight of the top or bottom of model	:	strength or breaking-weight of top or bottom of given tube.
As the leverage, or $\frac{l}{2d}$ in the given tube	:	the leverage, or $\frac{l}{2d}$ in the model				

And this will be a more convenient modification, since a cast-iron top is frequently applied to wrought-iron tubes, and the anomalies of tubes with thin tops may be specially dealt with.

Substituting the values of the expressions in the above proportions as derived from the last experiment with the

* It will be useful to have some simple rules for ascertaining the weight of tubes of wrought-iron. The following are on the assumption that one cubic foot of wrought-iron = 480 lbs.

First, The sectional area in square inches of any tube, bar, or plate of wrought-iron, multiplied by 10, will be the weight in pounds per yard of length.

Thus, an angle-iron whose section is 3·5 square inches, weighs 35 lbs. a yard.

In large masses or tubes, multiply the sectional area in square inches by the length in feet, and divide by 672 for the weight in tons.

To reduce cubic feet into tons, multiply by $1\frac{1}{3}$ and divide by 7.

A deduction of one-twentieth from these results will give a close approximation for cast-iron.

large model, we have, therefore, for the breaking-weight of any given tube, as regards the top,

$$W = \frac{\text{section of top of given tube} \times \text{depth}}{\text{length}} \times 59 \text{ tons.}$$

And as regards the bottom,

$$W = \frac{\text{section of the bottom} \times \text{depth}}{\text{length}} \times 74{\cdot}4 \text{ tons.}$$

These constants may be immediately deduced from the formula, $C = \frac{w\,l}{a\,d}$, and thus more briefly for the central breaking-weight of any analogous tube to this model, all the dimensions being in inches, we have—

$W = \frac{a\,d}{l}\,59$ tons, a being the sectional area of the top.

$W = \frac{a\,d}{l}\,74{\cdot}4$ tons, a being the sectional area of the bottom.

$W = \frac{a\,d}{l}\,26{\cdot}7$ tons, a being the sectional area of the whole tube.

The principles on which these analogies are founded are, it is hoped, explained in language sufficiently simple to leave no reader any excuse for employing such empirical formulæ without clearly understanding how far they may be applicable to his requirements.

We have remarked, in a note at page 188, that for the strain per square inch on either the top or bottom of a rectangular tube loaded at the centre with a weight W,

we have $$S = \frac{l\,W}{4\,a\,d};$$

and hence, in the formula $$W = \frac{a\,d}{l}\,C,$$

we have $C = S\,4$, *i. e.*, the constant for the breaking-weight, as regards either the top or the bottom of such a tube, is merely four times the ultimate strength of the material per square inch.

This furnishes a most convenient analogy for remembering these constants.

For instance, wrought-iron in the bottom of tubes bears (page 188) 18·6 tons per square inch; the constant is therefore $4 \times 18{\cdot}6 = 74{\cdot}4$, as above.

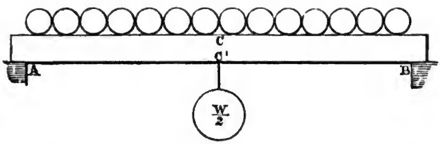
Similarly, it sustains 14·8 tons per square inch of compression; the constant is therefore 59·2, as before. And, moreover, we may find in the same way constants for any other material.

Thus, cast-iron bears tensilely about 6·5 tons per square inch, hence Mr. Hodgkinson's constant for cast-iron beams, $4 \times 6{\cdot}5 = 26$, these beams being perfectly analogous to the wrought-iron rectangular tube, and so for any other material used in constructing tubes.

	Tensile strength per square inch.	Compressive strength per square inch.	Constant to the bottom.	Constant to the top.	Weight of a cubic foot in lbs.
	Tons.	Tons.	Tons.	Tons.	
Brass, Cast..	8	4·6	32	18·4	525
Copper, Cast	8·5	..	34·1	..	538
Deal, Christiania middle	5·5	..	22	..	44
Glass, Plate	4·2	..	16·8	..	153
Cane	2·8	..	11·2	..	25
Lead, Cast..	·82	..	3·28	..	717
Mahogany ..	7·4	3·7	29·6	14·8	50
Steel, Soft ..	53·5	..	21·4	..	486
Wrought-iron	18·6	14·8	74·4	59·2	480
Cast-iron ..	6·5	48·	26	192·	450

We thus compare materials as to their adaptation to the construction of tubes; and the advantages of a cast-iron top and wrought-iron bottom are evident, while other combinations suggest themselves.

It must be remembered we are only treating of uniform tubes throughout, and of the strain at the centre section. With this limitation, we may still further simplify these practical rules for rapid approximation.



Let A B be a tube loaded equally all over, let W represent its own weight, together with that of its load, which is equivalent, as regards strain at the centre, to $\frac{W}{2}$ suspended there; then $\frac{W}{4}$ is the reaction at A.

Let the length l be taken in terms of the depth, that is, consider the depth unity, then S, the strain of compression at C, or of extension at C′, will be

$$\frac{W}{4} \times \frac{l}{2} = \frac{W\,l}{8};$$

i. e., the strain at top or bottom in the centre of any tube is merely one-eighth the length multiplied by the weight equally distributed along it, including its own weight. This is extremely convenient for calculation.

Supposing the Conway Tube uniform, its weight is about 1112 tons; the length being 16·8 times the depth, the strain on the bottom or top, from its own weight, will be

$$\frac{1112 \times 16{\cdot}8}{8} = 2335 \text{ tons};$$

there are 535 square inches in the bottom, therefore the strain per square inch is $\frac{2335}{535}$, or 4·36 tons.

A ton of load to the foot throughout would be 400 tons, the strain at the centre from this would be

$$\frac{400 \times 16{\cdot}8}{8}, \text{ or } 1{\cdot}5 \text{ ton per inch more.}$$

Again, tubes are generally constructed with their depth equal to about one-sixteenth of their length, in which case

$$l = 16, \text{ and } S = \frac{W\,16}{8} = 2\,W;$$

i. e. in such tubes the total strain at the centre, tending to crush the top or tear asunder the bottom, is merely twice the weight of the tube and its equally distributed load.

This is extremely convenient for rapid approximation, but if the depth is not one-sixteenth of the length, as, for example, one-fifteenth, then the strain found as above has only to be diminished in the proportion of 16 : 15.

Example.—Required approximately a section of bottom, section of top, and weight of a tube for 150 feet span, to carry 1 ton per foot.

Weight of the model 7 tons, weight of 150 feet span of a similar tube twice as large,

$$= 2^3 \times 7 = 56 \text{ tons.}$$

$$\text{Total load} = 150 + 56 = 206 \text{ tons.}$$

$$\text{Total strain at the centre} = 2 \times 206 = 412 \text{ tons.}$$

If, therefore, we intend adopting 5 tons per inch as the extreme strain,

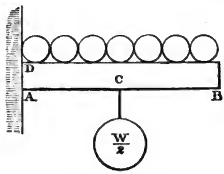
The section of the bottom will be $\frac{412}{5}$ = 82 square inches.
Section of top one-fourth more...... =102 ,, ,,
Depth = $\frac{150}{16}$.................... = 9 ft. 4 in.
Width at pleasure.
Area of the two sides about one-half the area of the bottom.

The weight may then be taken out more accurately, and the dimensions calculated more in detail.

Again, if we wanted to apply cast-iron to the top of this tube, assuming the weight of the tube, which is not large as compared with the whole weight, to remain unaltered by the change; —the compression at the top being 412 tons, if we

assume 15 tons per square inch as perfectly secure under the greatest strain to be expected, we have $\frac{412}{15} = 27{\cdot}3$ square inches as the section of the cast-iron top.

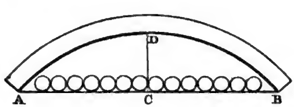
This relation between the strain and weight *equally distributed* over a girder, viz. $S = 2\,W$, is common to all structures on the principle of the beam where the depth is one-sixteenth of the length, and is applicable equally to the bracket by altering the constant 2 accordingly. For instance, in the bracket A B, let $W =$ the weight of the uniform bracket A B, together with that of its load equally distributed; the effect at A D is the same as though $\frac{W}{2}$ were suspended at the centre, A C being $= \frac{l}{2}$; l being taken in terms of the depth.



The strain at D, or A, is therefore $\frac{W}{2} \times \frac{l}{2} = \frac{W}{4}$; or one-fourth of the weight multiplied by the length. If the depth equal one-sixteenth of the depth, or $l = 16$, then $S = W\frac{16}{4} = 4\,W$.

We thus see the relation between the beam and the bracket, and have a ready means of arriving at the strain in the centre tower of the Britannia Bridge if the large tubes were cut through in the centre of each span.

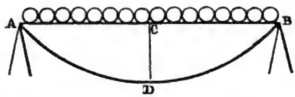
The strain caused at the summit of an arch, or at the centre of the chain of a suspension-bridge, by weight equally distributed, may be approximately ascertained by the same reasoning; for if the arch A B, instead of thrusting against



the abutments, be tied by a tension-rod A B, it becomes a beam of the depth D C; and the strain at D, from weight

equally distributed, is $S = \frac{W\,l}{8}$. And if the sectional area at D is a square inches, the compression from the equally distributed load W is $\frac{W\,l}{8} \div a = \frac{W\,l}{8\,a}$.

Similarly with a suspension-bridge,



If a strut A B, is used to resist the horizontal tension, A B becomes a beam of the depth C D, and the removal of these struts in both cases in no way affects the strain at D.

Example.—What strain approximately per inch would a line of locomotives, throughout the Menai Bridge, cause at the centre of the chains supporting one line of roadway, their section being 130 square inches, and the span being 580 feet? A line of locomotives weighs about a ton per foot run; total weight, 580 tons, and the length in terms of the depth $= \frac{580}{43} = 13{\cdot}5$.

$$S = \frac{W\,l}{8\,a} \text{ becomes}$$

$$S = \frac{580 \times 13{\cdot}5}{8 \times 130} = 7{\cdot}4 \text{ tons per square inch,}$$

in addition to the strain from the weight of the bridge.

It is better in every practical case to work out the problem for the particular case required rather than to employ a general formula: and it is in this respect that practical mechanics separates itself from theory; for the mathematician is always in search of the most general solution that his problem is capable of, whereas the engineer limits his attention to some particular case, and is often able to arrive at a solution by a much shorter route than by substitution or elimination in any complicated expression, while at the same time he can take into account circumstances peculiar to his par-

ticular problem, which are often incapable of being included in a formula essentially of general application.

We shall hereafter investigate theoretically the strength of the tube as a general problem, and the similarity of the results will shew how nearly we approach to all the requirements of practice by the most simple and elementary reasoning.

To recapitulate briefly the foregoing remarks; it would appear, that whenever space is crossed by any kind of structure, inducing no lateral thrust on the abutments;—that is, in all beams or tubes, whether round, oval, rectangular, or of whatever form, as also in all trussed roofs, trellis-bridges, bow-string arches, &c.;—the transverse strength of structures of similar section, but otherwise of any magnitude, is directly as their sectional area and depth, and inversely as their length.

We have been careful to observe that this is founded on the assumption of all these structures being of such dimensions as to preserve their form, and to fail by actual crushing or extension, and not by distortion.

We have seen also, that, even in the case of structures in which the horizontal thrust is resisted by the abutments, as in suspension-bridges and arches, this principle is practically available with a little modification. In fact, there can be no change in the direction of the vertical force at the supports without the intervention of a lever; and in the case of all these structures this force at each end of the beam is transferred into horizontal strain at the centre by an act of leverage; one arm being always the semi-length of the beam, and the other some fraction of the depth, which must in similar sections vary directly with the depth.

We have limited this law of beams to similar beams, to which alone it is strictly applicable. With a rectangular solid beam, however, the word "similar" is superfluous,

because we cannot increase the depth without increasing both the sectional area and depth in the same proportion, and the strength is therefore said to be as the square of the depth where the length and breadth remain unchanged; but with a rectangular tube we may increase the depth without increasing the section in the same proportion. For instance, we may double the depth, and in doing so add but little to the section, the sides being thin. In such a case the strength is not as the square of the depth, but nearly as the depth; but if we increase depth and thickness in the same proportion, the strength, with the tube as well as with the solid, is as the square of the depth, though it is evident that the constant in the formula for the solid will be of less value than in the formula for the hollow beam, *i.e.* we may have two beams of the same material, each of the same sectional area, and each of the same depth and length, and both of them subject to precisely the same law as to doubling or trebling all their dimensions; but the one may be three times as strong as the other under every corresponding magnitude, so that if the constant for one form were 9, the constant for the other would be 27.

Since the weight of the beam in similar beams is directly as the sectional area and the length, or as the cube of the length, while the strength is directly as the area and the depth, and inversely as the length, or only as the square of the length, no beam can be made of any form, but that some other similar beam of larger dimensions will fail by its own weight; there is thus a limit to the dimensions of any given form of beam or structure.

Thus it follows that solid square beams, or solid cylinders of any material, cannot be increased beyond a certain size. Any round tube, or square tube, has thus also a limit to its possible size.

But again; though this is true with respect to any assigned

form, we can assign no length so great but that we may make a form strong enough, theoretically, to support itself; for if we increase only the depth of a solid rectangular beam, then the weight will be simply as the depth, but the strength will be as the area and depth conjointly, or as the square of the depth, so that we may increase the strength *ad infinitum*, and consequently the length.

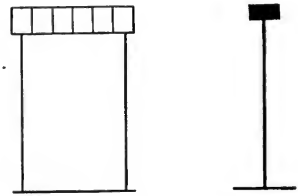
Similarly with a rectangular tube, we may increase the strength *ad infinitum*, by varying only the depth, for the weight will be increased in a less proportion than the strength. It becomes, therefore, very important to adopt a suitable prototype for any structure, which when enlarged to a similar tube of the dimensions required may not then be too weak.

The independent determination of the form of maximum strength for this prototype is involved in difficulties. As the depth is increased, the precautions requisite for maintaining the sides in shape become very formidable. The T-irons, and gussets, and stiffening plates for this purpose, in one of the Britannia Tubes, weigh 215 tons, or upwards of one-third of the whole weight of the sides.

The difficulty of determining the requisite thickness of the sides is very considerable, but it is evident they should be as thin as possible, consistent with the strain to which they are subject. If, in a tube with thick sides, we still retain the same depth, and the same quantity of material, but remove part of the material from the sides to the top and bottom, the strength will be greater than before, hence the constant for the latter form will be much greater than the constant for the first.

Perhaps the most simple method of arriving at the strength of any form generally, is to transfer all the material in a transverse section into a solid form, as we did in con-

sidering circular and elliptical tubes. The large model would then resolve itself into a solid girder of the following form,—



of which the strength might be obtained by determining the moment of inertia of the section, about an axis passing horizontally through its centre of gravity—(*See* Moseley's "Principles of Engineering," page 503,)—and employing the formula $W = \frac{f \cdot I}{l c} 4$, where W = the breaking-weight; I = the moment of inertia of the cross section; c the distance of the neutral surface from the flange which fails; l the length, and f the resistance to a direct tensile or compressive strain, determined from experiments on beams.

If the load be uniformly distributed, the breaking-weight is of course doubled, or the constant becomes 8. And if the weight of the beam be taken into the account, and represented by B, then—

For a load at the centre we have $W + \frac{B}{2} = \frac{2 f I}{l c}$.

For a load equally distributed $W + B = \frac{8 f I}{l c}$.

The value of I will be dependent on the section.

For rectangular solids $I = \frac{1}{12} b d^3$

For solid cylinders $I = \frac{1}{4} \pi r^4$.

For other forms the reader is referred to the complete treatise quoted above.

Some interesting considerations arise out of the assumptions on which we have been treating the strength of tubes. As regards the strain from the tube's own weight, which is the most important part of the strain in the Conway Bridge, it appears that this strain is not in the least altered by any alteration in the thickness of the plates, *i. e.* if it is five tons per square inch in the top, as at present constructed, it would still be five tons per square inch, whether the plates were all ten times as thick or ten times as thin; for the strain is inversely as the sectional area, and directly as the weight, hence if the plates were ten times as thick, the section would be ten times as great; but the weight would also be ten times as great, and the strain consequently would be still five tons per square inch, and similarly by lessening the thickness. The strength is, however, increased or diminished, as regards carrying any additional load, in direct proportion to the thickness; for if W be the absolute breaking-weight, and w the present weight of the tube, we have the load the bridge will carry at the centre $= W - \frac{w}{2}$; and if the plates are ten times as thick, then the load the bridge would carry would be,—

$$10\,W - \frac{10\,w}{2} = 10\left(W - \frac{w}{2}\right)$$

or ten times as great as before.

Again, if we make a tube similar to another, increasing every dimension except thickness, the absolute strength will be directly as the increase, that is to say, another tube twice the length, depth, and breadth of the Conway Bridge, but of the same thickness, would be just twice as strong; it would, however, be four times as heavy, and hence have four times the strain from its own weight, and would, therefore, soon come to a limit at which it would break itself.

This is evident by considering that with tubes of similar section, in which the thickness is not altered, the sectional area will be simply as the increase, and not as the square of the increase; the strength will therefore be simply as the lineal dimensions, instead of as their square.

But if we increase a tube in depth, and length, and width, and preserve its sectional area constant, that is, if the plates are thinner in the same proportion as the tube is enlarged, then the absolute strength of the enlarged tube *ad infinitum* will be the same as that of the first. So that by keeping the same sectional area as at Conway, and enlarging in the same proportions the length, breadth, and depth, we may make a tube of any length, equally strong, theoretically, with the Conway Tube. For the strength is directly as the sectional area into the depth, and inversely as the length, and the sectional area being constant, as well as the ratio $\frac{\text{depth}}{\text{length}}$, the strength will also be constant; but the weight of the tube, and hence the strain from its own weight, would increase as the length; and, consequently, if we suppose the strain to be five tons per square inch at present in the Conway Tube, another tube of the same sectional area, and of three and a half times the same length, breadth, and depth, would fail by its own weight. Such a tube would be 1400 feet long, and no increase of thickness would make such a tube bear more than its weight.

We have already alluded to the strength of the bamboo as an instructive natural example of the strength of a circular tube. The bones of animals are oval, the depth being always in the direction of the transverse strain. But the more special province of the bones appears to be their action as pillars, or struts, in forming immoveable fulcra for the reaction of the muscles; and since any yielding would involve a great increase of motion in the muscle itself, we find bone among the most incompressible of known substances.

The square form of stem characterises a very extensive natural family of plants — the labiate tribe, of which the beautiful dead nettle of the hedgerows is an example; though it is difficult to assign any mechanical reason for this peculiarity, which appears rather to be typical of the general developement of these plants. But in the feather-bearing part of the ordinary quill we have a most remarkable example of the strength of the rectangular form. Here, again, every dimension is tapered down in proportion to the strain, with an accuracy defying all analysis; the extended and compressed portions are composed of a horny substance of prodigious strength, though extremely light and elastic. The beam is not hollow, but to preserve its form it is filled with a pithy substance which replaces the clumsy gusset pieces and angle-irons of the tube without interfering with its pliability; the square shaft is peculiarly available for the attachment of the deep vanes which form the feather; and as the angular form would lacerate its active bearer, an exquisite transition to the circular quill at the base is another striking emblem of perfection. The imitation of such mechanics, so wonderfully adapted to such a medium, appears hopeless; but we are indebted to the flying philosopher, if his attempt only calls attention to such design, and induces us instructively to contemplate the beauty of a feather.

SECTION III.

GENERAL PRINCIPLES OF BEAMS.

CHAPTER I.

THE NATURE OF TRANSVERSE STRAIN.

It will be indispensable for the comprehension of our subsequent investigations, that the reader should be acquainted with the general principles on which the strength of beams is usually estimated.

We shall endeavour, first, for the benefit of the ordinary reader, to deduce and explain, without technicality, the laws of transverse strain, omitting nothing that is important to the full elucidation of the subject, but employing only geometrical reasoning of the most elementary character.

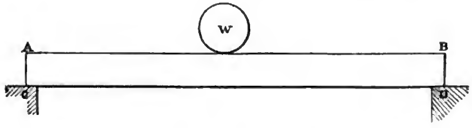
This method of treating the subject is almost invariable among engineers, but has not been adopted by any theoretical writers. It may so far be said to be novel.

The complete theory of a beam, in the present state of mechanical science, is involved in difficulties. The comparative amount of strain at the centre of the beam, where the strain is greatest, or at any other section, is easily determined, but the exact nature of the resistance of any given material almost defies mathematical investigation.

The top portion of any loaded beam is compressed, and the bottom extended; but to determine, under all conditions, at any given section of a beam, the exact proportions extended and compressed, and the varying amount of resistance offered to such extension and compression, at every layer of the depth, dependent as such resistance is on the nature of the elasticity of the material employed, is the general problem which has almost defied solution. The matter has been ably investigated by the most eminent mathematicians of all countries, and by no one more thoroughly than by Mr. Eaton Hodgkinson, whose valuable treatises on the subject are our standard text-books, and whose exact and numerous experiments, insufficient as they still are for the complete investigation of the subject, are yet our most important data.

But although the independent determination of the strength of any given beam is necessarily so complicated, there is little difficulty in arriving, from experiments on one beam, at the comparative properties of any other of the same material; and the matter is still further simplified if we have to deal only with similar beams: so that, in reality, the mathematician alone is at a loss, while the engineer finds little difficulty in obtaining a sufficient approximation for all his practical purposes.

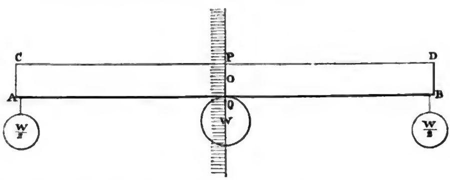
To illustrate this, let A B be a rectangular beam of



timber, supported at C D, and loaded with a weight, W, in the centre. Neglecting, for the present, the weight of the beam itself, let us consider the effect of the weight W.

In the first place, it is evident that the weight is wholly supported at the points C and D by the *vertical* reaction of the supports; and since the weight is at the centre, one-half of it is supported at C, and the other half at D, and the whole system is in equilibrium, the beam being in a state of strain.

Now, the beam will evidently be in precisely the same state of strain and equilibrium if it be inverted, and supported on W, in fig. 2, one-half the weight that was before at the centre being suspended at either end.



This simplifies our consideration of the subject; for let us now conceive the half C P to be placed in a vertical wall of water, which, by becoming solid ice, can in no way affect the strains in the remaining portion P D, to which we shall now confine our attention. We can thus form some idea of the general nature of the resistance at the section P Q. The portion P D will be a lever, the parts of the beam about P will be extended, and those about Q compressed; and at some point O, between P and Q, there will be neither extension nor compression. The weight is, in fact, acting at the end of the lever, P D, tending to turn it round the point O, but is restrained from doing so by the resistance to compression and extension of the portions between O Q and O P, respectively. Now, the action and reaction in these portions must be exactly equal: that is, the total resistance from compression must exactly equal

the total resistance from extension. Moreover, the strain at the section P Q is greater than at any other place, because the lever on which the weight is suspended, is there longest. If, therefore, the section of the beam were uniform throughout, it would break at P Q by sufficiently increasing the weight.

Again, it is evident that the horizontal layers of the beam near to P and Q will be more extended and more compressed than those near to O, and this in proportion to their distance from O; and since the strength of the beam at the section P Q arises wholly from the resistance to these compressions and extensions, it must vary with the nature of the elasticity of the material of which the beam is composed.

We may suppose, for instance, the nature of that elasticity to be such that it offers precisely the same resistance to compression as to extension, through any given space, up to the point of fracture; and that these resistances are also exactly in the ratio of its compression and extension: this is the condition of perfect elasticity, and is nearly the case with wrought-iron. A bar of this material, one inch square, is compressed or extended $\frac{1}{10000}$th of its length by one ton of direct compressive or tensile strain, $\frac{2}{10000}$ths by two tons, $\frac{3}{10000}$ths by three tons, and so on, until it is nearly destroyed; in other words, when compressed $\frac{3}{10000}$ths of its length, it would be reacting with three times the energy that it would when compressed $\frac{1}{10000}$th of its length.

Again: we may imagine the nature of the elasticity to be such that it will be partly destroyed after a certain strain; so that, for example, one ton shall compress or elongate the bar $\frac{1}{10000}$th of its length, the material in that state becoming permanently compressed or elongated, and its elasticity impaired, so that another half-ton only may be necessary in

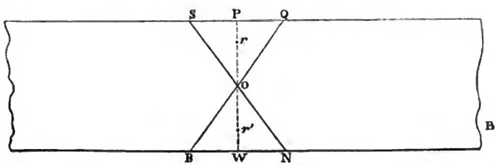
order to elongate or compress it a further $\frac{1}{10000}$th of its length. The elasticity of cast-iron is of this nature.

Lastly, the nature of the elasticity of a material may be such, that its resistance to extension shall follow one law, and to compression some other law.

Now, it is evident, that the determination of the position of the point O, which is the point separating the extended and compressed portions of the beam, is dependent on an exact acquaintance with the laws of elasticity of the material employed; and it is also evident that it may, under one particular strain, be at one part of the beam and at some other part when the strain is varied. In determining this point consists the difficulty of the independent investigation of the resistance of a beam.

If, in the beam we are considering, we suppose the elasticity of the material to be unimpaired by a strain short of that which breaks the beam, and to be of such a nature that under every strain it offers through any given space precisely the same resistance to compression as to extension; then O will be in the centre of the beam, because the same quantity of material will be employed in resisting extension as in resisting compression.

We will assume these conditions to exist.



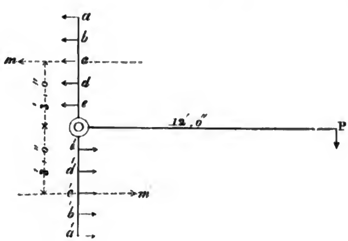
The beam will, it is evident, be broken either by the horizontal layers between P and O being torn asunder, or by those between O and W being crushed; and the beam will be on the point of breaking when the outside layers are strained to the greatest extent they can bear. If a line S Q

be taken to represent the amount of resistance of the outside layer at P, then similar parallel lines representing the resistance of the other layers between P and O, will be included between the straight lines S O, Q O, and the area of the triangle S Q O, will represent the total resistance to extension. A similar and equal triangle O R N will represent the resistance to compression.

Now, there is a point in each triangle at which we may suppose the resistances represented by that triangle to be replaced by a single equivalent force. These points will be at the centre of gravity of each triangle, that is, the point of action of all the resistances of the layers between O and P will be at the centre of gravity of the triangle O S Q, or at *r*, one-third of the distance O P from P; and, similarly, *r′* will be the point of action of the resistances of all the layers between O and W′.

Now, this being the case, we may, without regard to the point O, consider B, *r*, *r′* as a bended lever, either of the points *r* or *r′* being a fulcrum with respect to the other: the breaking-weight of the beam being that weight, which, acting at the end B of this lever, would cause strains at *r* and *r′* equal to the sums of the resistances of all the layers between P and O, and O and W respectively accumulated at *r*, and *r′*, when the layer S Q is strained to its ultimate limit.

To illustrate this still farther, let *a b c d e* represent five



men one foot apart, and each exerting 100 lbs. pressure at a capstan-bar to resist the pressure P at the end of the lever O P, 12 feet long. If there were no men at the other bar *a′ e′*, then O would be the fulcrum of the bended lever. Now, their effect on the bar would be the same as though one man of the same strength as all the five, that is, exerting 500 lbs. pressure, were placed at *m*, the place of the middle man; and since the leverage with which this imaginary middle man is acting is to the leverage of the power P as 3 feet to 12 feet, or as 1 : 4, it follows that 125 lbs. at P would be in exact equilibrium with 500 lbs. at *c*, the reaction now taking place at the centre O.

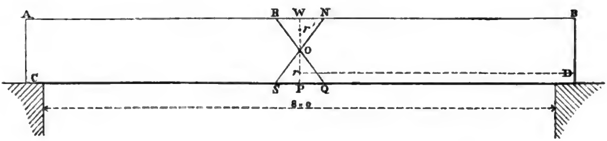
If we imagine five other men on the opposite bar *a′ e′*, we may similarly represent their effect by that of an imaginary man exerting a pressure of 500 lbs. at *c′*, and to restore the equilibrium of the system we should require another 125 lbs. at P; *i. e.* we have now 250 lbs. at P.

But in this state of things we might, without disturbing the equilibrium, (omitting the consideration of the forces parallel to the pressure P) withdraw the centre-pin O of the capstan, and consider *c* and *c′* as fulcra with respect to each other; we have, then, the same result as before, without respect to the centre point, viz. 250 lbs. at P produces an action of 500 lbs. at *c*, and an equal reaction of 500 lbs. at *c′*.

These men represent the extensions and compressions of the beam, with this difference, that in the case of the beam the men must not be considered as all exerting the same pressure; *a* must be considered a stronger man than *b*, and *b* stronger than *c*, &c., in proportion to their distances from O; and hence the point representing their joint action is not at *c* (the place of the middle man), but in a position further removed from O.

As an arithmetical example: Let it be required to find

what weight placed on the centre would break a beam 1 inch broad, 12 inches deep, 8 feet long, assuming the material to break with 20 tons per square inch of direct tensile or compressive strain. Now, by our first figure, we have seen that a weight pressing down on the centre of a beam is equal in effect to half the same weight pressing upwards at each end against a fixed obstacle in the centre; so we may entirely lose sight of the weight on the centre for the present, and inquire what the amount of that force will be, which, pushing upwards at each end of our beam, will be sufficient to overcome the resistances of the layers about the centre of it. We can at once see that the amount of this force will depend, 1. On the amount of the resistance to be overcome; and, 2. On the leverage at which the force acts to overcome it.

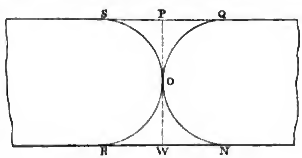


1. The amount of resistance to be overcome. We have seen that the area of the triangle O Q S is an exact representation of this resistance, Q S being taken as equal to the force that will just break one of the horizontal layers. Suppose the layers to be each 1 inch deep: the breadth of the beam being 1 inch, each layer will be a square inch in section; Q S therefore will equal 20 tons. The area of a triangle being equal to its base × half its height, that of O Q S equals $20 \times 3 = 60$; which number of tons is the whole resistance to be overcome at the point r.

2. The leverage. r and r', being at the centres of gravity of their respective triangles, are in this case 8 inches apart, and the whole length of the beam being 96 inches,

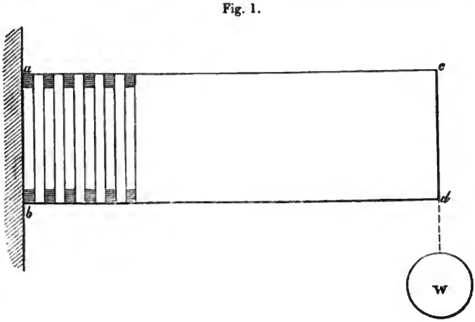
$B r : r r' :: 48 : 8$, or $6 : 1$; that is, a force of 10 tons at each end will produce a strain of 60 tons at the centre: but to produce an equal effect to 10 tons at the end, we must have 20 pressing downwards on the centre of the beam, which is, therefore, the breaking-weight.

Should the material resist compression and extension equally, but not in proportion to the space passed through, the line O S will become a curve, and the point of action of all the resistances will lie at the centre of gravity of the area O S P. The area O Q S will still be equal to the area O R N. When, however, the elasticity of a material is defective, as well as not proportional to the amount of the strain, the investigation of the point O, and of the curves S O, O N, is involved in difficulties, although the areas S O Q and O R N must still, under all circumstances, be exactly equal to each other.



Again, illustrating the subject in a different manner: Let *a b c d*, fig. 1, be a parallel semi-beam or cantiliver, rectangular in section, attached to a solid wall, and having

Fig. 1.

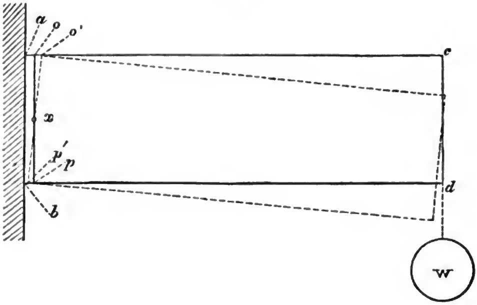


a weight, W, suspended from the extremity *d;* let this beam be constructed of plates of small thickness, combined by means of small distance-pieces, placed between them, and adhering to them at their upper and lower edges.

Now, *a b d* being a bent lever, turning upon a fulcrum at *b*, there will be upon the upper distance-piece at *a* a horizontal tension equal to *w*, increased in the ratio that *b d* bears to *a b*, and upon the distance-piece at *b* there will be horizontal compression to the like extent. In addition to these forces, which are equal and in opposite directions, there will be a vertical force equal to the weight *w*, tending to make the first plate slide upon the face of the wall. At every point in the length of the beam the strains may be similarly calculated, the horizontal strains being directly as the distance from the end *d*, the vertical strains remaining constant. For the present we will assume all vertical motion or sliding between the plates to be prevented by means of studs or other contrivances, so that we may have to consider no other than the horizontal strains. Instead of the distance-pieces at the top and bottom edges, let us interpose layers of some elastic substance, adhering to the plates throughout their entire depth; further, suppose that this elastic material is of such a nature that it is compressed and extended through equal spaces by equal weights, the spaces being as the weights; and let us investigate in what manner the horizontal strains act upon these elastic layers. It will be sufficient to consider this action upon any one layer, as it has been already seen that these strains are similar in their character throughout the whole length of the beam, varying merely in intensity as the leverage.

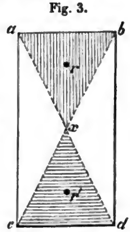
Let us take the one next to the wall (fig. 2), *a o p b:* the application of the weight will, as before, evidently produce pressure upon the lower side of this layer, and tension upon the upper, causing it to take a new form, *a o′ p′ b*,

Fig. 2.



the horizontal pressure below being equal and opposite to the horizontal tension above; and the elastic nature of the material being also assumed perfect, *o o'* will be equal to *p p'*, and the point of intersection *x* being a point where no motion takes place, will be in the centre of the beam: here the elastic layer will, as far as horizontal pressure is concerned, be perfectly inert, the compression and extension of each part of the layer increasing directly as the distance of that part from *x*. To exhibit the resistance to pressure and tension offered by the compression and extension of such a layer, let *b d* (fig. 3) represent the depth of the beam, and let *a b* represent the resistance to tension of the top filament of the elastic layer, which will be as its width and as its extension.

Fig. 3.



The resistance to tension of other filaments above *x* will be, their width being constant, simply as their extension or as their distance from *x*; the sum of their joint resistances may, therefore, be geometrically expressed by a triangle, having *a b* for its base, and *x* for its vertex. The position of the centre of resistance to tension will be coincident

with the centre of gravity of this triangle, and its area expresses the amount of resistance. The amount and the position of the resistance to compression may be similarly found.

Hence, it is easy to estimate the strength of a beam constructed of a material possessed of the assumed qualities, knowing the ultimate power of resistance of this material to extension and compression, or, rather, knowing that of the two which is the weaker. The strength being precisely equal to that of a beam in which the elastic layer next to the wall is replaced by distance-pieces at the points $r\,r'$, the ultimate powers of resistance of which to tension and pressure are equal to those of the triangles $a\,x\,b$, $c\,x\,a$.

The Neutral Axis in Flanged Girders.

It has been seen that the filaments of the elastic layers exert powers of resistance in proportion to their distance from the neutral line x; they also act with greater leverage; it is therefore evident, that if instead of making the cross section of the beam a rectangle we take away material from the vicinity of the neutral line, and add it laterally to the top and bottom, we shall have a beam of the same sectional area and of the same depth, capable of supporting a far greater weight without increasing the strain upon the top and bottom filaments.

Let $a\,b\,c\,d$ (fig. 4) represent the cross section of a rectangular beam; let the same area be disposed as shewn in fig. 5. The upper filament being assumed to be equally strained in both sections, the amounts of resistance to tension of each will be as its width, that is, as $a\,.\,b$ to $g\,.\,h$, and the sum of the diminishing resistances of the successive filaments, as they approach x, will, as before, be represented for the beam $a\,b\,c\,d$, by constructing the triangle $a\,x\,b$. In the

Fig. 4.

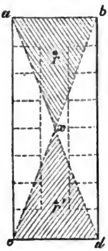
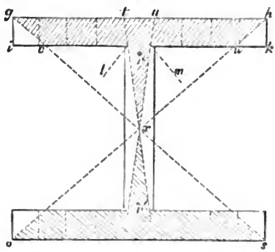


Fig. 5.



beam *g h o s* the resistances of the filaments composing the top flange, *g i h k,* will be identical with those of a similar portion of a rectangular beam, of which the dimensions are *g . o* by *o s,* and their sum will, therefore, be expressed by the figure *g v w h ;* the resistances of the filaments composing that portion of the middle web above *x* will be identical with those of a similar portion of a rectangular beam, of which the dimensions are *t u* by *g . o.* The sum, therefore, of the resistances to tension of the portion of the beam subject to this strain may be represented by the figure *g v l x m w h,* all parts of which offer as much resistance to tension as the upper filament. A similar figure represents the sum of resistance to pressure.

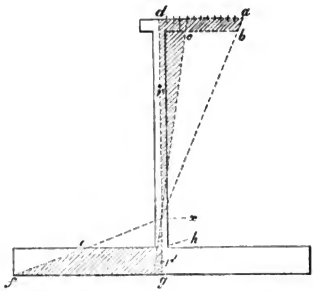
It will easily be seen that, not only have the areas of the figures representing the sum of resistances been increased, but that the leverage with which they are acting has also been increased, owing to the centres of resistance being further apart. Thus, by this new disposition of the material, a beam has been designed capable of bearing a far greater weight without increasing the strain upon the exterior filament.

In investigating the properties of the various materials

of which beams are constructed, it may appear that, notwithstanding the truth of the assumption, that the "elastic material is of such a nature that it is compressed and extended through equal spaces by equal weights, the spaces being as the weights," yet that the ultimate powers of resistance to pressure and tension in some such materials are not alike, this knowledge will influence us in disposing this material in the beam. Let us assume, for instance, that a certain material requires four times the amount of pressure as of tension to destroy it. This is equivalent to saying, that in order to reach the breaking point we may compress it through four times the space that we may extend it. Now, both the foregoing beams being symmetrical in section, the top and bottom filaments were shewn to be equally extended and compressed with a given weight, and would be still so at the instant preceding fracture, if constructed of the assumed material; therefore, at the instant such beams were about to break, owing to the extension of the top filament having reached its limit, the lower filament, although compressed as much as the top one was extended, would still be exercising only one-fourth of the resistance to pressure of which it is capable. In order to enable such a material to be performing its maximum duty both in the upper and lower filaments, the latter must be compressed four times as much as the former is extended. In fact, $o\,o'$ (fig. 2) must be one-fourth $p\,p'$, and, consequently, $o\,x$ will be but one-fourth of $p\,x$; or, in more general terms, the material in a girder should be so arranged, that the distances between the neutral line and the exterior filaments exposed to pressure and tension should be inversely as the capability of the material to bear such strains.

A girder of the section (fig. 6), supported at each end, would very nearly fulfil these conditions for our assumed

Fig. 6.



material, the neutral line x being four times as distant from the top of the beam as from the bottom, and the areas of the figures $a\,b\,c\,x\,d$ and $x\,h\,e\,f\,g$ being equal.

For the assumed material, therefore, this section is a better one than either of the foregoing ones, as its great power of resistance to pressure is brought into play.

Now, the properties that we have thus assumed very nearly approach those belonging to cast-iron, in which material the relative powers of ultimate resistance to pressure and to tension are about as five to one, and in which the variation in length due to pressure or tension is, when the metal is not strained beyond the limits of safety, very nearly as the strain; and were the proportions of a beam merely dependent upon these considerations, it would be easy to calculate the section best suited to this material. There are, however, two other considerations, which in cast-iron render it impossible to load that portion of the section exposed to pressure with the same proportion of its ultimate power as that portion exposed to tension.

It is found inadvisable to employ a section in which there is a very marked and sudden change in thickness, on account of the unequal strains produced by contraction in

cooling. It is also to be remembered that the portion of the section exposed to pressure is in the position of a column, and that, consequently, in most cases it is more likely to yield by being bent laterally than by being positively crushed. Owing to the increase of thickness and breadth hence requisite in the part exposed to pressure, it is inadvisable in practice that the ratio of the amount of pressure to that of tension exceed two and a half to one.

The reasoning which applies to the case of a beam, or bracket, attached to a wall and loaded at the extremity, will equally apply to the case of a beam attached to a wall and loaded uniformly, the only difference being, that in the latter the tendency of the imaginary plates to slide upon each other increases directly as their distance from the extremity of the beam, instead of being constant throughout the whole length as in the former case, and that the horizontal strains, instead of increasing directly as the distance from the extremity, increase as the square of that distance, as not only is the leverage increasing, but the weight to be borne also.

It is needless to repeat the well-known application of the reasoning upon beams attached at one end to the case of beams supported at both ends. It will be sufficient to state, that in beams supported at both ends, and loaded in the middle, the tendency of the plates to slide upon each other is constant between the centre and the points of support, and that the horizontal forces decrease directly as their distance from the centre, while in beams loaded uniformly the tendency of the plates to slide upon each other increases directly as their distance from the centre, and the horizontal forces diminish as the rectangles of the segments.

In the foregoing remarks the beams have been supposed to be built up of thin layers of an elastic material, confined between thin plates, for the purpose of being better able to describe the effects of the strains at the various parts; but

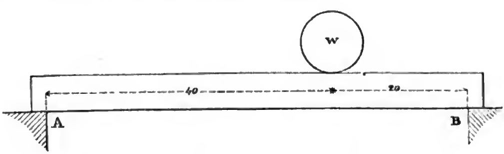
if we suppose the plates withdrawn, and the elastic layers to become united into one mass, the reasoning relating to the horizontal strains will still hold good; we, however, have hitherto, although giving their amount, purposely omitted the consideration of vertical strains, supposing them for the time to be met by contrivances attached to the plates.

These strains are, in reality, compounded with the horizontal forces whose direction is thereby altered. In beams of moderate span, wherein the effect of the vertical forces is inconsiderable, it may be safely omitted in calculation; in beams of great magnitude, such as those forming the subject of this work, wherein it is requisite to apportion exactly in each part the quantity of material to the work to be done, its effect has to be carefully taken into account.

Strain in Beams as dependent on the position of the Load.

We have hitherto been considering the effect of weight placed on the centre of a beam; we have now to consider, 1, the strain produced at any part of a beam by a weight placed at any other part; 2, the strain from several weights distributed; and 3, the strain from the weight of the beam itself—in our case a most important consideration.

1. The strain produced at any given point in a beam by a weight placed at any other point.



Let a weight = 100 be placed on any part of a beam, as at W, the reaction at A and B will be inversely proportional to the distance of W from the ends. Thus,

As $60 : 40 :: 100 : 66{\cdot}6$ pressure at B.
And $100 - 66{\cdot}6 = 33{\cdot}3 =$ pressure at A.

The strain at W may thus be represented by

$$66{\cdot}6 \times 20, \text{ or } 33{\cdot}3 \times 40 = 1333,$$

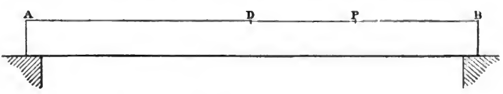
while the same weight placed at the centre of the beam would produce a strain thereof $50 \times 30 = 1500$.

In order, therefore, that the strain may be the same at each point, the weight must be in the inverse ratio of these strains, or as $1333 : 1500$.

Now W is 20 feet from one end and 40 feet from the other end, $20 \times 40 = 800$; and the centre is 30 feet from each end, and $30 \times 30 = 900$. And these products are in the same ratio as the above strains,

$$i.\ e.\ 800 : 900 :: 1333 : 1500.$$

Hence, in order that the strain may be the same wherever a weight is hung, such weight must be inversely as the product of its distances from the extremities, or inversely as the rectangle of the segments into which its position divides the beam. This property may be thus usefully expressed :—



If W be the greatest weight that a beam of uniform section will support in the middle D, the greatest weight W′, that it will support at any other point P, will be found by the proportion, as $AP \times PB : AD \times AD :: W : W'$.

We shall hereafter have an opportunity of demonstrating this analytically.

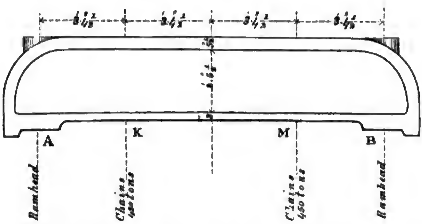
From this it follows, that if the beam is to be used where

a weight will pass along it, as a carriage over a bridge, and is of uniform section, it is unnecessarily strong at all other parts when it is of sufficient strength at the centre. A beam, therefore, which is intended for a passing load, may be diminished in strength towa:ds the ends in the ratio of the rectangles of the segments at every point.

If this diminution be made by altering the depth of the beam at every part, the strength being diminished as the square of the depth, the form of the curve will be an ellipse.

The same deductions, since we have been only treating of strains, hold good also with respect to tubular or any beams; and some highly interesting experiments on tubes, hereafter described, and made in order to test this law, were perfectly in accordance with the above theory.

Let us next consider the effect of two weights on a beam, as in the cross-head of the Britannia Bridge, and determine the strain at the centre section, the chains being equidistant from the centre and from each end.



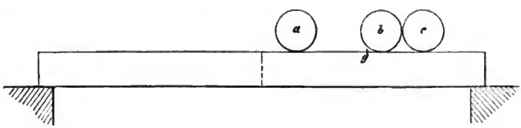
To find, first, the strain at the centre arising from one weight K. Since the spaces on each side of the chain are equal, we will call them each = 1, so that the whole length = 4.

Then as 4 : 1 : : 450 tons : pressure at B = 112·5; the strain at the centre may be represented, therefore, by 112·5 × 2 = 225.

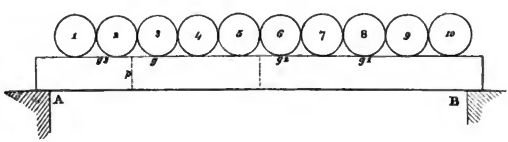
The strain at the centre from the other weight at M will also $= 225$; therefore $225 + 225 = 450$ will represent the total strain at centre.

Now, if 450 tons were placed on the centre, the strain would similarly be represented by $225 + 2 = 450$; so that the strain at the centre, under the circumstances shewn in the figure, is the same as it would be if 450 tons were hung on the centre. It must be remembered, however, that although the strain at the *centre* section is the same in both these cases, yet the strain at any other section is widely different, though it may be determined easily by similar reasoning.

Next, as regards the effect of weight distributed equally over a beam, or, which is evidently the same thing, of the weight of the beam itself if uniform.



If it were required to find the strain at the centre produced by the three weights a, b, c, it would be necessary to find their centre of gravity g, and suppose their total weight collected at that point, which is a case of precisely similar circumstances to the last. But if we would determine the strain at any other point, as at g intermediate between a and b, we must first determine the strain at g from the weight a, and then that from the two weights b and c, and the sum would be the whole strain required.



Let a beam be covered with equal equidistant weights, 1, 2, 3 . . . . 10, to find the strain at the centre.

The strain at the centre produced by the weights 1 . . . . 5, acting at their centre of gravity g, added to the equal strain produced by 5 . . . . 10, acting at g^1, will give the strain required. For the strain elsewhere, as at p, we should similarly find the strain at p, occasioned by the weights 3 . . . . 10, acting at their centre of gravity g^2; and also that at p, by the weights 1, 2, acting at their centre of gravity g^3; and the sum of these would be the required result.

As a numerical example, let each weight be 100 lbs., and let them be 1 foot apart, so that the beam is 10 feet long.

1. To find the strain at the centre—

The sum of the weights 1 to 5 = 500 lbs., their centre of gravity is at 2 ft. 6 in. from the end A, *i. e.* one-fourth the length of the beam. For the pressure on B—

We have as 10 : $2\frac{1}{2}$, or as 4 : 1 : : 500 : 125, the pressure at B.

The strain at the centre will therefore be represented by 125×5	=	625
Similarly the strain from the weights 6 to 10	=	625
The total strain on centre	=	1250

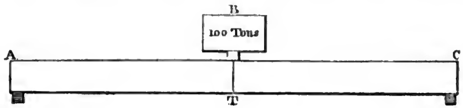
Now, if the whole of the weights were placed at the centre, the strain would equal $500 \times 5 = 2500$; that is, double the strain of the weights distributed. Hence we obtain a result which is commonly expressed as follows: The strain at the centre, from weight equally distributed over a beam, is the same as that produced by half the same weight placed at the centre.

Hence the strain at the centre of any uniform beam from its own weight is precisely the same as though half its weight were accumulated at the centre, the beam itself being sup-

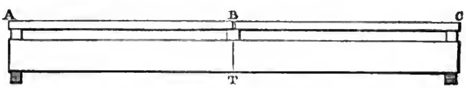
posed devoid of weight, although the strain at other parts, and also the deflection, is entirely different in the two cases.

This is sometimes not clearly understood by the pupil, who is puzzled to comprehend why the whole effect of the reaction at the bearings is not to be understood as inducing strain at the centre; *i. e.* if a given beam weighs 100 tons, he does not see why the strain at the centre should not be the same as though 100 tons were placed at the middle of the same beam, the weight supported at each extremity being precisely the same in both cases. The difference of the two cases will be clearly seen as follows:

Let the beam A C be supposed without weight, and let

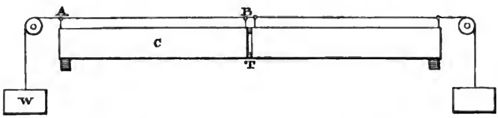


100 tons be placed at the centre. The pressure at each end is then 50 tons, tending to tear asunder the beam at T, the pressure at B being 100 tons.



Let *the same weight* be rolled into two bars, A B, B C bearing on the beam only at A, B, and C. It is evident in this case that only half the weight of each bar is now bearing on the centre B, the other half being a mere dead weight over the supports, and inducing no transverse strain in the beam. The pressure at B in this case is evidently only 50 tons, the reaction produced by which at each end is now only 25 tons; this is analogous to the strain from the weight of the beam itself, supposing it to weigh 100 tons.

Again: we may look at this matter in another light. Let us suppose A B to represent half of the Conway Tube, united



only by the chocks B and T to the other half, and supporting its own weight. It is evident that some weight, W, will be just sufficient to relieve the chock B of all pressure, and support the half A B without contact with the other half; and this weight will be just equivalent to the compression at B from the strain caused by the beam.

It is also immaterial whether the weight be attached at B or A, and we will assume it attached at A, the line A B being supposed coincident with the top of the beam.

The strain caused on the rope is due to the weight of the portion A T, which may be considered as acting at its centre of gravity C; and is therefore only half the amount of strain that would take place if the whole weight were acting at B, which it would be if the load were central.

In determining, therefore, the strain at the centre B, on the top of such a beam, from its own weight, the reaction at each end is only one-fourth of the weight of the beam instead of one-half, as would be the case if the weight were all accumulated at the centre.

Lastly, referring to fig. 2, at page 226, to ascertain the strain at p, from weight distributed equally over the beam.

The sum of the weights 3 to 10 = 800 lbs.

Their centre of gravity is at 4 feet from B.

Then as 10 : 4 : : 800 lbs. : 320 lbs. the pressure at A.

And, therefore, $2 \times 320 = 640$ will represent the strain at p from the weights 3 . . . . 10.

Again: the sum of the weights 1, 2 = 200 lbs.

Their centre of gravity is at 1 foot from A.

Then, as 10 : 1 : : 200 : 20, the pressure at B.

Therefore, $8 \times 20 = 160$, the strain at p from the weights 1, 2;

And, therefore, $640 + 160 = 800$, total strain at p.

Now, the strain at the centre from the same weight equally distributed = 1250; but the products of the respective distances of the centre, and of the point p, from each end, are to each other as $5 \times 5 : 2 \times 8$, or as 25 : 16 : : 1250 : 800.

In other words, the strain at the centre from a weight equally distributed over a beam is to the strain at any other point as the rectangle of the segments at the centre is to the rectangle of the segments at the given point. Also the strain at any part of a beam from the weight of the beam itself is proportional to the rectangle of the segments at that part.

We have seen, then, that in all uniform beams, whether rectangular, tubular, or of any other form, the strain at the centre produced by a weight equally distributed, or, what is the same thing, from the weight of the beam itself, is exactly the same as if half that weight were accumulated there; and, generally, that whether a beam is intended to carry, 1°, the same weight at any part of it; or, 2°, a weight equally distributed; or, 3°, merely its own weight, the strength should be everywhere as the rectangle of the segments.

But if the beam is to be loaded at one point only (its own weight not being taken into consideration), then the strength should be everywhere as the distance from the extremities.

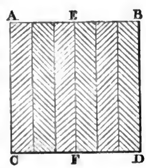
Practical Deductions and Observations.

The usual practical rules applied in calculating the strength of beams are very simple. Let A B represent a rectangular beam. If such a beam is loaded at the centre P, the strain at any other point X is evidently as D X to D P, or as its distance from the extremity.

Similarly if the points of support D C were removed to

twice their original distance from P, or the length of the beam were doubled, as in the dotted figure, the strain would be increased in the same proportion, *i. e.* the strength of a beam is inversely as the length when other dimensions remain constant.

Let us next consider the effect of an alteration in the breadth, and let A B C D represent a section through the centre.



Now the strain, and therefore the resistance, in beams takes place in *vertical* planes only, as A C, E F, B D, throughout the length of the beam, the resistance of each one of these planes being independent of the plane on either side of it; so that the beam, for the sake of illustration, may be looked on as consisting of a row of thin planks placed edgewise, side by side, the addition of another plank in no way interfering with the resistance of the rest, but merely adding its own strength to the mass: so that, by doubling or trebling the number of vertical planks or layers, we double or treble the strength; and thus the strength of a beam is directly as its breadth when other dimensions are constant.

Lastly, If we vary the *depth* of the beam. The resistance

acting, as it does, in vertical planes, will be affected in two ways by an increase in the depth of those planes. In the first place, the increase in depth causes an exactly equal increase in the actual quantity of material at any section of the beam, to resist compression and extension. If the beam be doubled in depth, there will be in it twice as much material to resist the transverse strain; and from this cause alone the beam will be doubled in strength. Secondly, the points r and r' (fig. 3, p. 217), whether we can accurately determine their position or not, are separated from each other exactly in proportion to the depth of the beam, and will be now double their former distance asunder: so that, not only have we twice the quantity of material to resist the transverse strain, but it is also acting with twice the leverage; and hence the strength of a solid beam varies as the square of its depth when other dimensions are constant.

Thus in a simple rectangular beam the strength varies as $\frac{b\,d^2}{l}$; or, in other words, the strength of one beam as compared with another is, first, directly as the section of fracture; secondly, directly as the depth; and, thirdly, inversely as the length. And it possesses these properties independent of the nature of its resistance, so that, by direct experiment with any given beam, we can determine the strength of any other beam of the same material.

Experiments have been made on beams of every kind of material, and to simplify the calculation of other beams from these, the results have been reduced to those which would be obtained from beams one inch square and one inch long. Thus, a beam of cast-iron of these dimensions breaks with 11 tons on it (Hodgkinson); of wrought-iron, 13 or 12·264 tons (Barlow); of red pine, 2·4. Knowing, then, the breaking-weight of this one-inch beam, to find the strength of any other similar beam we have simply, 1. Multiply this breaking-weight by the sectional area of the new beam; 2. By the

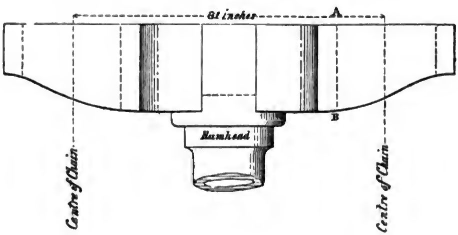
depth of the new beam; 3. Divide this product by the length, using inches for each dimension.

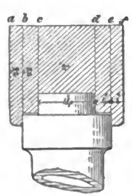
Taking, as an example, the beam at p. 214, 8 feet long, 12 inches deep, 1 inch broad, we have for wrought-iron

$$\frac{\overset{\text{Constant.}}{13} \times \overset{\text{Area of Section.}}{12 \text{ inches}} \times \overset{\text{Depth.}}{12 \text{ inches}}}{\text{Length } 96 \text{ inches}} = 19{\cdot}5 \text{ tons.}$$

It is frequently more convenient to divide by the length in feet instead of inches, in which case these constants must be divided by 12; and in this form they are usually tabulated in works on the subject. (*See* Barlow and Tredgold.)

As another example, we will take the cast-iron cross-head of the hydraulic press used for raising the Conway tube.





Constant 11 tons.

We have here to find the strength of each of the vertical

portions *a b*, *b c*, *d e*, *e f*, and add them together. In the portions *a b* and *e f* we have

Area (6 × 25) × depth (25)	= 3750
In the portions *b c* and *d e* area (8 × 22) × depth (22)	= 3872
In the centre portion area (17 × 14) × depth (17)	= 4046
Sum	11668
Multiply by the constant	11
Divide by the length..............	81)128348
Breaking-weight at centre......	1584 tons.

One-third of this quantity, or 528 tons, is as much as is safe in practice to load such a beam with. But we have supposed the weight to be exactly in the centre of the cross-head, whereas the ram is 14 inches broad, and the strength was thus in practice somewhat greater.

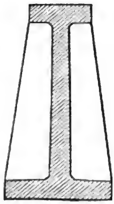
Two of these cross-heads, weighing nearly seven tons each, were employed in raising the Conway tubes, the weight borne by each being 650 tons. During the raising of the second tube, to the imminent peril of the structure itself, as well as of those employed in the operation, which was conducted by Mr. Robert Stephenson in person, one of the cross-heads was discovered to be fractured, the tube being at the time suspended 15 feet above the water. The tube was at once lowered on to the clams of the lifting-chain, and additionally secured by timber packing from the bed below. The fracture was evident at the top of the cross-head, running from the centre outwards through a space of seven inches, and had been for some time seen opening and closing as the weight was taken, by an assistant, who, thinking it of no consequence, had not mentioned it. Precautions were taken to ensure the tube from falling in case of complete fracture, and the remaining three feet of lift was accomplished without the cross-head breaking, though the fissure

spread a little further during the operation. A little more time would have, doubtless, completed the fracture, which had probably commenced at the raising of the first tube.

The cause of this mishap originated in the neglect of the founders, who, in casting the cross-head, not only did so with its top — or part to be extended — upwards instead of downwards, thereby endangering the quality of the metal in the most important part of the beam, but, moreover, had poured the metal into the mould at the very centre; and in stirring the metal at this place, to liberate the air, and supply the contraction, as is usual in large castings, had continued to move it after it had cooled to a semi-fluid state, so that at the part at which the fracture took place there was a core of metal about nine inches in diameter very slightly connected with the metal round it. And in addition to this cause of weakness, it was also found that from a defect in fitting, the cross-head did not bear on a shoulder all round the ram-head, but that contact took place only at the top of the ram itself.

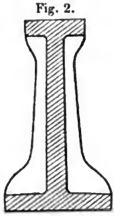
A very considerable margin should be allowed in all large masses of metal, since they are always more or less faulty, while giving no outward indication of such a state; and from unequal contraction in cooling alone it frequently happens that some portions are already in a state of considerable strain. An illustration of this fact was given at Conway. In the first design for the large 12-ton beams for carrying the presses, the brackets connecting the top and bottom flanges were made throughout of an equal depth with them, the section being as in fig. 1, and when cast, these beams broke to pieces spontaneously in cooling; upon this the section was modified to that of fig. 2, in which shape the beams have answered their purpose effectually.

Fig. 1.



Large castings, in which this tendency is not provided against, often break in cooling; while in some cases the fractures do not manifest themselves for several months.

Fig. 2.



In castings for beams care should be taken to have all the interior angles well rounded off, the brackets by no means very broad, and all the parts as nearly as possible of an equal thickness.

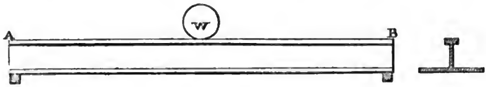
However, with all these, and similar precautions, large masses of cast metal cannot be depended on, and several small beams are much safer than one large one of the same calculated strength. The addition of metal, after a certain size has been reached, appears to add but little to the strength of a casting, of which an instance will be described in explaining the hydraulic press. In consequence of these anomalies, Mr. Stephenson determined on using wrought-iron in the apparatus for raising the Britannia Bridge.

We have given, in page 232, the constant for rectangular bars of cast-iron generally employed by practical men. It will be hereafter seen this constant is *much too high* for large castings.

The breaking-weight of a wrought-iron bar one inch square and one foot long is generally taken at 1·08 tons, which is, therefore, the constant for wrought-iron rectangular beams. These are seldom absolutely broken, but rendered useless by their flexure. They become stiffer as they are further bent; they may thus be improved by forging them in a curved shape, and straightening them by force, and employing them in the direction in which they are straightened.

Cast-iron flanged beams, as generally constructed, fail by the breaking of the lower flange; the sectional area of the lower flange only is, therefore, generally taken into account in

estimating their strength, the vertical rib not being included in the calculation, and the following method of determining their strength is almost universal as a rough approximation.



Let A B be a girder 12 inches long, the sectional area of the lower flange being 1 square inch; let the depth be also 1 inch. A weight W placed at the centre gives a reaction at each extremity of $\frac{W}{2}$, and as the half length is 6 inches, the strain in the lower flange at the centre is $\frac{W}{2} 6 = 3\,W$.

Now the area of the lower flange being 1 square inch, and the tensile strength of cast-iron being 6·5 tons per square inch, the beam will break when $3\,W = 6{\cdot}5$, or $W = \frac{6{\cdot}5}{3} = 2{\cdot}1666$, which is the breaking-weight of a cast-iron flanged beam 1 inch deep, 1 foot long, and the sectional area of the lower flange being also 1 square inch.

And since the strength of beams is assumed to be directly as the sectional area of the lower flange, and is directly as the depth, and inversely as the length, the strength of any cast-iron flanged beam will be

$$W = \frac{a\,d}{l} 2{\cdot}1666 \text{ tons},$$

a being the sectional area of the lower flange, and d the depth in inches, and l the length in feet.

If the length be taken in inches, then the constant 2·1666 becomes $2{\cdot}1666 \times 12 = 26$.

We have seen that the strength of box girders, or rectangular tubes, may be determined in the same way by considering only the sectional area of the bottom, and properly proportioning the top and sides on other considerations.

But in forming such rules we have leaped through all difficulties by assuming that we have obtained the proper proportion for the top flange and vertical rib, whereas in the construction of flanged cast-iron beams, the subject of greatest moment is the determination of this proportion. With respect to the vertical rib it would be miserable economy, in so inexpensive a material, to attempt to reduce its thickness to anywhere near the limits of its mere requirement; and, as will be seen in considering the duty of the sides of wrought-iron tubes, towards the end of a beam considerable strength is necessary in the vertical rib: nor do the brackets usually placed at the ends at all dispense with a necessity for increased thickness in the rib itself. It is customary to make the depth of cast-iron beams about one-fifteenth of their length, with more or less depth in proportion, other considerations will influence the strength: the stiffness is increased by increasing the depth, and therefore deeper beams are less suited for resisting impact.

If the vertical rib were unusually strong we might calculate its strength as a rectangular beam, and add the result to the strength obtained from the above formula.

It is sometimes convenient to remember that an ordinary flanged beam of moderate dimensions may safely deflect one-fortieth of an inch for every foot of length. Any approximation, however, in which the deflection is supposed proportional to the length must be extremely limited in its application.

CHAPTER II.

INVESTIGATION OF GENERAL FORMULÆ FOR THE STRENGTH OF BEAMS.

The general reasoning hitherto employed has been based on practical considerations of a very simple character. Without, however, some more general analysis than a mere rational explanation can possibly afford, the subject must of necessity be imperfectly elucidated. We shall, therefore, briefly investigate from general principles such theoretical formulæ as seem best adapted for determining the strength and flexure of bodies subjected to transverse strain, and shall thus have an opportunity, as we proceed, of deducing from more general expressions the useful, practical approximations and limited analogies which have been employed.

We shall adopt throughout the following investigation the principles and methods of calculation laid down in "Moseley's Mechanical Principles of Engineering and Architecture," and shall follow as closely as the nature of the problems under consideration will admit, the reasoning there contained.

It will be well to mention at the outset two assumptions by which the treatment of the subject will be much simplified.

1st, We shall assume that the material of the structures treated of is *perfectly elastic*; *i. e.* that the force necessary to keep any fibre extended or compressed is proportional to the amount of extension or compression, and is the same for a given amount of the

one as for an equal amount of the other—*ut tensio sic vis*. The complete theory of beams, taking imperfect elasticity into account, is complicated and difficult;* but it fortunately happens that wrought-iron, which we have most to do with in this work, complies so nearly with the above conditions, that it may be assumed perfectly elastic without much error; and indeed under certain restrictions and with certain precautions, the assumption be extended to cast-iron and other materials.

2dly, We shall assume that the position of all beams treated of is *horizontal*, and that all pressures applied to them are in a *vertical* direction. More general investigations may be referred to in the work above-named.

With respect to the theory of the strength of beams we may also make an introductory remark, which will much facilitate our subsequent investigations. The determination of the transverse strength of a beam is usually limited in practice to finding the breaking-weight. As a theoretical problem, however, it may be more accurately defined to consist in determining the ratio between the weight of a beam in supporting, and the longitudinal strain either of extension or compression, on any given fibre in the cross section of the beam. It has already been explained that when a beam is loaded, certain of its fibres become subject to a force of tension, and certain others to a force of compression; and if, therefore, we can establish the ratio between these tensile and compressive forces and the load on the beam, we thereby solve the problem of its strength, since we know by experi-

* Mr. Hodgkinson has given some valuable essays on this subject. See fourth edition of "Tredgold on Cast-Iron."

ments what degree of longitudinal strain the fibres of the given material will break with, or will bear with safety.

Let W represent the weight supported by a beam, and f the longitudinal strain caused by that weight on any given fibre per square unit of its area; then, if we can determine the value of f for any given value of W, or W for any given value of f, we have in reality solved the problem of the strength of the beam, since we can immediately find, by reference to experiment, whether or not any fibre of the beam is overstrained by a given weight, or what weight will correspond to a given maximum strain on the fibres. The value of f is usually obtained for either the extreme upper or extreme lower fibres of the beam, these being subjected to the greatest strain; and under these conditions we shall hereafter determine the value of $\frac{W}{f}$ for the different modifications of beams treated of.

The Neutral Axis.

Attention has already been drawn to the consideration that in a solid beam, since the upper fibres are compressed, and the lower extended, by the action of the load, there must be a part of the beam where the fibres remain *neutral*, being neither compressed nor extended: this part forms a surface, horizontal or nearly so, contained within the beam, and called the *neutral surface.* Now, taking a vertical transverse section of the beam at any part of its length, the line where this section is cut by the neutral surface is called the *neutral axis* of the section, all the fibres on one side of it being compressed, all those on the other side extended.

In the case of hollow or tubular beams, the neutral axis is, except where it cuts the sides, imaginary, just as is the centre of a wheel having a hollow nave. The place, however, of the neutral axis can be calculated for hollow beams

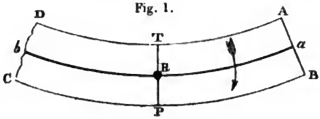
as well as for solid ones, and all the calculations which depend on its position will hold as good for one case as for the other.

Now if all the pressures applied to a beam be vertical, the forces of tension above, and compression below, the neutral axis, being the only horizontal forces, must be equal to each other; from which it follows that if the material be perfectly elastic, *the neutral axis will pass through the centre of gravity of the section*, and by this rule its place may be easily found for any form of beam.*

THE MOMENT OF THE ELASTIC FORCES EXERTED AT ANY GIVEN SECTION OF A DEFLECTED BEAM.

Let A B C D, fig. 1, be a longitudinal view of a portion of a deflected beam, and *a b* the neutral line. It is evident that at any section P T a force will be exerted, by the compression of the fibres from R to T, and the extension of those from R to P, tending to turn the part A B P T round

Fig. 1.



R as a centre, in the direction of the arrow. We purpose to find an expression for the value of this force.

Imagine the beam to be composed of longitudinal fibres parallel to *a b*. Now, the fibres in the section P T being strained longitudinally in proportion to their distance from R,

* For the investigation of the position of the Neutral Axis, *see* "Moseley," Art. 359.

let $f =$ the strain per square unit of area on any given fibre situate at a distance $= c$ from the neutral axis. Then the strain per square unit of area on any other fibre, at a distance ϱ from the neutral axis, will be $= f \frac{\varrho}{c}$. Make the area of this latter fibre $= \Delta k$; then the force exerted by it will be

$$= \frac{f}{c} \varrho \, \Delta k.$$

But in estimating the effect of any force in turning a body round any point, we must take into account also the *leverage* with which such force acts, and the usual way of doing this is to find what is called the *moment of the force* about such point, *i.e.* the product of the force itself multiplied into the perpendicular distance at which it acts. Now the perpendicular distance of the above fibre from the point R being $= \varrho$, and the elastic force with which this fibre acts being $= \frac{f}{c} \varrho \, \Delta k$, the *moment of this force* about R will be $= \frac{f}{c} \varrho^2 \Delta k$, which expression represents the moment of an elementary portion of the elastic force exerted at the section P T by a fibre, whose area is $= \Delta k$, and whose distance from R is $= \varrho$.

Applying this, therefore, to every fibre of the section, and adding the whole together, we shall obtain the sum of all the moments about R, of all the elastic forces exerted on the section PT. Let this moment of elastic forces be represented by Φ, then we have

$$\Phi = \frac{f}{c} \times \text{sum of all the } (\varrho^2 \Delta k),$$

or using Σ as a sign of summation,

$$\Phi = \frac{f}{c} \Sigma \varrho^2 \Delta k.$$

But $\Sigma \varrho^2 \Delta k$ represents the *moment of inertia* of the section P T about the axis R, which may be easily found for

any given form of section, as will be hereafter shewn. Let, therefore, this moment of inertia be represented by I, we have

$$\text{(I.)} \qquad \Phi = \frac{f}{c} \mathrm{I},$$

which gives the moment of the elastic force at any section of the beam, in terms of the longitudinal strain on any given fibre.

We now proceed to determine

THE VALUE OF THE MOMENT OF INERTIA FOR VARIOUS FORMS OF SECTION.

The moment of inertia is expressed by the general equation,

$$\text{(II.)} \qquad \mathrm{I} = \Sigma \varrho^2 \Delta k;$$

that is, conceiving the whole area of section divided into small portions, multiply the area of each portion by the square of its distance from the neutral axis, and the sum of the whole of these products will be the moment of inertia of the section. Hence by drawing the section, and dividing it off into small portions, the above operation will give an approximation to the value of I for any given form of section, sufficiently near for all practical purposes.

In many cases, however, where the form is regular, we may obtain the value of the moment of inertia more accurately by other means.

Suppose the section to be divided into layers parallel to the neutral axis, and indefinitely thin in depth; let the breadth of any one of these layers be represented by β, its distance from the neutral axis by ϱ, and its depth by $d\varrho$, then its area will be $= \beta\, d\varrho$, which corresponds with the symbol Δk in the above equation. Let, moreover, the distance of the top and bottom of the beam from the neutral

line $= h_2$ and h_1 respectively; then the expression for the moment of inertia will become

$$\text{(III.)} \qquad \mathrm{I} = \int^{h_1} \beta \varrho^2 d\varrho + \int^{h_2} \beta \varrho^2 d\varrho,$$

in which β, if not constant, must be expressed in terms of ϱ.

When the neutral axis is in the middle of the depth, as in all forms symmetrical with respect to the axis, $h_1 = h_2 = \frac{d}{2}$, where $d =$ the depth of the beam; therefore, in this case

$$\text{(IV.)} \qquad \mathrm{I} = 2\int^{\frac{1}{2}d} \beta \varrho^2 d\varrho.$$

We will now apply these rules to various regular forms of beams.

For the Section of a Solid Rectangular Beam.

Let b represent the breadth, and d the depth of the beam, then Equation IV. becomes

$$\mathrm{I} = 2b\int_0^{\frac{1}{2}d} \varrho^2 d\varrho = 2b \times \frac{d^3}{24} = \frac{b d^3}{12}.$$

If then a represent the sectional area $= b\,d$, we have,

$$\text{(V.)} \qquad \mathrm{I} = \frac{a d^2}{12}.$$

For the Section of a Solid Circular Beam.

In this case if $d =$ the diameter, we shall have, by the properties of the circle,

$$(\tfrac{1}{2}\beta)^2 = r^2 - \varrho^2;$$

or

$$\beta = 2\sqrt{r^2 - \varrho^2};$$

whence, by Equation IV.,

$$\mathrm{I} = 4\int_0^r \varrho^2 \sqrt{r^2 - \varrho^2}\, d\varrho,$$

or integrating and reducing, $I = \frac{\pi r^4}{4}$. Let a represent the sectional area $= \pi r^2$, and d = diameter or depth $= 2r$, then

(VI.) $$I = \frac{a d^2}{16}.$$

For a hollow Circular Beam, or Circular Tube.

Let r_1 represent the external, and r_2 the internal radius; then, since the moment of inertia of the tube will be equal to that of the external circle, minus that of the internal, we have, as above,

$$I = \frac{1}{4} \pi (r_1^4 - r_2^4).$$

Now, let t = thickness of the tube; then $r_1 = r_2 + t$; whence by substitution in the above equation,

$$I = \frac{1}{4} \pi (4 r_2^3 t + 6 r_2^2 t^2 + 4 r_2 t^3 + t^4).$$

When the thickness is very small in proportion to the radius, we may neglect the three last terms; and making d = the mean diameter, and a = sectional area $= \pi d t$,

(VII.) $$I = \frac{a d^2}{8}.$$

For a Rectangular Tube or Flanged Girder.

It is evident that the two sides of a rectangular tube, taken together, are equivalent to the vertical rib of an ordinary flanged girder. Confining, therefore, our attention to this latter form, we shall take two simple cases; in the first we shall neglect the vertical rib; and in the second we shall include the vertical rib, but assume the top and bottom flanges to be of equal area.

1st, In this case we suppose the vertical rib or sides very thin, so as merely to answer the purpose of keeping the top and bottom apart from each other. We have, therefore, only

to take the moment of inertia of the top and bottom plates, and if these are but of small depth, in proportion to the whole depth of the beam, the application of Equation II. in the following manner will give a sufficiently near approximation for all practical purposes.

Let the area of the bottom plate $= a_1$, that of the top plate $= a_2$, and the depth from centre to centre of the plates $= d$. Also let the distance of the bottom and top plates from the neutral axis be represented by h_1 and h_2 respectively. Then by Equation II. we have

$$I = a_1 h_1^2 + a_2 h_2^2.$$

But since the neutral axis is in the centre of gravity between the top and bottom plates, we have, by known rules,

$$h_1 = \frac{a_2}{A} d,$$

where $A = a_1 + a_2 =$ the area of top and bottom together.

Similarly $$h_2 = \frac{a_1}{A} d,$$

whence by substitution and reduction,

$$\text{(VIII.)} \qquad I = \frac{a_1 a_2 d^2}{A}.$$

2dly. In a flanged girder, of which the top and bottom flanges are of equal sectional area, the neutral line will be in the middle of the depth. Let $a =$ the sectional area of the top or bottom flange, $a' =$ that of the vertical rib, and $d =$ the depth from centre to centre of the flanges. Then the moment of inertia of the top or bottom flange is $= a \left(\frac{d}{2}\right)^2$ and of the vertical rib $= \frac{a' d^2}{12}$; whence,

$$\text{(IX.)} \qquad I = \frac{d^2}{12} (6a + a').$$

The *general* expression for the moment of inertia of a

flanged girder is very complicated.* When cases occur which cannot be included in either of the above classes, it is best first to find the position of the neutral axis, and then to take the moments of inertia of each part separately, and add the whole together.

Let a_1 and a_2 represent the areas of the bottom and top flanges respectively, and h_1 and h_2 their mean distances from the neutral axis. Also, let a_3 and a_4 represent the areas of the portions of the vertical rib below and above the neutral line. Then

$$\text{(X.)} \qquad I = \left(a_1 + \frac{a_3}{3}\right) h_1^2 + \left(a_2 + \frac{a_4}{3}\right) h_2^2 \text{ nearly.}$$

We shall hereafter give an example of the determination of the moment of inertia for a rectangular beam of complicated form, and this will serve as a guide for finding it, mechanically, for any form of section whatever.

We now proceed to the general problems of the strength of beams.

STRENGTH OF A BEAM SUPPORTED AT BOTH ENDS, AND LOADED IN THE MIDDLE.

The investigations of the strength and deflection of beams are founded on a principle of universal application, which it it may be well here, once for all, to enunciate. It is called *the principle of the equality of moments*, and is stated as follows:—

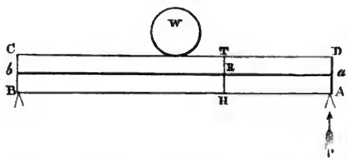
If any number of pressures in the same plane be in equilibrium, and any point be taken in that plane, from which their moments are measured, then the sum of the moments of those pressures which tend to turn the plane in one direction

* See "Moseley," page 503.

*about that point, is equal to the sum of the moments of those which tend to turn it in the opposite direction.**

Let ABCD, fig. 2, represent a beam supported at both

Fig. 2.



ends and loaded with a weight W in the middle, the weight of the beam itself not being taken into account.

Let R represent any point in the neutral line a R b of the beam, between the support A and the centre, and let a R $= x$. Also, let P = the reaction upon each of the supports A B.

Now let us apply the principle of the equality of moments to the portion A H R T D of the beam, taking R as the point from which the moments are measured, and round which that portion of the beam is to be considered in equilibrium. It will be observed that this portion of the beam is held in equilibrium by two forces, viz.—

1st. The pressure or reaction P on the support A, acting at a perpendicular distance $= x$ from the point R. The moment of this force round R is therefore $= Px$.

2d. The elastic forces called into operation on the transverse section H T of the beam. We have before designated the moment of these forces round R by Φ.

* Moseley, Art. 7.

These two forces tend to turn the portion of the beam in opposite directions round R, and therefore their moments must be equal to each other, *i. e.*

$$\text{(XI.)} \qquad \Phi = \mathrm{P}x.$$

But we know that since the weight W is borne equally by the two supports A and B, the reaction upon each will be = half the weight hung at the middle, *i. e.*

$$\mathrm{P} = \frac{\mathrm{W}}{2};$$

therefore

$$\text{(XII.)} \qquad \Phi = \frac{\mathrm{W}}{2}\,x,$$

which is the general equation of equilibrium for any point of the beam between either of the supports and the centre.

This may now be applied to ascertaining the strength of the beam.

It has already been explained that, in investigating the strength of beams, the problem is to discover the value of $\frac{\mathrm{W}}{f}$, *i. e.* the ratio between the weight a beam is supporting and the longitudinal strain on any of its fibres.

Now, by Equation I., we have $\Phi = \frac{f}{c}\,\mathrm{I}$; therefore, by substituting this value in Equation XII.

$$\frac{f}{c}\,\mathrm{I} = \frac{\mathrm{W}}{2}\,x,$$

or

$$\text{(XIII.)} \qquad \frac{\mathrm{W}}{f} = \frac{2}{cx}\,\mathrm{I},$$

which, applied to any section of the beam at a distance x from the end, expresses the relation between the weight W hung on the centre and the longitudinal strain f per square unit on any fibre of that section at a distance c from the neutral line.

Now, if we make the length of the beam = l, the strength

of the beam at the middle, where the weight is hung, will be found by making x in the above equation $= \frac{1}{2} l$, *i. e.*

$$\text{(XIV.)} \qquad \frac{W}{f} = \frac{4}{c\,l}\,\mathrm{I}.$$

Finally, substituting the value of I, as previously found, and making f apply to the extreme fibres at the top or bottom of the beam, we have the following formulæ for the strength of beams of different forms of section.

For Rectangular Beams.

Let a = sectional area, and d = depth; then by Equations V. and XIV.,

$$\frac{W}{f} = \frac{4}{c\,l} \times \frac{a\,d^2}{12} = \frac{a\,d^2}{3\,c\,l}.$$

But for the application of f to the extreme fibres of the beam, we must make $c = \frac{d}{2}$, whence we have

$$\text{(XV.)} \qquad \frac{W}{f} = \frac{2\,a\,d}{3\,l}.$$

For Solid Circular Beams.

Let a = sectional area, and d = diameter; then by Equations VI. and XIV.,

$$\frac{W}{f} = \frac{4}{c\,l} \times \frac{a\,d^2}{16}.$$

But for the extreme fibre $c = \frac{d}{2}$,

$$\text{(XVI.)} \qquad \therefore \quad \frac{W}{f} = \frac{a\,d}{2\,l}.^{*}$$

* A comparison of Equations XV. and XVI. shews that the strength of a circular beam is to that of a circumscribed square one as $\pi : \frac{16}{3}$, or as $1 : 1{\cdot}7$.

For Circular Tubes.

Let $r_1 =$ the external, and r_2 the internal radius; then

$$\frac{W}{f} = \frac{4}{c\,l} \times \frac{\pi(r_1^4 - r_2^4)}{4} = \frac{\pi}{c\,l}(r_1^4 - r_2^4);$$

but if the thickness t of the tube be small in proportion to the diameter d, we have by Equation VII., making $a =$ sectional area,

$$\frac{W}{f} = \frac{4}{c\,l} \times \frac{a\,d^2}{8},$$

or making $c = \frac{d}{2}$ for the top and bottom fibres, we have

$$\text{(XVII.)} \qquad \frac{W}{f} = \frac{a\,d}{l}.$$

For Rectangular Tubes, or I*-shaped Beams.*

Neglecting the sides, or vertical ribs, as elements of the strength, let $a_1 =$ the area of the bottom, $a_2 =$ area of the top, $A =$ total area $(= a_1 + a_2)$, and $d =$ depth of the tube; then by Equations VIII. and XIV.,

$$\frac{W}{f} = \frac{4}{c\,l} \times \frac{a_1\,a_2\,d_2}{A} = \frac{4\,a_1\,a_2\,d^2}{A\,c\,l}.$$

In resolving this equation further, by substituting for the value of c, we must bear in mind that c will have different values, according as the strain is estimated for either the top or bottom plates (or flanges); for, if a_1 be not $= a_2$, these will be at different distances from the neutral line. We shall have, therefore, two sets of equations, viz.—

For the Strain on the Bottom Plates.

Here,

$$c = \frac{a_2\,d}{A},$$

Therefore by substitution,

$$\text{(XVIII.)} \qquad \frac{W}{f} = \frac{4\,a_1\,d}{l}.$$

For the Strain on the Top Plates.

Here,

$$c = \frac{a_1\,d}{A},$$

whence

$$\text{(XIX.)} \qquad \frac{W}{f} = \frac{4\,a_2\,d}{l},$$

For a girder with flanges or plates of equal area at the top and bottom, including the vertical rib or plates as contributing to the strength, we have, by a combination of Equations Nos. IX. and XIV.,

$$\frac{W}{f} = \frac{4}{c\,l} \times \frac{d^2}{12}(6\,a + a');$$

but $c = \frac{d}{2}$; therefore,

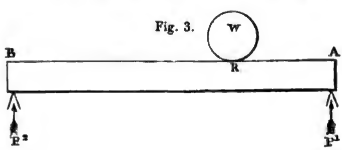
$$\text{(XX.)} \qquad \frac{W}{f} = \frac{4\,d}{l}\left(a + \frac{a'}{6}\right).$$

For flanged girders or rectangular tubes, in which the bottom and top plates are not of equal area, and the vertical plates are included, the moment of inertia must be calculated separately, and its value inserted in Equation XIV.

STRENGTH OF A BEAM SUPPORTED AT BOTH ENDS, AND LOADED AT ANY POINT OF ITS LENGTH.

Let A B (fig. 3) represent a beam loaded with a weight

Fig. 3.



$=$ W, at a point R. Let A R $= l_1$, B R $= l_2$, and the whole length A B $= l$. Also let P_1 and P_2 represent the resistance on the supports A and B respectively. Then applying Equation XI. to the section of the beam at R, or the position of the weight, we have, measuring from the end A, $\Phi = P_1 l_1$, and from the end B, $\Phi = P_2 l_2$, whence

$$\frac{P_1}{P_2} = \frac{l_2}{l_1},$$

but $$P_1 + P_2 = W,$$

therefore, $$P_1 = \frac{l_2}{l_1 + l_2} W = \frac{l_2}{l} W.$$

Substituting this value in the above equations for Φ, we obtain

$$\Phi = W \frac{l_2}{l} x,$$

which will give the elastic strain on any section of the beam between A and R, distant x from the end A. Similarly

$$\Phi = W \frac{l_1}{l} x$$

will give it for any section between R and B distant x from B.

To obtain the strength of the beam at the point where the weight is hung, make $x = l_2$ in the latter equation, then

$$\Phi = \frac{f}{c} I = W \frac{l_1 l_2}{l} *$$

or

(XXI.) $$\frac{W}{f} = \frac{l}{c\, l_1 l_2} I,$$

from which the strength may be found for any form of beam in the manner previously adopted for a weight hung in the middle.

* This equation shews that, if a weight be moved along a beam, the strain on the point immediately under the weight varies as the rectangle of the segments into which such point divides the length of the beam.

STRENGTH OF A BEAM SUPPORTED AT BOTH ENDS, AND LOADED UNIFORMLY OVER ITS WHOLE LENGTH.

In this case, of course, the weight of the beam itself, if uniform, may be considered as forming either part of the load, or, if requisite, the whole load.

Let, then, μ be the weight per lineal unit, distributed along the beam.

Proceeding as on page 249, and applying the principle of the equality of moments to the portion A H T D of the beam, and making $a\,R = x$, we observe that this portion is held in equilibrium by three forces, viz.

1st, The pressure or reaction P on the support A, the moment of which round R is $= P\,x$.

2dly, The portion of the load distributed between a and $R = \mu\,x$. This may be considered as collected at a point distant $\frac{1}{2}\,x$ from R; the moment of this, therefore, is $= \frac{1}{2}\,\mu\,x^2$.

3dly, The elastic forces called into action on the section of the beam at R, the moment of these being, as before, $= \Phi$.

Now the first of these three forces tends to turn this portion of the beam in one direction, and the second and third in the contrary direction; therefore,

$$P\,x = \tfrac{1}{2}\,\mu\,x^2 + \Phi,$$

or

$$\Phi = P\,x - \tfrac{1}{2}\,\mu\,x^2.$$

But we know that the reaction on each of the supports must be equal to half the load; *i. e.* $P = \frac{1}{2}\,\mu\,l$; whence

$$\text{(XXII.)} \qquad \Phi = \frac{\mu}{2}\,(l\,x - x^2),^*$$

* Or $\Phi = \frac{\mu}{2}\,(l - x)\,x$; which shews that in a beam loaded equally throughout, the strain at any point varies as the rectangle of the segments into which such point divides the length of the beam.

which is the general equation of equilibrium for any point of the beam.

Applying Equation I. to the above expression, we obtain,

$$\frac{f}{c}\,\mathrm{I} = \frac{\mu}{2}\,(l\,x - x^2).$$

In the middle of the beam, where the strain is greatest, $x = \frac{1}{2}\,l$,

$$\therefore \frac{f}{c}\,\mathrm{I} = \frac{\mu\,l^2}{8},$$

or

(XXIII.) $$\frac{\mu\,l}{f} = \frac{8}{c\,l}\,\mathrm{I},$$

from which we shall determine the strength for different forms of section, following the processes given in page 251. The quantity $\mu\,l$ will in all cases represent the total load distributed over the beam.

For Rectangular Beams.

Here,

(XXIV.) $$\frac{\mu\,l}{f} = \frac{8}{\frac{1}{2}\,d\,l} \times \frac{a\,d^2}{12} = \frac{4\,a\,d}{3\,l}.$$

For Circular Beams.

(XXV.) $$\frac{\mu\,l}{f} = \frac{8}{\frac{1}{2}\,d\,l} \times \frac{a\,d^2}{16} = \frac{a\,d}{l}.$$

For thin Circular Tubes.

(XXVI.) $$\frac{\mu\,l}{f} = \frac{8}{\frac{1}{2}\,d\,l} \times \frac{a\,d^2}{8} = \frac{2\,a\,d}{l}.$$

For Rectangular Tubes and Flanged Beams.

Neglecting the vertical rib,

$$\frac{\mu\,l}{f} = \frac{8}{c\,l} \times \frac{a_1\,a_2\,d^2}{\mathrm{A}},$$

or, making the necessary substitution for c,

For the Bottom Plates.

(XXVII.) $$\frac{\mu l}{f} = \frac{8 a_1 d}{l}.$$

For the Top Plates.

(XXVIII.) $$\frac{\mu l}{f} = \frac{8 a_2 d}{l}.$$

If the flanges or plates at top and bottom are equal in area, and the vertical rib is included, we have, by Equations IX. and XXIII.,

$$\frac{\mu l}{f} = \frac{8}{c l} \times \frac{d^2}{12} (6 a + a'),$$

or

(XXIX.) $$\frac{\mu l}{f} = \frac{8 d}{l} \left(a + \frac{a'}{6}\right).$$

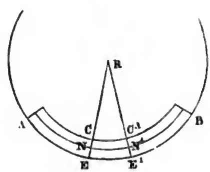
For the *general* case of rectangular tubes and flanged beams, the particular value of I must be substituted in Equation XXIII.

CHAPTER III.

THE DEFLECTION OF BEAMS.

The principles on which the curvature, and consequent deflection, of a rectangular tube is determined, may be easily understood. In the figure below let A B represent the Conway Tube deflected from its own weight, and let us suppose its curvature to be part of a circle, and the circle itself completed. Let N N′ be the neutral axis, and suppose the strain on the top at C C′ to be 5 tons per square inch, so that the small portion C C′ is compressed $\frac{5}{10000}$, or $\frac{1}{2000}$ of its length.

Fig. 1.



Now, if the portion N N′, which is not extended or compressed, be represented by 2000, C C′ will be equal to 1999; and we have—

$$N N' : C C' :: N' R : C' R.$$

Calling N′ R, or the radius of curvature r, and C′ N′ being = 13 feet, we have,

$$2000 : 1999 :: r : r - 13;$$

whence $\quad 2000 \times (r - 13) = 1999\, r,$

or $\quad (2000 - 1999)\, r = 2000 \times 13,$

$r = 26000$ feet, nearly 5 miles.

Secondly, knowing the radius of curvature O D, fig. 2, let A D B represent the tube, of which it is required to find the deflection or the versed sine C D,

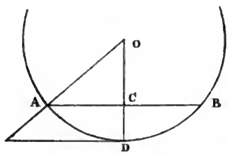
We have

$$O\,C = \sqrt{O\,A^2 - C\,A^2}$$

Now,

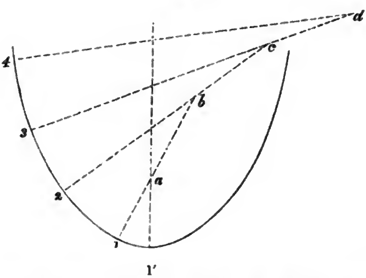
$$\begin{aligned} C\,D &= O\,D - O\,C \\ &= r - \sqrt{O\,A^2 - C\,A^2} \\ &= 26000 - \sqrt{26000^2 - 200^2} \text{ feet} \\ &= 9{\cdot}24 \text{ inches.} \end{aligned}$$

Fig. 2.



That this deflection is too great will be evident, if we consider that we have assumed the curvature to be the same throughout the whole length of the tube; but the curvature will vary with the strain, which is as the rectangle of the segments. The above radius of curvature belongs only to the centre of the beam; we might, however, similarly find the radius for any other portion, 1..2, or 2..3 (fig. 3), and we should find the radius of curvature to be longer as we recede from the centre, and at the extremities it would be infinite.

Fig. 3.



We might thus plot the curve, for we have only from centre a with radius 1 a to draw the small arc 1..1′; similarly, with radius 2 b, we might describe the arcs

1..2, and, with the radius 3 c, the arcs 2..3, &c. And if the radii are properly taken, this would give us the correct curve of deflection, the curvature being greatest at the centre. An equation to the curve may be obtained on these principles, and the deflection determined analytically, as will be done hereafter.

The following geometrical treatment of the subject of deflection is more in accordance with the elementary pretensions of this publication.

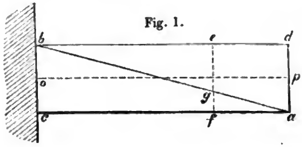
GEOMETRICAL INVESTIGATION OF THE DEFLECTION OF BEAMS.

In this investigation we will assume the same properties for the material of which they are constructed as in Chap. I. upon the Strength of Beams, viz., that it is compressed and extended through equal spaces by equal weights, the spaces being as the weights. We will also again neglect all consideration of the vertical forces, and consider the deflection as caused by the horizontal forces. The alteration in the length of leverage due to the deflection of the beam is also inappreciable in all practical cases.

Having, as before, resolved the sums of the horizontal equal and opposite forces into definite forces acting at a certain distance apart, which, in a beam of any uniform section, is constant throughout its length, let us cease to consider the resisting forces in the beam as acting at any other points than these.

In the case of a semibeam, $a\,c\,b\,d$ (fig. 1), attached at one end to a solid wall, and loaded at its extremity, we have already determined that the strains upon the centres of resistance close to the wall

Fig. 1.



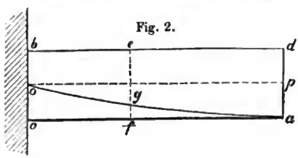
are directly as the amount of the weight, directly as the length of the semibeam, and inversely as the distance between the centres of resistance. We have also stated that the strains at other parts increase directly as the distance from the extremity; if the strain close to the wall be represented, therefore, by a line of given length, *b c*, the strain at any other section, *e f*, may be represented by the line *g f*; in fact, the triangle *b c a* represents the varying strains upon the centres of resistance.

Let us imagine the beam divided by vertical lines into an infinite number of thin laminæ: the amount of inclination between the vertical wall and the face of the adjacent lamina will depend upon the amount of the strain upon the centres of resistance, their distance asunder, and the elasticity of the material.

In treating of the deflection of a beam of uniform section, the two last elements being constant may be neglected; the inclination, therefore, is simply as the strain. But the same holds good for the inclination between the faces of any other laminæ in the beam: the sum of all the inclinations, or the inclination between the face of the wall and the end of the beam, is therefore composed of inclinations, each varying as the distance from *a*, and the number of which varies as the length of the beam or as the line *a c*. In other words, the final inclination is as the area of the triangle *a b c*.

If, instead of the semibeam being loaded at the extremity, it is uniformly loaded with the same weight, the strain upon the centres of resistance at the wall will be one-half what it was before, and this strain at other sections will vary as the square of the distance from the extremity *a*.

Fig. 2.

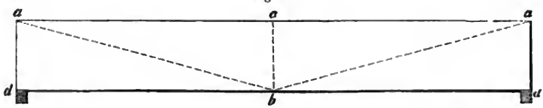


That is to say, *o c*

(fig. 2) will be one-half of *b c*, and the area of *a g o c*, the figure representing the varying strains upon the centres of resistance, will, as before, represent the final inclination.

The area of this figure, being the difference between the area of a parabola and its circumscribing rectangle *o c a p*, is one-third of the latter. But the triangle *b c a* is equal in area to the rectangle *o c a p*; therefore, the final inclination in a beam loaded uniformly is one-third of the final inclination produced by the same weight hung at the extremity.

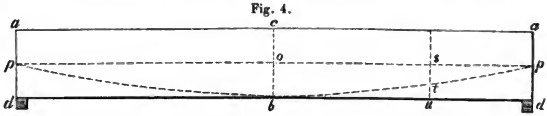
Fig. 3.



If, instead of the semibeam in fig. 1, we take, as in fig. 3, the case of a beam double its length, loaded in the centre with double the weight which was applied to the former, then each half-beam is in precisely similar circumstances to those of the main beam, as the load at *d* is the same as was there applied, the strains upon the centre of resistance at *c b* remain unaltered, as do also those strains between *c b* and *a d*: the inclination between *c b* and *a d* is, therefore, equally in extent in both cases.

But if, instead of loading this beam in the centre, we apply the same weight uniformly distributed, the strain upon the centres of resistance is at the centre section, *c b* (fig. 4), one-half of what it was, and this strain at any part,

Fig. 4.



s u, is compared to that at the centre as the rectangle of

the segments, or, in other words, as the ordinates of the parabola *b p*.

The figure *o b p*, therefore, which represents the final inclination between a central vertical section and the extremity is two-thirds of the rectangle *o b d p*; but the area of the triangle *a b c*, representing the final inclination in the beam (fig. 3), is likewise equal to the rectangle *o b d p*; therefore the final inclination of the extremity of a beam loaded uniformly is two-thirds of that of a beam loaded with the same weight in the centre.

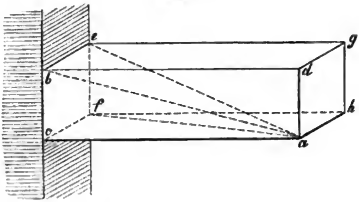
The final inclinations of these similar semibeams (figs. 2, 1, and 4), each strained to the same extent at the section *c b*, but loaded in the manner described, would therefore be compared with each other as the numbers 2, 3, and 4.

We have hitherto treated simply of the amount of inclination between sections of the loaded beams, we will now proceed to consider the amount of *deflection*. Recurring to our first example (fig. 1), it was seen that the inclination between any two adjacent laminæ varied as their distance from *a*; it was also seen that the final inclination was dependent upon the area of the figure *b c a*, and not upon its form; that, if the centres of resistance had been loaded throughout the length of the beam with a uniform strain, equal to half of *b c*, as the area *o c a p*, which would represent such strains, is equal to the area *b c a*, the final inclination would be alike in both cases.

This is, however, by no means the case as regards the *amount* of deflection, for whatever is the inclination between the wall and the adjacent plate it is communicated to the whole of the beam, whereas the inclination in any lamina midway between the wall and the extremity is merely communicated to half of the beam; wherefore, the value of the strain upon the centres of resistance at any particular

section varies as regards amount of deflection directly as the distance of this section from the extremity.

Fig. 5.

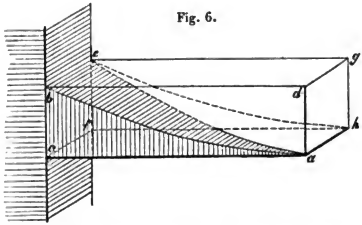


In the case of a beam loaded at the extremity we have seen that with a given load the strain upon the centres of resistance varies at any part directly as its distance from *a*: we now find that its value for producing deflection varies in the same ratio; therefore, the amount of deflection of the extremity caused by the inclination in any one lamina is as the square of its distance from *a*, and the sum of all these amounts may be represented by the pyramid *b c e f a*, the contents of which are equal to one-third of the prism *c g*.

In the case of a semibeam loaded uniformly, the strains upon the centres of resistance and the consequent inclination in any lamina vary as the square of the distance from *a*, and the surface *b c a* expressing this ratio is bounded by a parabola; but as the effect of the inclination in each lamina in producing deflection at the extremity varies as its distance from that point, it is evident that in a semibeam uniformly loaded the effect of inclination in any lamina in producing deflection at the extremity varies as the cube of this distance from that point. This ratio may be geometrically expressed by the solid *b e f c a*, wherein the vertical heights vary as the square of the distance from *a*, and the breadths vary directly as that distance.

This solid, therefore, correctly represents the sum of the

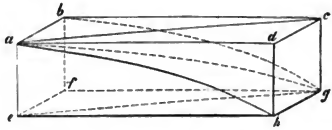
Fig. 6.



deflections caused by the varying inclinations, or, in other words, represents the actual deflections at *a*. The cubical content of such a solid is equal to one quarter of that of the prism *c g*.*

In the case of a beam loaded uniformly, and supported at both ends, it has been seen that the strains upon the centres of resistance, and the consequent amount of inclination in each lamina, varies as the ordinates of a parabola; applying

* The cubical contents of the two solids referred to may be calculated as follows:—

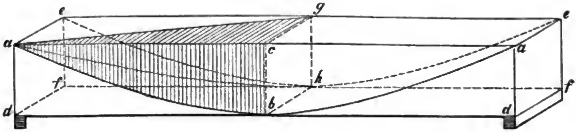


Let the line *a h* be a parabola, the surface *a e h d* is divided by it into two parts bearing to each other the proportion of 2 to 1; the parabolic surface *a b g h* divides the prism *a g* in the like proportion.

Again, the triangular prism *a c d e g h*, which is one-half of the prism *a g*, is bisected by the portion of the same parabolic surface which passes through it; for if we imagine that the rectangle *a d h e* revolves about *a e* as an axis, it will generate a cylinder; while the line *a h* will generate a paraboloid. But the content of a paraboloid is equal to one-half that of the circumscribing cylinder, and the same proportion will exist between any similar sections of these bodies, however small, even though the arc be so small that

to this case the law, that the value of the inclination in each

Fig. 7.



lamina for causing deflection at *a*, varies as its distance from *a*, we obtain the solid *a c b h g* as the representative of the sum of the deflections caused by the varying inclinations. The cubical content of such a solid is equal to $\frac{5}{12}$ of the prism *a h*. (*See Note.*)

If we compare the amount of the deflections of the semi-beam loaded in the manner shewn by figs. 7, 8, and 9, assuming their dimensions and the strain at *c b* to be the same in all cases, we find that they are represented by solids whose contents are respectively equal to $\frac{1}{3}$rd, $\frac{1}{4}$th, and $\frac{5}{12}$th of the same prism: they bear, therefore, to each other the proportions of 4, 3, and 5.

The alteration in the deflection due to an alteration in any one of the dimensions of the beam may be arrived at as follows:—

If the length of the beam fig. 5 be increased, the strains upon the centres of resistance, and the consequent inclination in each lamina, will be increased in the same ratio; the increase of length, also, directly increases the line *b e*, which

it does not differ from a straight line. But the triangular prism may be conceived to be made up of an infinite number of such segments of various radii, each of which would be bisected by surfaces not appreciably differing from the parabolic surface *a h g*, therefore the triangular prism *a e h g e d* is bisected by the surface *a h g*. But as the solid *a e h g* is thus proved to be one quarter of the entire prism *a g*, the solid *a b e f g*, which, together with the former, is equal to two-thirds of the same prism, must itself be equal to $\frac{2}{3} - \frac{1}{4} = \frac{5}{12}$ of it. The prism is thus divided into four parts bearing to each other the proportions of $\frac{1}{12}$, $\frac{5}{12}$, $\frac{3}{12}$, $\frac{3}{12}$.

represents the effect of the inclination of the laminæ in producing deflection: the base of the pyramid $b\,c\,e\,f$ is, therefore, increased in both its dimensions by the increase of length; but the lengthening of the beam also increases the length of the line $c\,a$,—the third dimension of the pyramid—which thus, being increased in each of its three dimensions directly as the increase in the length of the beam, has its cubical content, which represents the amount of deflection at a increased as the *cube* of the length.

The effect of increasing the width of the beam would be simply to diminish in an inverse ratio the length of the line $c\,b$, for by increasing the width we increase the area of the surface offering resistance, and therefore diminish its intensity. The pyramid being, therefore, diminished in one of its dimensions, but remaining unaltered in the others, its cubical contents, which represent the amount of deflection at a, would vary inversely as the width of the beam.

In considering the effect of increasing the depth of a beam, we must refer to page 217, fig. 3, the increase of depth evidently increases directly both the area of the triangles $a\,b\,x$ and $c\,d\,x$, offering resistance to pressure and tension, and also increases directly the distance between r and r'; the extent of the compression and extension of each lamina is, therefore, diminished as the depth of the beam is increased, owing to the strains being resisted by areas increasing in this ratio; secondly, the amount of strain itself is diminished in the same ratio, owing to the distance between r and r' being so increased, and the leverage with which the load is acting being thereby affected; and thirdly, the deflection caused by the compression and extension of each lamina is diminished in the same ratio, owing to the angle of inclination arising from any given amount of compression and extension varying as the depth of the beam. The deflection of a beam varies, therefore, as the cube of its depth.

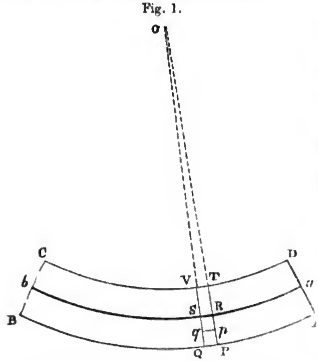
ANALYTICAL INVESTIGATION OF FORMULÆ FOR THE DEFLECTION OF BEAMS.

We shall now proceed to investigate the deflection of beams analytically, proceeding on the principles already adopted for the determination of the strength.

The deflection of a beam is most commonly taken only at one point, namely, at the place where it is greatest, or at the centre of the beam. The following method, however, will give the deflection for the whole length of the beam, *i. e.* will determine the complete deflection curve. This method of treating the subject is not only more general and satisfactory, but becomes, as will be seen, essential in the elucidation of the principles of *continuous* beams, to which it will hereafter be applied.

Resuming the consideration of the moment of the elastic forces exerted at any given section of a deflected beam (see p. 242), we shall deduce another value for this moment applicable to the deflection curve.

Fig. 1.



Let A B C D (fig. 1) represent a portion of a beam whose neutral line is *a b*. Let P T and Q V be transverse sections exceedingly near to each other, and perpendicular to the neutral line at the points R and S; and let O be the point where P T and Q V would intersect when produced, or the centre of curvature to the neutral line at R S.

Let $a\,\mathrm{R} = x$; $\mathrm{S\,R} = \Delta x$; and imagine the lamina P Q V T to be made up of fibres parallel to S R: then will Δx represent the length of each of these fibres before the deflection of the beam, since the length of the neutral fibre S R has remained unaltered by the deflection. Let δx represent the quantity by which the fibre $p\,q$ has been elongated by the deflection of the beam; then is the actual length of that fibre represented by $\Delta x + \delta x$.

Now, if E represent the *modulus of elasticity* of the material, and Δk the area of section of the fibre (or an exceedingly small element of the section P T), then the force which must have operated to produce the elongation δx in a fibre whose previous length was Δx, will be represented by $\mathrm{E}\,\frac{\delta x}{\Delta x}\,\Delta k$.

Let the radius of curvature O R be represented by R, and the distance $\mathrm{R}\,p$ by ϱ. By similar triangles $\frac{\mathrm{O}p}{\mathrm{OR}} = \frac{pq}{\mathrm{SR}}$, or $\frac{\mathrm{R}+\varrho}{\mathrm{R}} = \frac{\Delta x + \delta x}{\Delta x}$, or $1 + \frac{\varrho}{\mathrm{R}} = 1 + \frac{\delta x}{\Delta x}$; therefore $\frac{\varrho}{\mathrm{R}} = \frac{\delta x}{\Delta x}$. Substituting this value of $\frac{\delta x}{\Delta x}$ in the expression for the pressure which must have operated to produce the elongation of the fibre $p\,q$, and representing that pressure by $\Delta \mathrm{P}$, we have

$$\Delta \mathrm{P} = \mathrm{E}\,\frac{\delta x}{\Delta x}\,\Delta k = \frac{\mathrm{E}}{\mathrm{R}}\,\varrho\,\Delta k.$$

Proceeding as on page 243, the *moment* of this force is found $= \frac{\mathrm{E}}{\mathrm{R}}\,\varrho^2\,\Delta k$; and designating, as before, the sum of the moments of all the elastic forces, by Φ, we have,

$$\text{(XXX.)} \qquad \Phi = \frac{\mathrm{E\,I}}{\mathrm{R}},$$

where I = moment of inertia of the section about the neutral axis. This gives the moment of the elastic force at any section of the beam, in terms of the radius of curvature at that point.

We may now transform this expression into one involving the elements of the *deflection* curve.

Fig. 2.

Let a R b (fig. 2) be the neutral line of the beam, as deflected from its original horizontal position $a\,h\,b$.

Let $a\,h = x$, and h R, the deflection at this point, $= y$. Now, by the principles of the differential calculus, we have, when the curve is concave to the axis of x,

$$\text{Rad. of curvature} = \frac{d\,x^2 \left(1 + \frac{d\,y^2}{d\,x^2}\right)^{\frac{3}{2}}}{-\,d^2\,y}$$

or
$$\frac{1}{\mathrm{R}} = -\frac{d^2\,y}{d\,x^2}\left(1 + \frac{d\,y^2}{d\,x^2}\right)^{-\frac{3}{2}}.$$

But as the deflection of beams is usually very small compared to their length, the inclination to the horizontal of the tangent to the neutral line is, at all points, very small also; so that $\left(\frac{d\,y}{d\,x}\right)^2$ may be neglected as compared with unity, whence we may take

$$\frac{1}{\mathrm{R}} = -\frac{d^2\,y}{d\,x^2};$$

or, by substituting this value in Equation XXX., we have,

$$\text{(XXXI.)} \qquad \Phi = -\,\mathrm{E\,I}\,\frac{d^2\,y}{d\,x^2},$$

which expresses the moment of the elastic forces at any section of the beam, in terms of the co-ordinates of the deflection curve.

DEFLECTION OF BEAMS SUPPORTED AT BOTH ENDS AND LOADED IN THE MIDDLE.

This is determined by the combination of Equation XII. ($\Phi = \frac{W}{2} x$) with Equation XXXI., which gives the value of Φ in terms of the elements of the deflection curve. We thus obtain by substitution,

$$-EI\frac{d^2 y}{d x^2} = \frac{W}{2} x,$$

or,

$$EI\frac{d^2 y}{d x^2} = -\frac{W}{2} x,$$

where E is the modulus of elasticity, and I is the moment of inertia of the section.*

Integrating this last equation we have

$$EI\frac{d y}{d x} = -\frac{W}{4} x^2 + \text{Constant}.$$

To find the constant we must bear in mind that at the middle of the beam, where $x = \frac{l}{2}$, and where the deflection is at a maximum, $\frac{dy}{dx} = 0$; therefore

$$\text{Constant} = \frac{W}{16} l^2,$$

$$\therefore EI\frac{dy}{dx} = \frac{W}{4}\left(\frac{l^2}{4} - x^2\right)$$

Integrating again,

$$\text{(XXXII.)} \qquad EIy = \frac{W}{4}\left(\frac{l^2}{4} x - \frac{x^3}{3}\right)$$

which is the equation to the deflection curve; y being the deflection at any point at a distance x from the end of the beam.

In adapting this general equation to particular forms of

* This is supposed uniform throughout the length of the beam.

beams, we shall, for brevity's sake, confine our attention to the deflection at the centre of the beam, where $x = \frac{1}{2} l$; let this deflection be called D. Then

$$\text{(XXXIII.)} \qquad \mathrm{E\,I\,D} = \frac{\mathrm{W}\,l^3}{48}.$$

For Rectangular Beams.

If $a =$ area, and $d =$ depth, we have, by Equations V. and XXXIII., making $x = \frac{1}{2} l$.

$$\mathrm{E\,D}\frac{a\,d^2}{12} = \frac{\mathrm{W}\,l^3}{48},$$

or,

$$\text{(XXXIV.)} \qquad \mathrm{D} = \frac{\mathrm{W}\,l^3}{4\,\mathrm{E}\,a\,d^2}.$$

For Circular Beams.

If $a =$ sectional area, and $d =$ diameter, by Equations VI. and XXXIII.,

$$\mathrm{E\,D}\frac{a\,d^2}{16} = \frac{\mathrm{W}\,l^3}{48},$$

or,

$$\text{(XXXV.)} \qquad \mathrm{D} = \frac{\mathrm{W}\,l^3}{3\,\mathrm{E}\,a\,d^2}. *$$

For Circular Tubes.

Where the thickness bears but a small proportion to the diameter d, we have, by Equations VII. and XXXIII., making $a =$ sectional area,

$$\mathrm{E\,D}\ \frac{a\,d^2}{8} = \frac{\mathrm{W}\,l^3}{48},$$

or

$$\text{(XXXVI.)} \qquad \mathrm{D} = \frac{\mathrm{W}\,l^3}{6\,\mathrm{E}\,a\,d^2}.$$

* By comparing this with Equation XXXIV., it will be found that the deflection of a circular beam is to that of the circumscribed square beam as $16 : 3\,\pi$; or as $1{\cdot}7 : 1$.

For Rectangular Tubes and Flanged Girders.

Neglecting the vertical rib, and using the former notation,

$$E\,D\,\frac{a_1\,a_2\,d^2}{A} = \frac{W\,l^3}{48},$$

or

(XXXVII.) $$D = \frac{W\,A\,l^3}{48\,E\,a_1\,a_2\,d^2}.$$

If the vertical rib be taken into account, and the top and bottom flanges be of equal area,

$$E\,D\,\frac{d^2}{12}(6\,a + a') = \frac{W\,l^3}{48},$$

or

(XXXVIII.) $$D = \frac{W\,l^3}{4\,E\,d^2\,(6\,a + a')}.$$

DEFLECTION OF BEAMS SUPPORTED AT EACH END, AND LOADED UNIFORMLY OVER THEIR LENGTH.

Referring back to Equation XXII. page 255, and substituting in it the value of Φ obtained in Equation XXXI., we have,

$$-E\,I\,\frac{d^2\,y}{d\,x^2} = \frac{\mu}{2}\,(l\,x - x^2).$$

Integrating, and remembering that in the middle of the beam (where $x = \frac{1}{2}\,l$) the deflection is a maximum, and therefore $\frac{d\,y}{d\,x} = 0$, we have,

$$E\,I\,\frac{d\,y}{d\,x} = \frac{\mu}{2}\left(\frac{x^3}{3} - \frac{l\,x^2}{2} + \frac{l^3}{12}\right).$$

And integrating again,

(XXXIX.) $$E\,I\,y = \frac{\mu}{24}\,(x^4 - 2\,l\,x^3 + l^3\,x),$$

which is the equation to the deflection curve.

T

Making D = the deflection in the middle of the beam, we have,

$$\text{(XL.)} \qquad \text{E I D} = \frac{5\,\mu\,l^4}{384}.*$$

Adapting this to different forms of beams as before, we obtain,

For Rectangular Beams.

$$\text{(XLI.)} \qquad \text{D} = \frac{5\,\mu\,l\,.\,l^3}{32\,\text{E}\,a d^2}.$$

For Circular Beams.

$$\text{(XLII.)} \qquad \text{D} = \frac{5\,\mu\,l\,.\,l^3}{24\,\text{E}\,a\,d^2}.$$

For thin Circular Tubes.

$$\text{(XLIII.)} \qquad \text{D} = \frac{5\,\mu\,l\,.\,l^3}{48\,\text{E}\,a\,d^2}$$

For Rectangular Tubes.

Neglecting the sides,

$$\text{(XLIV.)} \qquad \text{D} = \frac{5\,\text{A}\,\mu\,l\,.\,l^3}{384\,\text{E}\,a_1\,a_2\,d^2}.$$

If the sides be taken into account, and the top and bottom plates be of equal area,

$$\text{(XLV.)} \qquad \text{D} = \frac{5\,\mu\,l\,.\,l^3}{32\,\text{E}\,d^2\,(6\,a + a')}.$$

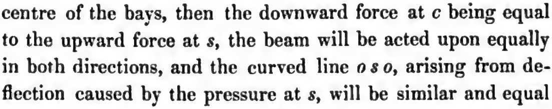
* Or $= \frac{5}{8} \times \frac{\mu\,l\,.\,l^3}{48}$. A comparison of this with Equation XXXII. shews that the deflection of a beam loaded uniformly over its length is $\frac{5}{8}$ths of the deflection when the same load is hung on the centre.

CHAPTER IV.

ON CONTINUOUS BEAMS.

We have hitherto treated only of beams supported at the extremities. If the extremities of a beam are fixed, or if the beam is continued beyond the bearings, or supported at many parts of its length, its strength and flexure will evidently be modified. Previous to a more general investigation, we shall first illustrate this subject in an elementary manner, by determining geometrically the strains upon a beam of uniform section supported at regular intervals.

If there be a beam, of which *a b* represents a portion, supported at regular intervals at *s s s*, and if this beam be



assumed to be without weight, but to be loaded at *c c*, the centre of the bays, then the downward force at *c* being equal to the upward force at *s*, the beam will be acted upon equally in both directions, and the curved line *o s o*, arising from deflection caused by the pressure at *s*, will be similar and equal in length to the curve *o c o*, arising from the pressure at *c*.

At *o*, which may be called the point of contrary flexure, the material of which the beam is composed is evidently free from any strain, excepting a vertical one equal to half the load at *c*; in fact, were the beam severed at the points *o*, and were the central portions *o c o* of the severed beam suspended from

these points, no alteration in the form of the portions of the beam, nor in the strains upon them, would take place.

The strength of a beam thus supported, as far as regards loads applied centrally between the supports, would, as the virtual length of each beam is but one half the span, be compared to a similar beam severed at the points of support, as 2 to 1.

The deflection of the beam *o c o* being half the length of an unconnected beam resting upon the supports at *s s*, would be but one-eighth of that of the latter, but the beam *o s o* having deflected to a like extent, the total deflection at *c* would be in a continuous beam one quarter of that in a disconnected beam.

If, instead of applying central loads, we suppose the beam to be loaded uniformly, or to have merely its own weight to support, the case is materially altered.

The beam *o c o* has now to support its own weight equally distributed, the semibeam, or cantilever, *s o*, has to support half the weight of the beam *o c o* suspended from its extremity, and, in addition to this, its own weight distributed.

Let us suppose that we have found the position of the point of contrary flexure, and have severed the beam as before, the severed face of the beam *o s o* evidently makes, with a vertical line, the same angle as does the severed face of the beam *o c o*.

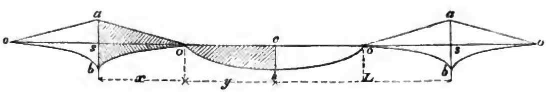
The knowledge of the peculiar strains due to this distribution of the load, and of the identity of the inclination of the severed faces, enables us to determine the position of the point of contrary flexure.

It will simplify the investigation if we continue to consider the compressive and tensile strains to be concentrated in the centres of resistance, the distance between which we shall call *d*, and which, in the case of a beam of uniform section, is constant throughout the length.

We shall also continue to assume that, within the limits to which it is safe to strain the material of which a beam is made, equal compression and extension are produced by equal weights, the extent being as the weight.

The horizontal strains in the semibeam *s o*, it has been already stated, are due, firstly, to the half weight of the beam *o c o* suspended from the point *o*; secondly, to the distributed weight of the semibeam itself. The strain arising from the former increases as the distance from the point of suspension *o*, and the sum of its effect in producing inclination upon the vertical laminæ of the semibeam, or, in other words, the final inclination due to this cause, as has been demonstrated in Chapter III., may be represented by a trough, of which the base is *s o* and the depth is a line *s a*, representing the horizontal strains at *s* and *a*, arising from the half weight of the beam *o c o*.

The final inclination arising from the weight of the semibeam *s o*, it has also been demonstrated, may be represented by the surface *s b o*, in which *b o* is a parabolic curve, and in



which the line *s b* represents the horizontal strains at *s* and *b* due to the weight of the semibeam.

In the semibeam *o c* the final inclination arising from the distributed load may be represented by the figure *o c h*, in which the line *o h* is a parabolic curve, and in which the line *c h* represents the horizontal strain at the centre due to the distributed load, which is equal to that caused by half the same load if applied in the centre, or by one quarter of the same load if applied upwards at the extremity *o*.

We thus find that the final inclination of the semibeam *s o* is represented by the combined areas of the figures *a s o*

and $b\ s\ o$, while that of the semibeam $c\ o$ is represented by the area of the figure $o\ c\ h$, but the final inclinations of the semibeams are necessarily equal, therefore

$$a\,s\,o + b\,s\,o = o\,c\,h \qquad \text{(No. 1.)}$$

Let us now, by reducing this equation, ascertain the actual position of the point o, the distance of which from the support we shall call x, and from the centre of the opening y.

The value of $a\ s = \frac{y \times x}{d}$ (No. 2.)

The value of the area $a\ s\ o = \frac{y \times x}{d} \times x \times \frac{1}{2} \frac{y\ x^2}{2d}$ (No. 3.)

The value of $s\ b = \frac{\frac{x}{2} \times x}{d}$ (No. 4.)

The value of the area $o\ s\ b = \frac{\frac{x}{2} \times x}{d} \times x \times \frac{1}{3} \frac{x^3}{6d}$ (No. 5.)

The value of the $c\ h = \frac{\frac{y}{2} \times y}{d}$ (No. 6.)

The value of the area $o\ c\ h = \frac{\frac{y}{2} \times y}{d} \times y \times \frac{2}{3} = \frac{y^3}{3d}$ (No. 7.)

The equation may now, therefore, be rendered:

$$\frac{y\ x^2}{2d} + \frac{x^3}{6d} = \frac{y^3}{3d} \qquad \text{(No. 8.)}$$

Multiply by d: $\frac{y\ x^2}{2} + \frac{x^3}{6} = \frac{y^3}{3}$ (No. 9.)

Let L = half the span, or $s\ c$,

Then, $y = \mathrm{L} - x$ (No. 10.)

Substituting this value of y in the Equation No. 9.

$$\frac{(\mathrm{L} - x)\ x^2}{2} + \frac{x^3}{6} = \frac{(\mathrm{L} - x)^3}{3}$$

By expanding, $\frac{L x^2 - x^3}{2} + \frac{x^3}{6} = \frac{L^3 - 3 L^2 x + 3 L x^2 - x^3}{3}$

Multiplying by 6, $3 L x^2 - 3 x^3 + x^3 = 2 L^3 - 6 L^2 x + 6 L x^2 - 2 x^3$

Cancelling and transposing, $3 L x^2 - 6 L^2 x = - 2 L^3$

Dividing by 3 L, $x^2 - 2 L x = - \frac{2}{3} L^2$

$$x = \frac{L^2}{\sqrt{3}}$$

And $$y = \frac{L}{\sqrt{3}}$$

If $L = 50$ feet, then $x = 21{\cdot}14$ feet, and $y = 28{\cdot}86$.

The points of contrary flexure in a continuous beam, supported at intervals of 100 feet, are therefore at distances of 21·14 feet from the supports.

The length of the beam *o c o* is therefore $2 \frac{L}{\sqrt{3}}$ but as the strains upon the centres of resistance of beams loaded in proportion to the length vary as the square of their length, these strains in the continuous beam are to those in a detached beam with the same bearing as

$$\left(2 + \frac{L}{\sqrt{3}}\right)^2 : 2 L^2$$

Or as $$\tfrac{4}{3} L^2 : 4 L^2$$

Or as $$1 : 3$$

These strains at that section of the beam over the bearing are, as has been already explained, due to the half weight of the beam *o c o*, hung at the extremity of the semibeam *s o*, in addition to the half weight of the semibeam *s o*, suspended from the same point, or, the weight being as the length, are due to a weight equal to

$$\frac{L}{\sqrt{3}} + \frac{L - \frac{L}{\sqrt{3}}}{2}.$$

Thus, for the calculation of these strains, we may look upon *o s o* as a beam whose length is $2\left(L - \frac{L}{\sqrt{3}}\right)$ loaded in

the centre with a weight of $2\frac{L}{\sqrt{3}} + L - \frac{L}{\sqrt{3}} = L + \frac{L}{\sqrt{3}}$ and comparing the strains due to these data with those due to the data for the detached beam, whose length is 2 L, we find that the ratio is

$$\text{As} \quad \left(L + \frac{L}{\sqrt{3}}\right) \times 2\left(L - \frac{L}{\sqrt{3}}\right) : L \times 2L$$

$$\text{Or as} \quad L^2 - \frac{L^2}{3} \quad : \quad L^2$$

$$\text{Or as} \quad 2 \quad : \quad 3$$

The relative horizontal strains, therefore, in the three positions of which we have treated, are as follows :

At the centre section of a detached beam of uniform section	3
At the section over the bearing of a continuous beam of similar section, and of the same length between bearings....	2
At the centre section of the same continuous beam	1

Analytical Investigation of the Strength and Deflection of Continuous Beams.

We now proceed to apply to continuous beams the analytical methods of investigation we have already adopted for detached beams; and in order to render the investigation less intricate, we shall here confine our attention to beams loaded uniformly over their length, and shall include the following cases, viz. :—

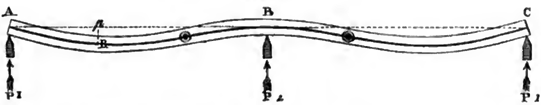
1st. The case of beams supported at three points;
2d. ,, ,, ,, four points;
3d. ,, ,, ,, five points;

And lastly, we shall add some observations on the case already treated, where a beam is supposed perfectly continuous, and supported at regular intervals throughout.

A BEAM SUPPORTED AT THREE POINTS.*

Let Fig. 1 represent a beam of uniform section sup-

Fig. 1.



ported at three points, A, B, and C, and loaded uniformly with a weight $= \mu$ per lineal unit, over its whole length. Let $AB = BC = l$, then $\mu l =$ the weight distributed over each opening. Let $P_1 =$ the reaction or resistance at each of the supports A and C, and $P_2 =$ that at B. Let $Ap = x$, and $pR = y$, the co-ordinates of the deflection curve; this curve will be concave upwards near the ends of the beam A and C, but convex upwards for some distance on each side the centre support B. Let O O be the points where the convexity ends and the concavity begins, or *the points of contrary flexure.*

Now proceeding in the same manner as on page 255, it is obvious that the portion A R of the beam must be held in equilibrium by three forces, viz.:—

1. The resistance P_1, whose moment about $R = P_1 x$.

2. The load μx on the portion A R of the beam, the moment of this being $= \frac{1}{2} \mu x^2$.

3. The elastic forces called into operation on the transverse section of the beam at R; let the moment of these, as before $= \Phi$.

* For a more extended investigation of beams of this kind, see discussion on the subject of "Torksey Bridge," published in the Minutes of the Institution of Civil Engineers, Session 1850.

Then, by the principle of the equality of moments,

$$P_1 x = \frac{1}{2} \mu x^2 + \Phi,$$

or,

(XLVI.) $$\Phi = P_1 x - \frac{1}{2} \mu x^2.$$

The value, however, of P_1 is yet unknown, and must be found in the following manner :

Substituting for Φ its value in Equation XXXI.,

$$E I \frac{d^2 y}{d x^2} = \frac{\mu x^2}{2} - P_1 x.$$

Or, integrating,

$$E I \frac{d y}{d x} = \frac{\mu x^3}{6} - \frac{P_1 x^2}{2} + \text{Constant}.$$

Now in order to find the value of the constant, we must recollect that at the point B, over the centre support, the tangent to the curve will be horizontal, *i. e.*, when $x = l$, then $\frac{d y}{d x} = 0$. Whence

$$E I \frac{d y}{d x} = \frac{\mu x^3}{6} - \frac{P_1 x^2}{2} + \frac{P_1 l^2}{2} - \frac{\mu l^3}{6}.$$

Integrating again, we have

(XLVII.) $$E I y = \frac{\mu}{24} x^4 - \frac{P_1}{6} x^3 + \frac{l^2}{2} \left(P_1 - \frac{\mu l}{3}\right) x.$$

But at the point B, where $x = l$, y is $= 0$, therefore substituting these values in the above equation, and reducing, we obtain

(XLVIII.) $$P_1 = \frac{3}{8} \mu l.$$

And since $2 P_1 + P_2 = 2 \mu l$

(XLIX.) $$P_2 = \frac{5}{4} \mu l.$$

We are now able to find the strength and the deflection of the beam.

Strength of the Beam.

Let Figure 1 be again referred to in order to illustrate the variation of the elastic strain in different parts of the length of the beam.

Beginning at the point A, where this strain $= 0$, it will increase gradually as far as a point near the middle of the beam, after which it will diminish till it again becomes equal to nothing at the point of contrary flexure O; beyond this point it increases again as we approach the centre support B, on the other side of which it undergoes corresponding variations. We have then to find expressions for the strength of the beam at three places; viz. at the points of maximum strain near the middle of each opening, and over the centre support.

To find the place of maximum strain between A and O, we may take Equation XLVI., and inquire what must be the value of x, so that Φ may be a maximum. By the ordinary process, making $\frac{d\,\Phi}{d\,x} = 0$, we find

$$P_1 - \mu x = 0,$$

or
$$x = \frac{P_1}{\mu};$$

but by Equation XLVIII. $P_1 = \frac{3}{8}\mu l$

$$\therefore x = \frac{3}{8} l;$$

that is, the greatest strain between A and O is at a point whose distance from A is three-eighths of the length of the opening.

Substituting this value of x in Equation XLVI., and giving P_1 its value previously found, we have

$$\Phi = \frac{9}{128}\mu l^2.$$

But by Equation I., page 244, $\Phi = \frac{f}{c} I$; therefore,

$$\frac{f}{c} I = \frac{9}{128} \mu l^2,$$

or

$$\text{(L.)} \qquad \frac{\mu l}{f} = \frac{128}{9 c l} I.$$

For the strength over the centre pier we have merely to make $x = l$ in Equation XLVI., giving P_1 its proper value; bearing in mind, however, that beyond the point of contrary flexure the direction of the elastic strain will change, and therefore Φ will have a contrary sign. Then

$$\Phi = \frac{1}{2} \mu l^2 - \frac{3}{8} \mu l^2,$$

or

$$\frac{f}{c} I = \frac{1}{8} \mu l^2,$$

or

$$\text{(LI.)} \qquad \frac{\mu l}{f} = \frac{8}{c l} I.$$

Comparing Equations L., LI., and XXIII., we obtain the following result.

The greatest strain in each of the spans is to the strain over the centre pier as $\frac{9}{128} : \frac{1}{8}$, or as $9 : 16$; the latter being, moreover, equal to the strain on the centre of an independent beam spanning one opening only.

Deflection of the Beam.

Substitute the value of P_1 in Equation XLVII., and we have

$$\text{(LII.)} \qquad E I y = \frac{\mu}{24} x^4 - \frac{\mu l}{16} x^3 + \frac{\mu l^3}{48} x,$$

from which the deflection at any point may be found.

If D represent the deflection at the middle of the opening, where $x = \frac{1}{2} l$, we obtain,

(LIII.) $$E I D = \frac{\mu l^4}{192}.$$

Comparing this with Equation XL., which gives the deflection of an independent beam, it is seen that the effect of the continuity is to reduce the deflection in the middle of the opening in the proportion of $\frac{5}{384}$ to $\frac{1}{192}$, or $5 : 2$. The greatest deflection, however, occurs at a point a little farther removed from the centre support, as may be found by trial.

To find the *points of contrary flexure* O . O, make the elastic strain $\Phi = 0$ in Equation XLVI., then, substituting for P_1 its proper value,

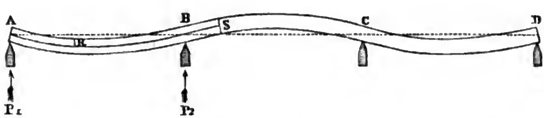
$$\frac{3}{8} \mu l x - \frac{1}{2} \mu x^2 = 0,$$

or $$x = \frac{3}{4} l;$$

that is, the distance B O is equal to one-fourth the length of the opening A B.

A BEAM SUPPORTED AT FOUR POINTS.

Let A B C D represent a beam extending over three equal



openings,* the width of each of which, as A B or C D = l. Let P_1 represent the reaction or resistance on each of the two outside supports A and D, and P_2 that on each of the two middle ones, B and C. Let x and y be the co-ordinates of the deflection curve at any point R, as before, and μ = the load per lineal unit distributed over the beam. Then pro-

* This case is solved more generally in "Moseley," art. 376.

ceeding as on pages 281 and 282, we have for the portion A B of the beam,

$$\mathrm{E\,I}\frac{d^2 y}{d x^2} = \frac{1}{2}\mu x^2 - \mathrm{P}_1 x,$$

Or integrating

$$\mathrm{E\,I}\frac{d y}{d x} = \frac{\mu x^3}{6} - \frac{\mathrm{P}_1 x^2}{2} + \text{Constant}.$$

To find the value of the constant, let β = the angle with the horizontal, made by the tangent to the curve at B, then $\frac{d y}{d x} = \tan \beta$, when $x = l$. Correcting the integral accordingly, and integrating again,

$$\text{(LIV.)} \qquad \mathrm{E\,I}\,y = \frac{\mu x^4}{24} - \frac{\mathrm{P}_1 x^3}{6} + \left(\mathrm{E\,I}\tan\beta - \frac{\mu l^3}{6} + \frac{\mathrm{P}_1 l^2}{2}\right)x.$$

Now at the point B, where $x = l$, $y = 0$; substituting, therefore, these values, and reducing

$$\text{(LV.)} \qquad \mathrm{E\,I}\tan\beta = \frac{1}{8}\mu l^3 - \frac{1}{3}\mathrm{P}_1 l^2,$$

which is a first value for E I tan β, involving, however, the yet unknown quantity P_1.

Directing attention now to the portion B C of the beam, let it be observed, that if x and y be taken to represent the co-ordinates of a point S in this portion of the beam, the pressures applied to A S, are, the elastic forces upon the section at S, the pressures P_1 and P_2, and the load μx; we have therefore

$$\mathrm{E\,I}\frac{d^2 y}{d x^2} = \frac{1}{2}\mu x^2 - \mathrm{P}_1 x - \mathrm{P}_2 (x - l).$$

Integrating, and observing that when $x = l$, the value of $\frac{d y}{d x}$ is represented by tan β, we have

$$\mathrm{E\,I}\frac{d y}{d x} = \frac{1}{6}\mu(x^3 - l^3) - \frac{1}{2}\mathrm{P}_1(x^2 - l^2) - \frac{1}{2}\mathrm{P}_2(x - l)^2 + \mathrm{E\,I}\tan\beta.$$

Now it is evident, that since the supports B and C are placed symmetrically, the deflection curve will be horizontal

at the middle point between them; *i. e.* when $x = \frac{3}{2} l$, then $\frac{dy}{dx} = 0$. Substituting and reducing, therefore, we have

$$\text{(LVI.)} \qquad E\,I \tan \beta = \frac{5\,P_1 + P_2}{8} l^2 - \frac{19}{48} \mu\, l^3.$$

Since, moreover, the resistances at C and D are equal to those at A and B, and the whole load upon the beam is sustained by these four resistances, we have

$$\text{(LVII.)} \qquad 2\,P_1 + 2\,P_2 = 3\,\mu\, l.$$

Hence, eliminating between the three Equations LV., LVI., and LVII., and reducing

$$\text{(LVIII.)} \quad P_1 = \frac{2}{5} \mu\, l; \quad P_2 = \frac{11}{10} \mu\, l; \quad \text{and } E\,I \tan \beta = -\frac{1}{120} \mu\, l^3.$$

To find the deflection, we may substitute these values in the equations of the curve previously found, whence we obtain from A to B,

$$\text{(LIX.)} \qquad E\,I\,y = \frac{\mu}{24} x^4 - \frac{\mu\, l}{15} x^3 + \frac{\mu\, l^3}{40} x,$$

from B to C,

$$\text{(LX.)} \quad E\,I\,y = \frac{\mu}{24} x^4 - \frac{\mu\, l}{4} x^3 + \frac{11\,\mu\, l^2}{20} x^2 - \frac{21\,\mu\, l^3}{40} x + \frac{11}{60} \mu\, l^4.$$

If D represent the deflection in the middle of either of the side spans, where $x = \frac{1}{2} l$, we have

$$E\,I\,D = \frac{13}{1920} \mu\, l^4.$$

Comparing this with Equation XL. it is seen that the centre deflection of one of the side spans A B is to the deflection of the same beam, if considered detached, as

$$\frac{13}{1920} : \frac{5}{384}, \text{ or as } 13 \text{ to } 25.$$

The strength of the beam may be found from the following relations deduced from the preceding reasoning :—

Let strain at centre of beam A B, if detached ...... = 25
Then maximum strain in side spans of continuous beam
(at distance $= \frac{2}{5} l$ from end) .... = 16
" strain over middle supports .... = 20
" strain at centre of middle span .... = 5

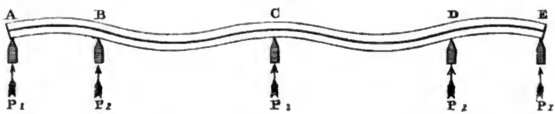
The distances of the points of contrary flexure from the middle supports B and C will be,

In the side spans .... = $0{\cdot}2\, l$
" centre span.......................... = $0{\cdot}276\, l$

A BEAM SUPPORTED AT FIVE POINTS.

Let A B C D E (Fig. 3) represent a beam extending con-

Fig. 3.



tinuously over four openings; and to assimilate this in some measure to the case of the Britannia Bridge, we will suppose the two middle openings to be twice the length of the two outside ones, *i. e.*

Let $\quad A B = D E = l$; then $B C = C D = 2 l$.

Let μ = Load per lineal unit, as before;
P_1 = Resistance on each of the supports A and E ;
P_2 = That on B and D ; and
P_3 = That on the centre support C.

Also, let β represent the inclination of the deflection curve to the horizontal, at the points B and D.

The process of investigation being very similar to that adopted in the last case, we shall merely state the results of each step, without giving the details.

1. Taking the equation to the portion A B of the curve, and making $x = l$, and $y = 0$, we obtain

$$\text{E I} \tan \beta = \frac{l^2}{24}(3\mu l - 8\text{P}_1).$$

2. From the equation to the portion B C of the curve we obtain, in like manner,

$$\text{E I} \tan \beta = \frac{l^2}{6}(4\text{P}_2 + 10\text{P}_1 - 9\mu l).$$

3. Since the beam will be horizontal over the centre support, make $\frac{dy}{dx} = 0$ at that point, which gives

$$\text{E I} \tan \beta = l^2(4\text{P}_1 + 2\text{P}_2 - \frac{13}{3}\mu l).$$

4. Since the whole load is borne by the five supports,

$$2\text{P}_1 + 2\text{P}_2 + \text{P}_3 = 6\mu l.$$

Whence, by elimination in these four equations,

(LXI.) $$\text{P}_1 = \frac{1}{4}\mu l, \quad \text{P}_2 = \frac{27}{16}\mu l, \quad \text{and} \quad \text{P}_3 = \frac{17}{8}\mu l;$$

Also $$\text{E I} \tan\beta = \frac{1}{24}\mu l^3.$$

The equations for the *deflection* are, from A to B,

(LXII.) $$\text{E I}\, y = \frac{\mu}{24}(x^4 - l x^3).$$

From B to C, it is simpler to consider B as the origin of the abscissæ x, so that at the point B, $x = 0$. Then,

(LXIII.) $$\text{E I}\, y = \frac{\mu}{24}\left(x^4 - \frac{15}{4} l x^3 + 3 l^2 x^2 + l^3 x\right).$$

The deflection in the centre of the span B C is *one-fourth* that of an independent beam of the same length.

The *strength* of the beam may be found from the following comparative results:—

Let strain at centre of beam B C, if detached	= 8
Then strain over support B of continuous beam	= 4
,, over support C	= 6
,, at centre of span B C	= 3
Greatest strain in span B C (distant $\frac{15}{16}l$ from B)	= $3\frac{1}{32}$

The points of contrary flexure will be,

In the large span, at distances = $0{\cdot}322\,l$ from B, and $0{\cdot}447\,l$ from C.
In the small span, midway between A and B.

A LONG CONTINUOUS BEAM SUPPORTED AT REGULAR INTERVALS.

The general nature of this case having been already explained, and the strains determined, we shall merely add here a few remarks upon the *deflection*.

Let C C′ be the points of contrary flexure, then if A B $= l$, C C′ $= l_1$, and A C $= l_2$, we have by the former investigation—

$$l_1 = \frac{l}{\sqrt{3}}$$

$$l_2 = \frac{l - l_1}{2} = \frac{l}{2}\left(1 - \frac{1}{\sqrt{3}}\right)$$

Also, let μ = the load on each lineal unit of the beam.

Now, it has already been explained that the length A B may be considered as divided at the points C C′ into three portions, the middle portion being simply suspended from the

ends of the small beams, or cantilevers, A C and B C′. We may, therefore, find first the deflection of the portion A C, which may be called D_2; and, secondly, that of C C′, which call D_1; the sum of the two giving the total deflection = D.

For the portion A C. Taking any point R, and making A $r = x$, r R = y, we may observe that the portion C R is held in equilibrium by three forces, viz.,—

1st. The weight suspended at the end C, *i.e.* half the load on the portion C C′, or, $\frac{1}{2}\mu\, l_1 = \frac{\mu\, l}{2\sqrt{3}}$. Let this for the present = W. The moment of this about R is therefore $W\,(l_2 - x)$.

2d. The load $\mu\,(l_2 - x)$ on the portion C R, the moment of this being $= \frac{\mu}{2}\,(l_2 - x)^2$.

3d. The elastic forces induced on the section R, the moment of which = Φ.

The two first of these tend to turn the portion of the beam in a contrary direction to the third. Therefore,

$$\Phi = W\,(l_2 - x) + \frac{\mu}{2}\,(l_2 - x)^2.$$

The curve being *convex* to the axis of x, we have (see page 270),

$$\Phi = E\,I\,\frac{d^2 y}{d x^2}.$$

Whence

$$E\,I\,\frac{d^2 y}{d x^2} = W\,(l_2 - x) + \frac{\mu}{2}\,(l_2 - x)^2.$$

Integrating, and bearing in mind that at the point A the curve is horizontal; *i. e.* when $x = 0$, $\frac{d y}{d x} = 0$; also when $x = 0, y = 0$, we have

$$E\,I\,y = \frac{\mu\, x^4}{24} - \frac{W + \mu\, l_2}{6}\,x^3 + \left(\frac{W}{2} + \frac{\mu\, l_2}{4}\right)\,l_2\, x^2,$$

which is the equation for the curve from A to C.

Making $x = l_2$, and the deflection at the point $C = D_2$,

$$\text{(LXIV.)} \qquad E\,I\,D_2 = \left(\frac{\mu\,l_2}{8} + \frac{W}{3}\right) l_2^3.$$

For the portion C C′ we may adopt Equation XL.; and making the deflection at the middle point $= D_1$, we have

$$\text{(LXV.)} \qquad E\,I\,D_1 = \frac{5}{384}\,\mu\,l_1^4.$$

Adding together Equations LXIV. and LXV.

$$E\,I\,D = \left(\frac{\mu\,l_2}{8} + \frac{W}{3}\right) l_2^3 + \frac{5}{384}\,\mu\,l_1^4.$$

Or substituting for l_1, l_2, and W, their proper values, we have for the total deflection

$$\text{(LXVI.)} \qquad E\,I\,D = \frac{\mu\,l^4}{384}.$$

A comparison of this with Equation XL. shews, that the *deflection of each span of a perfectly continuous beam is one-fifth that of an independent beam spanning the same opening.*

SECTION IV.

SPECIFIC EXPERIMENTAL INQUIRIES.

CHAPTER I.

INTRODUCTORY OBSERVATIONS.

Many of the experiments employed in illustrating the following specific disquisitions, viz., all those which were conducted by Mr. Hodgkinson, have, with the permission of Mr. Stephenson, been printed by that gentleman in the "Report of the Commissioners appointed to Inquire into the Application of Iron to Railway Structures."

They are here combined with other information on the subject of beams, to illustrate as popularly as possible the general properties of wrought and cast-iron when applied to purposes of construction; and where further information was required, many experiments on a large scale have been subsequently made by Mr. Stephenson, to render the subject as complete as possible.

Rather than perplex the reader with the various views entertained by engineers on some of these subjects, the Author only professes to illustrate the conclusions that have been arrived at, and more immediately acted upon in the construction of the Tubular Bridges; and some entirely novel and valuable

practical information will be found embodied in these inquiries.

In all the first experiments the various models were designed with especial reference to the practical construction of a large tube for the proposed bridge. The parts that failed were continually strengthened or replaced by stronger forms; and the result of such a tentative process would necessarily be a *model of maximum strength for a given quantity of material in a given form.* Had the experiments been made on tubes of the full intended size, such a method of investigation would have been perfect. This would, however, have been tantamount to the construction of several large bridges. Attention was therefore directed to these more particular objects, first, the determination of the best form or prototype; and secondly, the investigation of laws for the safe reduction of smaller to larger structures.

Supposing the form of maximum strength, with a given quantity of material in any particular form of tube to be arrived at, as in the large model, then there was required for the Britannia Bridge a similar tube six times as large in every respect—six times the length, depth, width, and thickness—and without any experiments, the most elementary reasoning was sufficient for the *approximate* determination of the strength or deflection of such a structure. Such reasoning, however, must assume no change of shape, and must rely on the constancy of the laws on which it is based. It was necessary, therefore, to analyse not only the simple relations which exist between similar bodies, but the elements on which those relations are founded. It was the probing of old foundations, which, although sufficient for the building removed, might prove an inadequate base for the great superstructure now to be reared upon them.

Moreover, the best form for the *large* beam was still

undetermined. The requisite arrangements for the preservation of its shape, the effect of its weight, independent of its properties as a beam, the requisite strength for the sides, and the effect of the increase of width, had all to be considered; it became also important to investigate the limits of such a beam, and to ascertain whether they were approached.

By increasing the depth *ad infinitum*, we should assume an infinite increase of strength; by widening the top or bottom until the plates are infinitely thin, we assume no loss of strength; and even a moderate extension of such principles would lead to a tube that we know would be destroyed by its own weight.

This consummation was prophesied by some of the most eminent mathematicians and greatest mechanics; and Mr. Stephenson did not presume to commence such a structure on such empirical data alone. At the close of an investigation of the subject, the late Rev. E. Sibson, a sound mathematician, observes:

"In whatever form the tube is made, it must be so constructed, that in every part its strength may be in proportion to the stress; and all the parts must be firmly compacted, that the whole tube may vibrate nearly as a musical string.

"The tube must be braced by strong iron cables, so that the wind may not blow it away, and that a sudden jerk of the carriages may not break it.

"But an iron tube 450 feet long, to carry a train of carriages 150 tons in weight, at the rate of 40 miles an hour, *is a chimera*.

"It is said that the strength and form of the tube have been satisfactorily determined; that the stone piers are to be commenced immediately; and that the whole work will be completed in two years. Almost wherever the statement was read there was a startling apprehension that a great and expensive work, involving the reputation of many scientific men, and *perilling* the public safety, was to be undertaken on very slight and imperfect data."

The whole of this gentleman's remarks on the conclusion of these preliminary experiments are here quoted :—

" 3d April, 1846.

" *Objection to the Principle.*—Suppose a train of carriages 150 tons weight travelling at the rate of 40 miles an hour, which is at the rate of 60 feet in a second, then the effect of the vibrations produced by the momentum of the train on the line of the railway will be very considerable, even when the railway passes over a solid and hard surface. For a person standing, for instance, at the Newton Station will feel the vibrations of the ground very sensibly both in the station-house, and even in the Leigh Arms Hotel. When, therefore, the railway passes over arches of a large span, and more especially when it passes through a horizontal iron tube of great length, the vibrations of this tube cannot be contemplated without serious apprehension.

" *Experiments.*—In making experiments, the great and principal aim will be to assimilate them as nearly as possible to all the conditions and circumstances of the tubular bridge to be actually constructed.

" In a statical experiment, one-half the length of the tube may be fixed, and the other end loose.

" In an experiment to shew the vibrations, both ends of the tube must be loose, and placed on rollers. It is, however, hard to say how the experiments can be conducted so as to appreciate the effect of these vibrations. For, in making such experiments, when a tenth part of the length of the tube is taken, and one-tenth part of the weight of the train, together with that of the materials of the tube, is taken, still it will be necessary to use the *high* velocity of 60 feet in a second; for though the momentum would be the same with a weight of 150 tons, and the velocity of six feet in a second, yet the pressure on the tube, and the friction on the tube, would both be increased tenfold, from 15 to 150. Possibly a number of experiments might be made in which different weights of *the train* were used, the velocity of the train always remaining 60 feet in a second, and the strength of the tube remaining invariable; and these experiments might be continued with increasing weights of the train, either till

the train jumped off the railway by the vibrations of the tube, or till the tube itself was broken by the vibrations.

"If the experiments be made on a hollow flat surface of iron, it would appear that if the strength and flexibility of this flat surface be equal to those of the tube, then the vibrations of the flat surface will be much greater than those of the tube.

"*Form of Tube.*—The cylindrical seems to be the worst of all forms. For the true form must be such that the strength of each part of the tube shall be exactly in proportion to the strain acting on that part of the tube. But in the cylindrical, the moment or strain of the extreme upper part or the extreme lower part is the greatest possible; because that part is at the greatest possible distance from the axis passing through the neutral line; and at each of these extreme points on the surface of the cylinder the strength of the cylinder is the least possible, for at each of these extreme points the curved surface of the cylinder is infinitely short.

"The rectangular form seems the best, with a number of Mr. Hodgkinson's beams adapted to wrought-iron placed near to each other and parallel, so that one series of these beams shall be connected by cramps and interlaced fastenings and form the upper plane surface of the rectangular tube; and so that also the other set of beams connected by interlacings in the same manner shall form the lower surface of this rectangular tube. The distance between this upper and lower surface should not be greater than is necessary for head-room for the train; for if this distance be considerable the rods which connect the two surfaces will cause considerable vibration in the lower surface. It would appear desirable that there should be two lines of railway in the same tube and a foot-road between them; and that, therefore, the upper and lower surfaces of the tube be connected by four lines of vertical rods.

"From the experiments that have been already made, it has been found almost invariably to happen that the upper surface of the tube creases and gives way by contraction, while the lower surface of the tube does not appear to have suffered in any respect from extension. Now, that the tube may be equally strong both at its upper and lower surface, the strength of the upper surface must be so adjusted that the tube will be disposed to tear as soon at the bottom as to crease at the top. The condition has been contem-

plated in the experiments which have been already made; but the basis on which these experiments were made was rather that of conjecture than that of judicious and well-directed design. Before the form of the upper surface of the tube can be determined, it should be ascertained, by a series of well-conducted experiments, how the contraction of malleable-iron bars is affected by the weights which contract them. And then only the strength of the upper surface of the tube will be verified by experiment, and no longer attempted to be determined at random by conjecture."

The necessity for chains was advocated on all hands. Mr. Hodgkinson observes, "I would beg to recommend that suspension-chains be employed as an auxiliary, otherwise great thickness of metal would be required."

General Pasley still maintained the opinions he had expressed in his report on that subject.

Many doubted the effective value of rivets in uniting such a mass of plates; some foretold the most fatal oscillation and vibration from a passing train, sufficient even to destroy the sides of the structure; while the lateral strength was asserted to be insufficient to resist the wind.

In fact, with few exceptions, scientific men generally either remained neutral or ominously shook their heads and hoped for the best.

Even the most sanguine waited for further experimental investigation.

"I have no doubt," said one of our most eminent philosophers, "that the strength of the tubular bridge, when new, supposing ordinary care to be used in most parts, and considerable attention given to accuracy in those parts which are to resist crushing, will be more than abundantly sufficient. I think also the permanent strength of the bridge may be quite sufficient; but I am not so decided upon this. These opinions would be scarcely affected by a moderate alteration in the thickness of the plates. But these considerations apply only to the consideration of weights placed quietly upon the bridge: and I should not think myself justified in express-

ing any opinion which could be supposed to apply as to the capability of the bridge to resist the sudden introduction of a weight at railway speed. On this subject experimental information is wanting to me. All that I can tell, is, that a stiff structure would be likely to suffer more from it than a flexible one. My opinion also depends entirely upon the assumed accuracy of those parts which are to resist compression. If, in the tubes intended for this purpose, there are no transversal stops or frames, their strength for resisting thrust will be very greatly a matter of accident. My notion would have been, to rely upon tube structure for stiffness, and upon something else,—as chain suspension, for absolute permanent strength."

Again, reasoning on the principles we have alluded to, a sound practical man observes, that if the tube be thought not strong enough, and the metal of the bridge be increased everywhere any given number of times, its strength will be increased exactly the same number of times, but the weight of the beam is also increased the same number of times; so that we cannot in a similar tube increase the ratio between the breaking-weight and the weight it will have to carry.

This position is illustrated by the reduction of Exper. 1. The weight of the model, with the top somewhat reduced, would be 4·5 tons. The breaking-weight + half its own weight = 38 tons.

Assuming the Conway tube to be similar in every respect, and $5\frac{1}{3}$ times greater, we have

The weight of the Conway tube = $(5\frac{1}{3})^3 \times 4{\cdot}5$ =	683	tons
And the weight of a train....................	133	,,
Total..........	816	,,

Therefore the weight acting at the centre will be 408 tons.

The load such a tube would bear, including its own weight, would be

$$(5\tfrac{1}{3})^2 \times 38 = 1080 \text{ tons.}$$

And therefore $\frac{1080}{408}$, or 1 : 2·6, is the ratio of the weight it will have to bear to that of its breaking load.

Now, the sectional area of the model at the bottom was 9·2 square inches; therefore the sectional area of the bottom of the tube for Conway would be

$$(5\tfrac{1}{3})^2 \times 9{\cdot}2 = 262 \text{ inches.}$$

But it was then intended that the sectional area of the bottom of the Conway should be 420 square inches.

Supposing every dimension increased in the same ratio, the weight of the Conway, instead of 683, as above, would be

$$\frac{420}{262} \times 683 = 1091 \text{ tons.}$$

And the weight of the load	133	,,
Total............	1224	,,
And the weight at the centre	612	,,

Similarly, the breaking-weight would be

$$\frac{420}{262} \times 1080 = 1731 \text{ tons.}$$

And the ratio of the weight it will have to bear to that of its breaking-load is now only

$$\frac{1731}{612}, \text{ or } 1 : 2{\cdot}8\,;$$

and this with an additional weight of material of 408 tons; and if we now double the weight of this tube, the above ratio will be but little altered. Thus, very little strength is gained by increasing the thickness of the tube, all the other dimensions remaining in proportion.

Hence, it was argued: "It appears to be absolutely impossible to give the requisite strength to the tube alone; and that it is therefore absolutely necessary that the suspension-chains be permanent."

However erroneous this last deduction may be, the whole of the argument is a most interesting illustration of the great importance of the inquiry, as to *what form of model* should be employed for reduction to a larger size. With but little additional weight the very model alluded to subsequently carried 89 tons at its centre; but even when, by constant alteration, this model was made of such just proportions as to be ready to fail simultaneously at every part, it by no means followed that this was the best, or even a good form for the Conway Bridge; and still less does it follow that this form of increased dimensions should retain the same proportion between all its parts.

A great deal of much practical importance was doubtless accomplished. Nearly all that the practical man could do was done. A good form for construction was arrived at, the efficacy of riveting was demonstrated, and the practical details of construction were matured; a model of magnificent dimensions, and strong enough for railway traffic, was, in fact, completed. The laws of one particular phenomenon had been experimentally analysed; but the theory necessary to their application on a larger scale was almost untouched, and remained the peculiar object of further and higher investigation.

The experiments made by Mr. Hodgkinson were immediately of this theoretical character. His models had no direct reference to the structure in hand. They were designed entirely to meet theoretical requirement, regardless of practical consideration.

The principal subjects of inquiry may be classed as follows: First, to ascertain the limits and the laws of the resistance of wrought-iron to compression and extension, as applied to the tube. Secondly, to determine the laws of resistance of plates, and of circular, square, and rectangular cells of wrought-iron, to a crushing or buckling strain.

Thirdly, to test some of the usually-received theories of beams as applied to tubes. And lastly, to determine the proper distribution of the material in an enlarged model, and to investigate laws for the transverse strength of such a structure. As to the construction of the top of the tube there was still a complete absence of all experimental or practical information, while the prominent phenomenon of buckling, which has been described, rendered such knowledge peculiarly indispensable.

CHAPTER II.

COMPRESSION, FLEXURE, AND CRUSHING OF MATERIALS UNDER DIRECT PRESSURE.

THE immediate effect of transverse strain in a beam or tube is to compress all those portions of the beam which lie above the neutral axis. As the strain is increased up to the limit of failure, failure may take place in the centre of the beam, by destruction of the top layer from the effect of compression.

It is thus an important inquiry to investigate the behaviour of materials under these circumstances. In order to confine our attention more definitely, and simplify the investigation, we will first consider the effect of this strain on a cubic inch of wrought-iron.

Now, on compressing or placing a moderate weight, say one ton, on a cubic inch of wrought-iron, the immediate effect will be to shorten its height, which is simultaneously accompanied by some increase of its bulk horizontally. Omitting the bulging at present, with respect to the amount of compression, or the shortening of the cube, it is found by experiment that it is nearly proportional to the weight placed upon it: one ton shortens the cube one-tenthousandth of an inch; five tons will shorten it five-tenthousandths. Moreover, if we double or treble the surface compressed, we have, as we should naturally expect, one-half or one-third the amount of compression. The cube is thus a mere spring; but it is not a

perfect spring; it does not return to its natural dimensions on the removal of the weight.

It is certain that, with light weights, we can detect no permanent set on their removal; but as the weights approach the crushing weight, a permanent set becomes obvious, the permanent set under additional loads not being proportionate to the load, but increasing in a higher ratio, probably as the square of the weights, as is exactly the case with cast-iron. The weight being the ordinate of a parabola, the set will be the abscissa; this goes on until the material is crushed.

Though we cannot detect it, it is highly probable that under any weight, however small, some permanent set will take place in our pillar, provided it be a new unstrained pillar.

This in no way leads to the conclusion, that the pillar is damaged by repeating a similar load; for if we place five tons on a new cubic inch of wrought-iron, it is compressed five-tenthousandths of an inch; and it takes a certain permanent set due to this compression. On removal of the five tons, it partly recovers itself; but it never returns to its original height. The cube is in a new condition. The subsequent replacing and removing of these five tons any number of times after this first set never alters these new conditions nor increases the permanent set. It is still elastic through a certain space, although five tons no longer compresses it so much as it did at first, but compresses it less by the amount of its permanent set. This applies equally to the extension of the material from a tensile strain. It may not be theoretically exact, but is a valuable empirical consideration for practical purposes, and may assist our imagination if we conceive the ultimate particles of the cube as it cooled down from the furnace, to come to some arrangement in which they are in equilibrium among each other; but so delicate

is the equilibrium, that the slightest force disturbs it, and a new arrangement adapted to its altered circumstances takes place, time being an important element in this permanent change.

It follows from this, that if we measure the practical use of a pillar by the amount of its compression, a pillar that has already been severely strained is better than a new pillar;—workmen call it stronger, which is only true if we measure its strength by the amount of its compression.

The Britannia Tube would thus have had less deflection, had it been possible to put a severe strain on the top and bottom previous to uniting them with the sides. Thus, also, wire that has been stretched stretches less on subsequent use, which the bell-hanger is well aware of; and hammered brass becomes highly elastic partly from this cause. The mechanic will call to mind daily phenomena which this consideration will explain, and may often avail himself of the fact.

One remarkable practical illustration occurred as follows. Messrs. Easton and Amos, of London, who constructed the hydraulic presses, &c. for raising the Britannia Bridge, carry on a manufacture in which lead in a semifluid state, and sometimes nearly or quite solid, is placed in a cylinder internally 22 inches deep and 4 inches diameter. The lead, by means of a piston, accurately fitted to the cylinder, and moved by a powerful hydraulic press, is then forced through a contracted orifice in the bottom of the cylinder. The pressure required is prodigious, amounting in the lead cylinder to 60 or 70 tons per circular inch, and in the press itself to 3, and sometimes 3½ tons on the circular inch. The practical difficulty of getting any cylinder to withstand this pressure was almost insurmountable. Cast-iron cylinders, 12 inches in thickness, were quite useless, they began to open in the inside, the fracture as gradually extending to the outside, and increased

thickness beyond moderate limits giving no increased strength. Cylinder after cylinder thus failed, and Messrs. Easton and Amos at length constructed a cylinder of wrought-iron 8 inches thick. After using this cylinder the first time, the internal diameter was so much increased by the pressure, that the piston no longer fitted with sufficient accuracy. A new piston was made to suit the enlarged cylinder, and a further enlargement occurring again and again with subsequent use, the new pistons became as formidable an obstacle as the cylinders. The wrought-iron cylinder was on the point of being abandoned, when Mr. Amos, having carefully guaged the cylinder inside and out, found, to his surprise, that, although the internal diameter had increased considerably, the external diameter retained precisely its original dimensions. He consequently persevered in the construction of new pistons; and confirmatory of the views we have taken above, he found ultimately that the cylinder enlarged no longer, and to this day continues in constant use. Layer after layer having attained additional permanent set, sufficient material was at length brought into play, with sufficient tenacity to withstand the pressure, and thus an obstacle, apparently insurmountable, and which threatened at one time to render much valuable machinery useless, was entirely overcome. The workman may be excused for calling the stretched cylinder stronger than the new one, though it is only stronger as regards the amount of its yielding to a given force.

To return to our cubic inch. It is evident that on increasing the load, the permanent set, which increases as the square of the load, becomes at length very great: it ends in the destruction of the cube.

With a wrought-iron inch cube the set becomes so great with 12 tons, that its shape and proportions begin to suffer; and where these are of any consequence, as in most practical cases they are, we come to the limit of its utility.

It is not, however, yet destroyed until the load is about 16 tons. It then oozes away beneath additional strain, as a lump of lead would do in a vice, or like a red-hot rivet under the pressure of the riveting machine, and to some extent obeys the laws of liquids under pressure.

If prevented from bulging or oozing away, the softest metal would bear an infinite weight, like water in a hydraulic press; and when the cube of metal becomes a very large plate, but still only an inch thick, it would no longer follow the laws of the cubic inch, as the plate may be considered so many compressed concentric rings, acting simultaneously with their resistance to crushing in confining the central portions from oozing away. Thus a few soft dèal planks carry the tube; and a sheet of lead is placed under the beams by which the tube was raised.

If instead of a cubic inch of wrought-iron we employ one of cast-iron, we meet with this important fact, that while the relation between its compression and permanent set is similar to that of wrought-iron, its amount of compression, or yielding under similar weight, is, for a considerable range, *twice* as great, and its ultimate resistance to destruction *three* times as great, being equal to 50 tons per square inch. It will therefore be compressed about two-tenthousandths of its height by a single ton, or more nearly $\frac{1}{4260}$.

This remarkable property is beautifully exemplified by Mr. Hodgkinson in the following experiments. It is evidently impossible to observe the compression of so short a pillar as our one-inch cube; he accordingly employed columns of Low Moor cast and wrought-iron, as nearly as possible one inch square, but ten feet long, so that the compression was 120 times as great as in our cube, but still bearing the same proportion to the length, *i. e.* in wrought-iron, still one-tenthousandth of the length of the pillar for every ton on the square inch of surface. Moreover, as these pillars,

on account of their length, would bend under the weight, according to certain laws that we shall presently examine, unless restrained in the line of pressure, mechanical means were employed to keep them straight, though not to prevent their bulging. The table is so arranged that *nearly* equal diminutions in length of the two pillars compared, stand alongside each other; and by reference to the corresponding weight producing that compression, it will be found it is throughout nearly twice as great for the wrought-iron bar as for the cast-iron bar.

The following Table shews the comparative forces exerted by bars of cast and wrought-iron 10 feet long and 1 inch square nearly, when subjected to compressions differing but slightly:—

Cast-iron Bar. Area of Section, 1·031 × 1·029.		Wrought-iron Bar. Area of Section, 1·025 × 1·025.		Cast-iron Bar. Area of Section, 1·028 × 1·047.		Wrought-iron Bar. Area of Section, 1·016 × 1·02.	
Weight laid on the Bar.	Decrease of Length by that Weight.	Decrease of Length of the Bar.	Weight producing that Decrease.	Weight laid on the Bar.	Decrease of Length by that Weight.	Decrease of Length of Bar.	Weight producing that Decrease.
Lbs.	Inch.	Inch.	Lbs.	Lbs.	Inch.	Inch.	Lbs.
..	..	·028	5098	..	..	·027	5098
5054	·054	·052	9578	5098	·043	·047	9578
7316	·078	·073	14058	..	..	·067	14058
..	..	·085	16298	9578	·082	·089	18538
..	..	·096	18538	11818	·102	·100	20778
9578	·102	·107	20778	..	..	·113	23018
..	..	·119	23018	..	..	..	..
11818	·126	·130	25258	14058	·123	·128	25258
..	..	·142	27498	..	..	·143	27498
14058	·151	·154	29738	..	..	..	..
..	..	..	..	18538	·166	·163	29738
20778	·173	·174	31978	..	..	..	..
..	..	..	..	20778	·189	·190 to	31978 ,,
21898	·210	·214	34218	23018	·212	·261	in ½ hour.
23018	·247	..	..	27498	·254	·269 to	31978 ,,
27498	·300	..	..	31978	·302	·282 to	in ¼ hour. ,,
31978	·357	..	..	36458	·355	·328	repeated.
36458	·423	..	..	40938	·414	From ·190 to ·328 in 1¼ hour.	
40938	·503	..	..	45418	·481		
45418	·565	..	..	49898	·558		
49898	·694	..	..	54378	·667		
..	·749			Discontinued the experiment after this weight.			
54378	·865						
63338	Uncertain.						

The set or permanent decrement of length with the cast-iron bar after the weight was removed was found to be as the square of the compression. It will be seen that the compression of one-tenthousandth of the length in wrought-iron, for every ton per square inch, is nearly correct, even as we approach the crushing strain, and, under ordinary limits, very approximate.

Now the mathematicians, for the purpose of rendering the subject a little technical, call the weight that would extend an inch bar through a space equal to its length, supposing the elasticity perfect, its coefficient or *modulus of elasticity;* and still farther, the height of a column of the given material in feet, which will extend a bar of any length through a space equal to its length, is called the modulus of elasticity for the given material in feet. Thus, since one ton extends an inch bar of wrought-iron one-tenthousandth of its length, it is evident that, at the same rate, 10,000 tons would extend it through its whole length, and 10,000 tons, or 22,400,000 lbs., is the modulus of elasticity of wrought-iron, on this assumption. But an inch bar of iron, to weigh 22,400,000 lbs. at 3·3 lbs. per foot, would be 6,787,878 feet in length, which is the modulus of elasticity in feet. The modulus, according to Tredgold, is 24,920,000 lbs. As may be imagined, experimenters differ widely as to this modulus, which has one particular advantage, that a few millions more or less make but little difference.

Wrought-iron is permanently injured by a compression of 26,933 lbs. or 12 tons per square inch, causing a decrease of length in a 10-foot bar of ·139 inch; *i. e.* an iron bar is nearly destroyed by being impressed $\frac{1}{863}$rd part of its length. And if 12 tons per square inch compress a bar $\frac{1}{863}$rd of its length, 863 times 12 tons will compress it through half its length, this quantity, or 23,243,179 lbs., or about 24,000,000

lbs., or 10,356 tons, is the modulus of elasticity employed by Mr. Hodgkinson.

It will be perceived, the experiments stop short at a weight far under the crushing weight of cast-iron; but the wrought-iron was sinking away under the last weight laid on, or with 15 tons per inch. Thus, to recapitulate, the mean ultimate resistance of wrought-iron to a force of compression, as useful in practice, is 12 tons per square inch, while the crushing-weight of cast-iron is 49 tons per square inch; but for a considerable range, under equal weights, the cast-iron is twice as elastic, or compresses twice as much, as the wrought-iron, and the wrought-iron compresses ·0001 of its length per ton per inch of sectional area.*

Although with wrought-iron the compression is nearly as the weight applied, this is not the case with cast-iron; *i. e.* the ratio of the weight to the compression is not constant, the compression becoming more per ton as the weight in-

* A remarkable illustration of the effect of intense strain on cast-iron was witnessed by the Author at the works of Messrs. Easton and Amos. The subject of the experiment was a cast-iron cylinder $10\frac{3}{8}$ inches thick and $14\frac{1}{2}$ inches high, the internal diameter being 8 inches.

It was requisite for a specific purpose to reduce the internal diameter to $3\frac{1}{2}$ inches, and this was effected by the insertion of a smaller cast-iron cylinder into the centre of the large one; and to insure some initial strain, the large cylinder was expanded by heating it, and the internal cylinder being first turned too large, was thus powerfully compressed.

The inner cylinder was partly filled with pewter, and a steel piston being fitted to the bore, a pressure of 972 tons was put on the steel piston. The steel was "upset" by the pressure, and the internal diameter of the small cylinder was increased by full $\frac{3}{16}$ths of an inch! *i. e.* the diameter became $3\frac{11}{16}$ inch. A new piston was accordingly adapted to these dimensions, and in this state the cylinder continues to be used, and to resist the pressure; the internal layer of the inner cylinder was thus permanently extended $\frac{1}{38}$th of its length. In fact, it can only be regarded as loose packing, giving no additional strength to the cylinder.

Under these high pressures, when confined mechanically, cast-iron, as well as other metals, appears, like liquids, to exert an equal pressure in every direction in which its motion is opposed.

creases. Thus, a cast-iron bar, one square inch in section, was compressed $\frac{1}{5900}$ of its length by one ton of direct pressure, but with 17 tons, instead of being compressed $\frac{17}{5900}$, it was compressed upwards of $\frac{20}{5900}$. This will be seen in the following table :

TABLE VI.

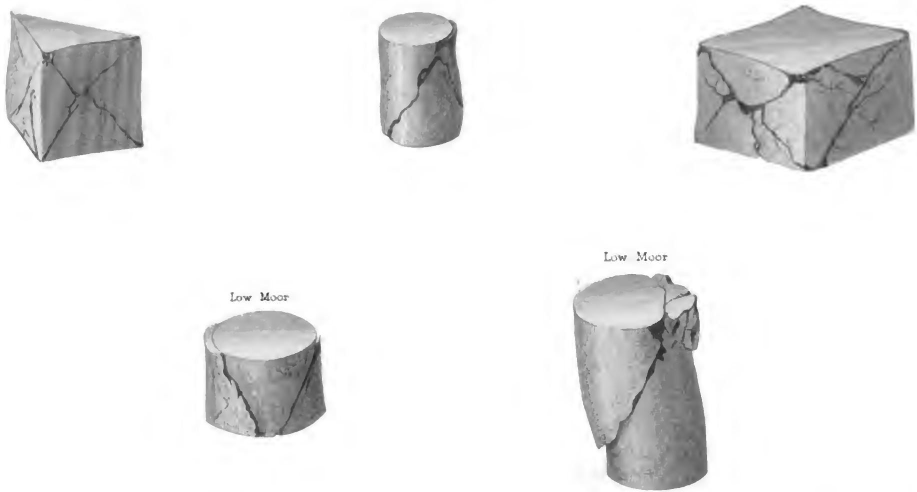
Compression of a Cast-iron Bar 10 feet long and 1 inch square.

	Compression per ton.	Total Compression.	Total Permanent Set.
Tons.	Inch.	Inch.	Inch.
1	·020338	·020338	·000510
2	·021038	·042077	·002452
3	·021618	·064855	·004340
4	·021369	·085479	·006998
5	·021594	·107872	·009188
6	·021752	·130513	·011798
7	·021950	·153654	·015243
8	·022154	·177235	·018572
9	·022374	·201373	·024254
10	·022477	·224774	·028126
11	·022567	·248237	·032023
12	·022802	·273632	·037653
13	·023014	·299187	·043318
14	·023523	·329330	·052640
15	·023539	·353092	·060905
16	·024409	·390558	·080256
17	·024805	·421695	·086298

The compression per ton increases with the weight, as shewn in column 2.

More minute information on the compression of this material, will be given in a subsequent page, where its tensile and compressive strengths are compared, and the relations between the compression and permanent set and the weight producing them are given. Under moderate strain, the compression appears to be about $\frac{1}{5425}$ of the length for every ton per square inch of section, the set or permanent decrement of

CRUSHING OF CAST IRON



Day & Son Lith^rs to the Queen

length being very nearly given by the following formula, viz. Set $= \cdot 543\, d^2 + \cdot 0013$, where d is the compression in inches.

But not only does the cast-iron cube differ so widely in elasticity and strength from the wrought-iron one, but the method of its ultimate failure is equally distinct. It at length crushes suddenly, by the sliding off of the corners in wedge-shaped fragments. Being a crystalline mass, without sufficient ductility to allow of its bulging horizontally, the outward tendency, combined with the compression, results in this method of failure, the angle of rupture at which these wedges slide off being tolerably constant, and varying from 48° to 58°.

The method of failure of several forms of section will be seen in the accompanying figures. And further illustrations will be found, in the Report above referred to, from experiments made by Mr. Hodgkinson. The ultimate compressive strength of various qualities of cast-iron will also be given in a subsequent page.

These laws are common to any sized cube of these materials, or to any cylinder whose height is equal to its diameter; *i. e.* the strength is as the transverse area.

We remarked in the experiments with the 10-feet bars that it was necessary to keep them straight in the line of pressure, by mechanical means, or otherwise they would bend under the weight instead of crushing. It is requisite to examine into the circumstances of this bending, which is evidently dependent on the ratio of the length of the bar to its diameter.

First, It has been determined experimentally that this flexure will not commence until the one-inch cube becomes about 5 inches high, or the height of any square or circular pillar becomes equal to five times its diameter. Up to this point, all we have stated of cubes holds good.

Beyond this height up to 25 inches, with our bar, or in

any other column up to 25 times its diameter, fracture takes place, partly by crushing, and partly by the bending of the column, so that it is, as it were, broken across; and with any height beyond 25 times the diameter, the crushing forms but little or no part of the phenomenon of fracture, which arises purely from the bending of the column, as with a transverse strain.

Three sets of laws, consequently, regulate these three cases. The crushing we have investigated. We have next to examine the case where the pillar breaks purely from flexure; and, lastly, the case in which fracture is a sort of compound fracture, arising partly from crushing and partly from flexure.

First, Pillars in which the height is above 25 times their diameter break by bending. This property of long pillars will be familiar; but the great increase of strength obtained by restraining the pillar from bending, and which may be done by a very small lateral force, is not so generally considered. The timber scaffolding at the Britannia Bridge, 100 feet high, and supporting a weight of eight tons on every foot of its length, owes its strength entirely to the cross-bracing, which preserves its lofty pillars of deal from flexure. The laws which regulate the bending of long pillars formed a subject for exquisite mathematical analysis for the immortal Euler; and he determined theoretically that their strength or their breaking-weight is inversely as the square of their length, and as the fourth power of their diameter; consequently, as regards its diameter, the breaking-weight follows the same law as the deflection of a beam of given length loaded transversely; for with a given weight, the deflection of a beam is as the cube of the depth into the breadth; and if the breadth is equal the depth, as in a square pillar, then the deflection is as the fourth power of the depth or diameter. But it differs from the beam as regards variation in length;

for the deflection of a beam is as the cube of the length, whereas in the pillar the breaking-weight is inversely as the square of the length. If we, therefore, make an experiment on a column of any given material, we have data for the strength of any other column of the same material.

The powers of the diameter and length to which the breaking-weight is proportional, are only the fourth powers and square as above, on the supposition of the material being perfectly incompressible. As this is not the case, these exponents require to be varied, and it has been determined by Mr. Hodgkinson experimentally, that the strength of long columns of cast-iron varies as the 3·6 power of the diameter, and inversely as the 1·7 power of the length. The constant, moreover, or the strength of a column reduced to one foot long and one inch diameter, was found to be 46·16 tons. Thus, for columns of cast-iron of any length exceeding 25 times their diameter, and of any diameter, we have the following practical formulæ:

$$\text{Breaking-weight} = 44{\cdot}16\,\frac{D^{3\cdot6}}{l^{1\cdot7}}\text{ tons.}$$

And for a hollow cast-iron column:

$$\text{Breaking-weight} = 44{\cdot}3\,\frac{D^{3\cdot6} - d^{3\cdot6}}{l^{1\cdot7}}\text{ tons.}$$

When D and *d* represent the external and internal diameters in inches, and *l* the length in feet.

The breaking-weight of long square columns of wrought-iron, as regards lateral dimensions, may be derived from the experiments on the flexure of plates used as pillars, at page 322, by reducing the strength, in the ratio of the breadth to the thickness. The comparative results thus obtained by Mr. Hodgkinson were as follows:—

TABLE VII.

Length of Bars compared.		Lateral Dimensions (or side of square) of Bar.	Strength or Weight of ultimate Resistance.	Power (n) of the Lateral Dimensions, on which the Strength depends.
Ft.	In.	Inches.	Lbs.	
10	0	·766 1·51	1948 23025	3·56
10	0	1·00 1·5	4245 23025	4·17
7	6	1·02 1·53	10236 45873	3·69
7	6	·5 1·00	583 9873	4·08
5	0	·5 1·00	1411 18038	3·67
2	6	·502 1·00	4216 27212	2·69
2	6	·502 ·76	4216 15946	3·28
			Mean . .	3·591

The mean is 3·59, which agrees very nearly with the above law for cast-iron pillars.

In similar pillars, the strength is nearly as the square of the diameter, and consequently as the area of the transverse section. Thus, if one pillar be made of twice the lineal dimensions of another, it will be twice as high, four times as strong, and eight times as heavy.

Among other remarkable laws affecting pillars, it has been also shewn by Mr. Hodgkinson, that the strength of a pillar with its ends flat, as in our experimental bar, is three times greater than when both the ends are turned round like the end of an egg, where the strain passes up the centre; and consequently, for pillars with round ends, the constants given in our formulæ must be divided by three. Hence the importance of a sound base to a column, and,

where possible, of securely fixing the end, as in the case of the guide-rods of the presses, which are fitted into deep sockets at the top of the cylinder; the strength of the column to resist flexure is thereby still further increased in a manner analogous to the fixing of a beam at the ends.

The masts and bowsprit of a ship are other instances of pillars much strengthened by the security of their base; and we *feel* most instructively the value of this precaution in thrusting a light stick into the ground.

Again: having found the strength of a cast-iron column of any dimensions, the strength of a wrought-iron column of the same dimensions will be greater, in the proportion of 1000 to 1745; a column of cast-steel will be stronger in the proportion of 1000 to 2518; and with Dantzic oak and red deal the proportion will be as 1000 : 108·8, and 1000 : 78·5, respectively.

Thus we are now enabled to find the strength of our one-inch bar, when of any greater length than 25 inches, and this from a constant independent of its crushing-weight alone.

To complete the history of the bar, we have to find the laws of its failure between the length of 5 inches and 25 inches, or the failure of a column whose height is between five and twenty-five times its diameter.

It is first evident, that when 6 or 7 inches high, the strength of our bar will be closely allied to its crushing-weight, and at the other extreme at 23 or 24 inches high, it will not depart much from the laws of flexure that we have determined; but these laws intermingle gradually, since the strength to resist flexure is as the fourth power of the diameter, and the resistance to crushing increases only as the square of the diameter. In fact, the flexure is accompanied by a partial crushing. There is, consequently, a falling-off in the strength of the pillars as determined by our formulæ, nearly in proportion to the reduction of length.

The formulæ must now evidently contain quantities dependent both on flexure and crushing, and is thus given by Mr. Hodgkinson and confirmed by experiment.

First, obtain the strength necessary to destroy the pillar by flexure (B lbs.); secondly, the crushing-force of the transverse section as a cube (C lbs.). Then

$$\text{The breaking-weight} = \frac{B\,C}{B + \frac{3}{4}\,C}$$

whether for hollow or solid pillars.

The reasoning on which these formulæ are based will be found in the "Philosophical Transactions" of the Royal Society, Part. II. 1840. The practical results are merely collected here as a link in our subject. Further experiments on inch bars will be found in the next table of experiments on the resistance of plates and bars to compression.

Resistance of Plates and Bars to Crushing.

The laws of regular pillars, though an important link in the subject, are evidently not directly applicable to the circumstances under which the top of a tubular beam is destroyed by compression. We have ascertained the behaviour of a cubic inch of iron under such conditions, and also of any square or circular pillar. We have yet to investigate the effect of such a strain on plates of any length, width, or thickness; and lastly, the subject will become more intricate when we consider the combination of these plates into the form of cells, whether circular or square.

Thus much, however, is settled: we have ascertained the limit which no combination can exceed. Wrought-iron is literally crushed with 16 tons per square inch of area; and under no circumstances can any pillar whose length exceeds its diameter safely carry a greater weight than this

limit. We shall find, however, in crushing plates, that we may nearly approach this limit, as their thickness becomes considerable compared with their length; that is, when they are thick enough to resist the flexure or buckling, that took so prominent a position in the experiments before described on thin circular and elliptical tubes.

If we take a wrought-iron plate of considerable length in proportion to its thickness, or use it as a pillar lengthwise, and imagine its dimensions varied, we have to inquire what corresponding variation will take place in its resistance to flexure. We can only make it wider, longer, or thicker; and we will separately consider each such variation.

First, if we make it wider, its resistance to flexure will evidently be proportionate directly to its width. If we make it, for instance, twice as wide, each half exerts the same resistance to flexure, whether they are separated or united; and consequently, doubling the width is merely doubling the amount of material to be curved, or doubling the strength.

Secondly, if we make it longer, it will follow from the laws of pillars that its strength will be inversely proportionate to the square of its length; for the strength of a pillar varies inversely as the square of its length; the plate may be considered as a number of square pillars placed side by side, the deflection being merely confined in direction by their junction, but the resistance of each pillar to flexure remaining unaltered, and consequently the resistance of the whole. Thus, an inch plate, 12 inches broad and 12 feet long, is evidently the same thing, as regards its bending under these circumstances, as 12 separate pillars, each one inch square and 12 feet long, placed side by side and bending in a similar direction.

Thirdly, as regards its thickness, this may also be inferred from the laws of square pillars, though not so evidently at first sight.

Let us suppose an inch plate of given length and width, say 12 feet by 12 inches, and we wish to ascertain the increase of its strength when employed as a pillar after doubling its thickness; and let A B be the given plate in section, that is,

looking down on the top; and let C D be the addition of thickness. Divide A B into 12 square pillars, each one inch square; then the resistance of pillar 1 to flexure is, by the laws of pillars, as the fourth power of its diameter A M; so that if its diameter A M is doubled, and becomes A G, the new pillar A H is 2^4 as strong as the pillar A G. But the new pillar A H can only deflect flatways as regards the plate, and in that direction may be considered as two pillars, A N, M H, standing side by side and separate. The strength of each of them will evidently be, therefore, $\frac{2^4}{2}$, or 2^3 times as strong as the original inch pillar. The original pillar 1 has therefore, in that direction, had its strength increased in the proportion of the cube of the thickness, A E. And similarly will pillars 2, 3, 4, be also increased in the same proportion; and the strength of the whole plate will therefore be increased, as regards flexure, in the proportion of the cube of its thickness.

So also, if the plate be made any number (n) times as thick, the new pillar A H will then be of n times the diameter of pillar 1, and its strength will be n^4 times as great; and this pillar may be conceived to consist of n pillars side by side, the strength of one of which will be $\frac{n^4}{n}$, or n^3 times that of the original pillar; and therefore the strength of the whole plate will be n^3 times as great. This is evidently general, whatever value we give to n.

Without, however, arriving at this result by this species

of reasoning, Mr. Hodgkinson, with the concurrence of Mr. Stephenson, submitted a number of plates to strain in the direction of their length, and fully demonstrated these laws. The following table contains his results. The plates are made to vary in length an exact number of times, so that they are convenient for comparison, the deflections and weights being recorded with that minute attention to accuracy which characterises all his experiments.

Results of Experiments made to determine the Resistance of Plates (or Bars) of Wrought Iron to a force of Compression; the Plates being placed in a Vertical Position during the Experiments, with their ends made perfectly flat, so as to be well bedded against two Parallel and Horizontal Crushing Surfaces.

Vertical Length of the Plate.	Lateral Dimensions of the Plate.	Mean from Lateral Dimensions.	Weight of the Plate.	Deflection of the Plate in the Middle.	Weight producing that Deflection.	Weight of greatest Resistance or Breaking Weight.	Mean from Weights of greatest Resistance.	Weight, per Square Inch of Section, borne by Plate at time of Fracture.	Remarks.
Ft. In.	Inches.	Inches.	Lbs. Oz.	Inch.	Lbs.	Lbs.	Lbs.		
10 0	2·98 × ·503	2·98 × ·497	..	..	..	1222			The bars being 10 feet high and only ½ inch thick, there was a great difficulty in getting them to stand vertically without support; and as the smallest deviation from the vertical produced flexure, the results would probably be too small; the larger breaking-weight is therefore only taken, in calculating the strength per square inch.
10 0	2·98 × ·492	=1·481 sq. in.		·14	438		1099	= ·364 ton from the first experiment.	
				·26	672				
				·40	868	976			
10 0	3·01 × ·763		76 13	·024	1494		..	..	The weights were in all cases laid on gradually, by small increments, carefully avoiding vibration.
				·02	4238				
				·02	6590				
				in an opp. direct. ·07	6982				
		3·01 × ·766 =2·306 sq. in.		in an opp. direct. ·072					
				in 5 minutes ·12	7374	7786			
10 0	3·01 × ·77		76 2½	·02	2670				
				·052	4238		7793	=1·508 tons	
				·10	5806				
				·15	6590				
				·31	7374	7800			

5 0	3·01 × ·765	3·01 × ·767 = 2·309 sq. in.	37 14	Just perceptible ·025 ·07 · 75 in 5 minutes	11686 23446 28150 . .		29955	= 5·79 tons	
5 0				·09 ·21 ·22 in 1 minute	28934 30502 . .	30620			
	3·01 × ·77		38 8½	Just perceptible ·02 ·05 ·08 ·12 ·15 and ·16 ·26	12148 19316 22900 25588 27380 28276 29172	29290			
5 0	3·02 × ·994	3·01 × ·995 = 2·995 sq. in.	49 7	Perceptible ·02 ·035 ·07	30114 42466 50530 53218	. .	. .	. .	
5 0	3·00 × ·996		49 10½	Perceptible ·03 ·05 ·085	19362 39778 48738 53218	53218 55010	54114	=8·066 tons	

Vertical Length of the Plate.	Lateral Dimensions of the Plate.	Mean from Lateral Dimensions.	Weight of the Plate.	Deflection of the Plate in the Middle.	Weight producing that Deflection.	Weight of greatest Resistance or Breaking Weight.	Mean from Weights of greatest Resistance.	Weight, per Square Inch of Section, borne by Plate at time of Fracture.	Remarks.
Ft. In.	Inches.	Inches.	Lbs. Oz.	Inch.	Lbs.	Lbs.	Lbs.		
7 6	2·97 × ·504		38 7	Perceptible ?	1614	..	..	..	This bar had been hammered, and was somewhat irregular in form.
				·06	2394				
				·29	2898	2928			
7 6	2·98 × 5·03	2·983 × ·5023 =1·4983 sq.in.		·08	3738		..	..	The deflection increased from ·20 to ·28 in about 5 minutes, when the bar gave way.
				·20	4074	4074			
				·28 in 5 minutes	..		3614	=1·076 tons	
7 6	3·00 × ·500		37 6½	·01	1614				Sunk by flexure as before.
				·03	2790				
				·09	3574				
				·18	3770	3840			
7 6	3·00 × 1·00		75 8	Perceptible	6052	..	..	..	
				Ditto increased	10364				
		3·005 × ·9955 =2·9915 sq.in.		·035	12716				
				·15	25260				
				·39	29180	29572			
7 6	3·01 × ·991		64 4	·03 nearly	26978		29619	=4·425 tons	This bar was very irregular in thickness.
				·08 nearly	29218	29666			

7 6	3·00 × 1·53	3·00 × 1·53 = 4·59 sq. in.	116 13	Perceptible ·01 ·06	57698 84578 89954	. . 91746	. . 91746	. . =8·923 tons	
7 6	5·86 × ·995	5·86 × ·995 =5·8307 sq.in.	148 3	·11 ·14 ·19 ·26 ·48 ·50 in ½ a minute	39778 43362 46946 50530 54114 . .	. . 54114	. . 54114	. . =4·143 tons	
5 0	1·024 × 1·024	1·024 × 1·024 =1·0486 sq.in.	16 13	Perceptible ·055 ·18 ·21 in 5 minutes	5178 14162 17746 . .	. . 17746	. .	. .	
5 0	1·024 × 1·024		16 14½	Bent ·02 nearly ·10 ·12 in 3 minutes ·14	5178 8834 17794 . . . . repeated	18466	. . 18106	. . =7·709 tons	

Vertical Length of the Plate.	Lateral Dimensions of the Plate.	Mean from Lateral Dimensions.	Weight of the Plate.	Deflection of the Plate in the Middle.	Weight producing that Deflection.	Weight of greatest Resistance or Breaking Weight.	Mean from Weights of greatest Resistance.	Weight, per Square Inch of Section, borne by Plate at time of Fracture.	REMARKS.
Ft. In.	Inches.	Inches.	Lbs. Oz.	Inch.	Lbs.	Lbs.	Lbs.		
5 0	2·97 × ·500		24 13	Perceptible	3700	. .	. .	. .	
				·02	4484				
				·08	6052				
				·13	6836	7620			
				Perceptible	2524				
			25 2	·03	3308				
5 0	2·98 × ·509	2·98 × ·507 = 1·511 sq. in.		·055	4484				
				·080	6052				
				·14	7620				
				·14	. . repeated				
				·25	8404	8502	8469	= 2·502 tons	
5 0	2·99 × ·512		25 6	Perceptible	2524				
				·015?	3700				
				·03	5268				
				·05	6836				
				·095	8404				
				·21	9188	9286			

5 0	3·01 × ·765		37 14	Just perceptible	11686				
				·025	23446				
				·07	28150				
				· 75 in 5 minutes	..				
		3·01 × ·767 = 2·309 sq. in.		·09	28934				
				·21	30502				
5 0				·22 in 1 minute	..	30620			
	3·01 × ·77		38 8½	Just perceptible	12148				
				·02	19316		29955	= 5·79 tons	
				·05	22900				
				·08	25588				
				·12	27380				
				·15 and ·16	28276				
				·26	29172	29290			
5 0	3·02 × ·994		49 7	Perceptible	30114	..	..	..	
		3·01 × ·995 = 2·995 sq. in.		·02	42466				
				·035	50530				
				·07	53218				
5 0	3·00 × ·996		49 10½	Perceptible	19362	53218			
				·03	39778		54114	= 8·066 tons	
				·05	48738				
				·085	53218	55010			

Vertical Length of the Plate.		Lateral Dimensions of the Plate.	Mean from Lateral Dimensions.	Weight of the Plate.		Deflection of the Plate in the Middle.	Weight producing that Deflection.	Weight of greatest Resistance or Breaking Weight.	Mean from Weights of greatest Resistance.	Weight, per Square Inch of Section, borne by Plate at time of Fracture.	REMARKS.
Ft.	In.	Inches.	Inches.	Lbs.	Oz.	Inch.	Lbs.	Lbs.	Lbs.		
5	0	5·84 × ·996	5·84 × ·996 =5·8166 sq.in.	101	0½	·03 nearly	39778	..	..	..	
						·055	75618				
						·115	93538				
						·160	98018				
						·35	102946	102946	102946	=7·901 tons	
2	6	1·024 × 1·024	1·0235×1·0235 =1·0475 sq.in.	8	8½	·0	19362	..	..	..	
						·10 nearly	26530	26530	26530	=11·307 to.	
2	6	1·023 × 1·023		8	6½	·0	22946	26530			
2	6	2·98 × ·503	2·9867 × ·5026 =1·5011 sq.in.	12	15	Not perceptible	22050	..	..	..	
						Perceptible	25634	29218			
2	6	2·98 × ·500		12	13	..	..	25526	25299	=7·524 tons	
2	6	3·00 × ·500		12	6	·09 nearly	19362	21154	..	..	
2	6	3·01 × ·764	3·01 × ·763 =2·297 sq. in.	19	5	Perceptible	46196	61994	..	..	Sunk by flexure with great violence.
2	6	3·01 × ·762		19	1	Perceptible; not perceived with a less weight.	48884		63786	=12·396 to.	
						·05 nearly in opposite direct.	58740				
						$\frac{1}{16}$ nearly.	64564	65578			Sunk as before with great violence.

2 6	3·00 × ·992	3·00 × ·996 =2·988 sq. in.	24 4½	·0	57698	..	..	..	
				Perceptible	66658				
				·02 ?	75618	83682	88610	=13·239 to.	
2 6	3·01 × 1·00		24 12	·0 ?	84578	93538			Began to bend with 93538 lbs. and sunk immediately.
1 3	1·023 × 1·023	1·023 × 1·023 =1·0465 sq.in.	4 3	Perceptible	31906	34146	..	..	It bent diagonally, and was crushed at the ends.
							36162	=15·426 to.	
1 3	1·023 × 1·023		4 3	Not perceptible	30114				
				Slightly bent	33698	38178	..	..	Bent diagonally as before. With 33698 lbs.; it was reduced in length ·047 inch.
0 7½	1·023 × 1·023	1·023 × 1·023 =1·0465 sq.in.	2 1½	·0	26530	..	..	..	With 26530 lbs. it was reduced in length ·024 inch, and with 37282 lbs., ·079 inch.
				Slightly bent	33698				
				·10 nearly	37282	50274			
0 7½	1·023 × 1·023		2 2	·0	30114		50946	=21·733 to.	
				Perceptible	33698				
				Very little bent, but crushed	37282				
				·02 nearly	40866				
				·05	44450	51618	..	..	With 26530 lbs. it was reduced in length ·020 inch.
0 3¾	1·023 × 1·023	1·023 × 1·023 =1·0465 sq.in.	1 0½	·0	48034	..	..	Weight of greatest resistance not obtained. =23·549 to. Weight sustained without fracture.	With 55202 lbs. the experiment was discontinued.
				·0	51618				
				·51	55202				

Abstract of the above Results.

Number of Experiment.	Length of the Plate or Bar.		Weight of the Plate.		Lateral Dimensions of the Plate.	Breaking Weight, or Weight of greatest Resistance.	Weight of greatest Resistance per Square Inch of Section.	Number of Experiments from which the Means were derived.
	Ft.	In.	Lbs.	Oz.	Inches. Sq. Inches.	Lbs.	Tons.	
1	10	0	..		2·98 × ·497 = 1·481	1099, or 1222 from first experiment.	·364	..
2	10	0	76	7¾	3·01 × ·766 = 2·306	7793	1·508	2
3	10	0	100	0	2·99 × ·995 = 2·975	12735	1·911	2
4	10	0	151	15	3·00 × 1·51 = 4·53	46050	4·538	1
5	7	6	25	10¾	1·024 × 1·025 = 1·0496	10236	4·354	2
6	7	6	37	14¾	2·983 × ·5023 = 1·4983	3614	1·076	3
7	7	6	69	14	3·005 × ·9955 = 2·9915	29619	4·425	2
8	7	6	116	13	3·00 × 1·53 = 4·59	91746	8·923	1
9	7	6	148	3	5·86 × ·995 = 5·8307	54114	4·143	1
10	5	0	16	13¾	1·024 × 1·024 = 1·0486	18106	7·709	2
11	5	0	25	2	2·98 × ·507 = 1·511	8469	2·502	3
12	5	0	38	3¼	3·01 × ·767 = 2·309	29955	5·79	2
13	5	0	49	8¾	3·01 × ·995 = 2·995	54114	8·066	2
14	5	0	101	0½	5·84 × ·996 = 5·8166	102946	7·901	1
15	2	6	8	7½	1·0235 × 1·0235 = 1·0475	26530	11·307	2
16	2	6	12	11½	2·9867 × ·5026 = 1·5011	25299	7·524	3
17	2	6	19	3	3·01 × ·763 = 2·297	63786	12·396	2
18	2	6	24	8¼	3·00 × ·996 = 2·988	88610	13·239	2
19	1	3	4	3	1·023 × 1·023 = 1·0465	36162	15·426	2
20	0	7½	2	1¾	1·023 × 1·023 = 1·0465	50946	21·733	2
21	0	3¾	1	0½	1·023 × 1·023 = 1·0465	..	Weight of greatest resistance not obtained 23 549 = weight sustained without fracture, or much flexure, though the length was considerably reduced.	

Now the method by which the laws we have already announced were derived by Mr. Hodgkinson from these experiments was as follows, and will serve as an example of practical analytical reasoning:—

First, to determine the powers of the length to which the breaking-weight was proportional, the other dimensions being supposed to remain constant.

Selecting from the foregoing table those experiments in which the section was the same, and in which the plates or bars only differ in length, and taking, first, a mean from those experiments where the plates were each 3 inches broad and 5 inches thick, we find that when these plates were $7\frac{1}{2}$ feet long they were bent beyond further resistance by a weight of 3501 lbs., and when they were $2\frac{1}{2}$ feet long by 27,372 lbs.

Now, supposing the resistance to be inversely proportional to l^n, that is, to some constant power (n) of the length, we have to determine the value of n. We shall evidently have:

$$(2\tfrac{1}{2})^n : (7\tfrac{1}{2})^n :: 3501 : 27372$$

Or $$1^n : 3^n :: 3501 : 27372$$

$$\therefore 3^n = \frac{27372}{3501} = 7\cdot81834$$

And $$n \log. 3 = \log. 7\cdot81834$$

$$\therefore n = \frac{\cdot8931144}{\cdot4771212} = 1\cdot87.$$

Taking from the table another set, where each plate was of the same lateral dimensions as before, when the plates were $2\frac{1}{2}$ feet long, we have, as above, the breaking-weight $= 27,372$ lbs.; and when the bars were 5 feet long it was 8469. Hence, as before,

$$(2\tfrac{1}{2})^n : 5^n :: 8469 : 27372$$

Or $$\left(\frac{5}{2\frac{1}{2}}\right)^n = \frac{27372}{8469} = 3\cdot23202$$

$$\therefore n \log. 2 = \log. 3\cdot23202$$

And $$n = 1\cdot69.$$

Again, selecting from the table bars each 3 inches broad and 1 inch thick, when 5 feet long the breaking-weight was 55,010 lbs., and when 10 feet long, 11,932 lbs.; hence—

$$1^n : 2^n :: 11932 : 55010$$

Or $$2^n = 4{\cdot}60945$$

$$\therefore \log. 2^n = \log. 4{\cdot}60945$$

Whence $$n = 2{\cdot}20.$$

Again, with bars each 3 inches broad and $1\frac{1}{2}$ inch thick, when 10 feet long the breaking-weight is 46,050 lbs.; when $7\frac{1}{2}$ feet long, 91,746 lbs.

$$\therefore (7\tfrac{1}{2})^n : (10)^n :: 46050 : 91746$$

$$\therefore \left(\frac{10}{7\frac{1}{2}}\right)^n \text{ or } (1\tfrac{1}{3})^n = 199231$$

And $$n \log. 1\tfrac{1}{3} = \log. 199231$$

Whence $$n = 2{\cdot}39.$$

Lastly, Selecting bars each 3 inches wide and 1 inch thick, when 10 feet long the breaking-weight is 11,932, and when $2\frac{1}{2}$ feet long, 83,682 lbs.

$$\therefore (2\tfrac{1}{2})^n : 10^n$$

Or $$1^n : 4^n :: 11932 : 83682$$

$$\therefore 4^n = 7{\cdot}01324$$

$$n = 1{\cdot}40.$$

Now, adding together these five values of n and taking the mean, we get

$$n = \frac{1{\cdot}87 + 1{\cdot}49 + 2{\cdot}20 + 2{\cdot}39 + 1{\cdot}4}{5} = 1{\cdot}91;$$

or the power of the length to which the breaking-weight is inversely proportional is the 1·9 power, or the square, very nearly as we before assumed.

To determine, secondly, the power of the thickness to which the breaking-weights obtained by experiment were proportional.

Selecting from the table some bars of the same length, but differing in thickness, we find that two bars, each $7\frac{1}{2}$ feet long, but which were respectively 3 inches broad and $\frac{1}{2}$ inch thick, and 3 inches broad and $1\frac{1}{2}$ inch thick, broke the first with 3501 lbs., the second with 91,746 lbs.

Now, supposing the resistance to the proportional to d^n, that is, to some constant power n, of the thickness d, we have to determine the value of n. We have, therefore,

$$\tfrac{1}{2}^n : (1\tfrac{1}{2})^n$$

Or $$1^n : 3^n :: 3501 : 91746$$

$$\therefore 3^n = \frac{91746}{3501} = 26{\cdot}20565$$

Whence $$n \log. 3 = \log. 26{\cdot}20565$$

And $$n = \frac{\log. 26\ 20565}{\log. 3}$$

$$= \frac{1{\cdot}4183950}{{\cdot}4771213} = 2{\cdot}97.$$

Again; selecting two bars, each 7 feet long, the one whose section was $2{\cdot}97 \times 1{\cdot}05$ inches bore 11,932 lbs.; the other, whose section was $3{\cdot}0 \times 1{\cdot}51$ inches, bore 46,050 lbs.: the depth varies as 1 : 1·51 nearly.

$$\therefore 1^n : 1{\cdot}51^n :: 11932 : 46050$$

$$\therefore \quad 1{\cdot}51^n = 3{\cdot}85937$$

$$n = \frac{\log. 3{\cdot}85937}{\log. 1{\cdot}51} = 3{\cdot}27.$$

Lastly; selecting two bars, each 5 feet long, the one whose section was $2{\cdot}98 \times {\cdot}509$ inches bore 8469 lbs.; the other, whose section was $3{\cdot}0 \times 1{\cdot}0$ inches, bore 55,010 lbs. the depths being, when reduced as to breadth, as 1 to 1·97, we have—

As $$1^n : 1{\cdot}97^n :: 8669 : 55010$$

Or $$1{\cdot}97^n = 6{\cdot}4954$$

Or $$n = \frac{\log. 6{\cdot}4954}{\log. 1{\cdot}97} = 2{\cdot}76.$$

Again; adding together these three values of n to obtain the mean, we have

$$n = \frac{2{\cdot}97 + 3{\cdot}27 + 2{\cdot}76}{3} = 3 \text{ exactly},$$

that is, the resistance varies exactly as the cube of the thickness, as we have before assumed.

But square pillars have their strengths, as the fourth powers of their diameters, or as the cube of their thickness, into their width, regarding them as narrow plates; hence all long pillars, or plates used as pillars, have their breaking-weights nearly proportional to their width and to the cube of their thickness directly, and to the square of their length inversely, both theoretically and experimentally. And also all similar pillars, or plates used as pillars, have their strengths proportional to the square of their like lineal dimensions.

The last very important deduction at once follows from what we have demonstrated, for if we suppose a plate in every respect similar to another, and of n times the dimensions, *but only of such dimensions*, that each fails by flexure when used as a pillar, the strength of the second plate will be n^3 times as great on account of its increase of thickness, n times as great on account of its increase of width, and $\frac{1}{n^2}$ times as great on account of its increase of length. It will thus be altogether $\frac{n^3 n}{n^2}$ or $\frac{n^4}{n^2}$, or n^2 times as strong.

We thus see how erroneous a conclusion would be drawn from *thin* models, as to the buckling of the top, in larger and similar structures, as was the case with the preliminary experiments on thin circular and elliptical tubes, which were condemned on this account; for the top of these tubes was merely a pillar failing from flexure, and had similar tubes been constructed of larger dimensions, then the strength of the top to resist buckling or flexure would have

been as the square of the lineal dimensions of increase, until the pillars or thickness became of sufficient dimensions to resist buckling altogether, and to fail by crushing. The curvature in the top in these tubes was, in fact, favourable to the resistance of flexure, a flat plate would have buckled with less strain.

The limit of 16 tons per inch cannot, however, be exceeded practically, whatever the form. This limit, it will be observed, is nearly approached in the inch square bars in Experiment No. 16, which were 15 inches long; in the next experiment the bar was practically crushed with about 16 tons, though it carried considerably more; and we shall hereafter see that with the large model with cells the top was destroyed by compression with about 14·8 tons per square inch.

However valuable, in numberless practical cases, these interesting experimental confirmations of the theory of pillars may prove, they are evidently not directly applicable to the structure which led to them. The top of the Britannia Bridge, though closely analogous, is not a simple pillar or plate subjected to compression. When failure takes place in the top of an ordinary tube by *actual compression*, it will be found to have sustained little more than 16 tons per square inch of its transverse section, and no alteration of its manner of construction would be of any avail. But should it fail by buckling or flexure, its breaking strain would fall short of this quantity; and any contrivance or method of construction which would prevent flexure, would approximate it to this limit. The importance of some such contrivance is thus evident with thin tubes. The necessity for preventing flexure in a column has already been alluded to, and (as in a beam broken transversely) the comparative depth, or leverage with which the material resists flexure, has been shewn to be the most important consideration. We may double the width

of a plank, and when we stand on the centre, it bends only half as much as before; but if we double its thickness, we have only one-eighth part of its original deflection. It, indeed, assumes another name, and we call it a beam, though in both cases the quantity of material is precisely the same.

A plate employed as a pillar similarly resists flexure in a much higher ratio than in the simple proportion of its thickness, such stiffness or strength being analogous to the transverse stiffness of a beam. Hence, as in the beam, it will also be highly advantageous to distribute any given material in a pillar in such a manner as to ensure the greatest possible depth in the direction in which it is liable to bend. If the pillars are short as compared with their diameter, such precautions are useless, the cubic inch of wrought-iron cannot be put in better form, but if it were rolled into a long and very thin plate one inch broad, and placed on edge, the smallest force would bend it. If we shorten this thin plate by increasing its thickness, but maintaining the same height of one inch, we shall increase its resistance to flexure in proportion directly to the cube of the thickness, and in proportion inversely to the length, since the length will diminish in the same proportion as the thickness increases, therefore the strength will increase directly as the square of the increasing thickness, or inversely as the square of the decreasing length, until the plate arrives at such a thickness that it will fail partly by crushing. The law will now begin to vary as we go on increasing the thickness at the expense of the length; and ultimately, as we approach to the cube itself again, the strength, instead of varying as the square of the increasing thickness, will cease to vary at all with the thickness; its strength will, therefore, have varied during these changes, as every power of the thickness between *o* and the square of the thickness, while the resistance itself would be represented progressively by every quantity between 16 tons and *o*, the

quantity of material or section and height, having remained constant, so that n square inches of sectional area on the top of a tube may resist any compression between 0 and $n \times 16$ tons, according to the form in which it is applied.

We should thus use the thickest plates we can get for the top of a tube, until their thickness was such that any variation in the thickness causes no corresponding variation in their resistance to compression,—beyond this we get no further advantage. If, however, we are compelled to use thin plates, we should arrange them so as to ensure depth to resist buckling. If one cubic inch, when rolled out into a long strip so as to fail by flexure, were, for instance, formed into corrugations, it would in this form support considerably more than in the

form of a straight plate, for instead of being a mere line in section with no depth, it would now possess a depth equal to the versed sine of the corrugations, or equal to the distance between each convexity; and in this corrugated form we should attain the maximum resistance to pressure, viz. 16 tons, with our plates much thinner than when used straight. The depth would be still further increased if we folded our corrugated plate round upon itself, so as to complete a series of tubes, taking care to unite carefully the points of contact.

There are numberless familiar examples of stiffness obtained by such method of construction. An ordinary paper fan, and many household articles in tin, though constructed of thin and pliable material, are extremely strong and rigid from the depth acquired by the bending of the material. The domestic tea-board and dust-shovel are striking examples. It

thus becomes a question, with a given section of material of given thickness, how to construct the strongest form of pillar to resist crushing, and how near we can with this form approach to the limit of 16 tons per square inch.

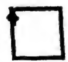
Since a flat plate, for the reasons explained, will bend sooner than a curved plate; it would be concluded, naturally, that a round tube, of moderate dimensions and of given thickness and section, would be a stronger form than the same plate in a rectangular form, in which the resistance to crippling must depend solely on the four angles; and since the rigidity afforded by the angles is extended throughout the four sides of a rectangular tube, in some manner proportionate to the distance from the angles, it would be concluded that a square tube  would be stronger than a rectangular tube  constructed with the same plate, inasmuch as the central portions of the longer sides of the rectangle will be less maintained in form on account of their greater distance from the angles; similarly increased strength might be expected from this form . These assumptions were all submitted to experiment and confirmed.

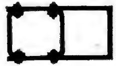
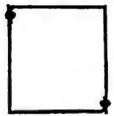
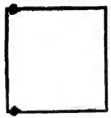
For this purpose a number of tubes or cells of wrought-iron were constructed, all 10 feet long and either 4 or 8 inches square, or of rectangular form about 4×8 inches; their ends were perfectly flat, and they were compressed by the intervention of a lever between two parallel disks of steel, with arrangements for maintaining the pressure perfectly vertical, the cells being unsupported laterally. The direct object was to ascertain the value of each particular form of cell, and to ascertain the resistance per square inch of section in each case. The lateral dimensions of these cells are so large, that with a length of 10 feet the pillars were not destroyed by flexure, as in a long pillar, but by absolute buckling or crushing; the strongest possible form should, therefore, give about 16 tons per square inch of section.

The results obtained with rectangular cells are given in Table IX.

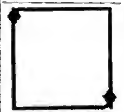
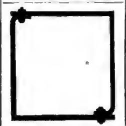
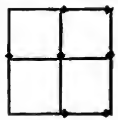
Similar experiments were then made with circular cells under precisely similar circumstances for comparison. The cylinders varying from $1\frac{1}{2}$ to 6 inches in diameter, the diameter being so small in some cases as compared with the length; some of these pillars failed by flexure, and followed the laws of long pillars, the resistance increasing nearly inversely as the square of the length; but where the diameter was 6 inches, the length being 10 feet, flexure could not take place, and the cells failed by buckling or crushing, as in all the rectangular pillars, and in such pillars the strength is independent of the length. The results are given in Table X.

TABLE IX.—*Resistance of Rectangular Tubes, all Ten Feet Long, to a Force of Compression in the direction of their Length.*

Number of Experiment.	Weight of the Tube.		External Dimensions of the Tube.	Thickness of the Plates of the Tube.	Weight with which Buckling, or perceptible Undulation, was observed.	Weight of greatest Resistance.	Form of Section of the Tube.	Area of Section of the Tube.	Weight per Square Inch at which Buckling or Undulation was perceptible.	Weight per Square Inch of greatest Resistance.
	Lbs.	oz.	Inches.	Inch.	Lbs.	Lbs.		Inches.	Lbs. Tons.	Tons.
1	21	2½	4·1 × 4·1	·03 nearly.	. .	5534		·504	. .	4·902
2	43	14½	4·1 × 4·1	·06 nearly.	. .	19646		1·0200	. .	8·5986
3	60	6½	4·25 × 4·25	·083	29290	37354		1·484	19742=8·814	11·237
4	80	6	4·25 × 4·25	·134	46314	51690		2·3947	19340=8·634	9·636
5	65	8	8·175 × 4·1	·061	13209	23289		1·532	8622=3·85	6·786

6	231	0	$8{\cdot}5 \times 4{\cdot}75$	·264	..	197163		7·326	..	12·015
7	233	0	$8{\cdot}4 \times 4{\cdot}25$	·26 & ·126	99916?	206571= 92·2 tons.		6·89 nearly.	14502=6·474?	13·3845
8	82	0	$8{\cdot}1 \times 4{\cdot}1$	·059	37401	43673		1·885	19842=8·857	9·877
9	290	0	$8\frac{1}{2} \times 4\frac{3}{4}$	$\frac{1}{4}$ nearly.	..	..		8·3466	..	(Not crushed with 11·12 tons.)
10	91	8	$8{\cdot}1 \times 8{\cdot}1$	·06 nearly.	15897	27545		2·070	76797=3·428, or more observable with 4·267	5·926
11	162	0	$8{\cdot}37 \times 8{\cdot}37$	·1390	82475	100395= 44·81 tons.		4·9262	16742=7·474	9·098

Resistance of Rectangular Tubes, all Ten Feet Long, to a Force of Compression in the direction of their Length—(continued).

Number of Experiment.	Weight of the Tube.	External Dimensions of the Tube.	Thickness of the Plates of the Tube.	Weight with which Buckling, or perceptible Undulation, was observed.	Weight of greatest Resistance.	Form of Section of the Tube.	Area of Section of the Tube.	Weight per Square Inch at which Buckling or Undulation was perceptible.	Weight per Square Inch of greatest Resistance.
	Lbs. oz.	Inches.	Inch.	Lbs.	Lbs.		Inches.	Lbs. Tons.	Tons.
12	296 0	8·5×8·375	·2191	..	198955= 88·8 tons.		7·7367	..	11·48
13	333 0	8·5×8·4	·245 & ·238	..	..		8·4665	..	Not crushed with 11·015 tons.
14	157 4	8·1×8·1	·0637	56630	70070		3·551	15947=7·119	8·809
" repeated.	..	8·1×8·1	·0637	46635	82027		3·551	13133=5·863	10·312

CRUSHING OF TUBES OF WROUGHT IRON. (Referred to also on page 352)

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Exper. 8. (P. 347)
Length 1.7½

Exper. 1. (P. 346)
Length 2.6

Exper. 9. (P. 358)
Length 7.6

Exper. 10. (P. 359)
Length 5 feet

Exper. 14 (P. 361)
Length 5 feet

Exper. 15. (P. 362)
Length 2.4.3

TABLE X.

Resistance of Circular Tubes, all Ten Feet Long, to a Force of Compression in the direction of their Length.

Number of the Tube.	Weight of the Tube.		External Diameter of the Tube.	Internal Diameter of the Tube.	Thickness of the Plates of the Tube.	Weight of greatest Resistance.	Area of Section of Tube.	Weight per Square Inch of greatest Resistance.
	Lbs.	oz.	Inches.	Inches.	Inches.	Lbs.	Inches.	Tons.
1	15	8	1·495	1·292	..	6514	·4443	6·55
2	20	15½	1·964	1·755	..	14158	·6104	10·35
3	28	4½	2·49	2·275	..	23958	·8045	13·29
4	53	14	2·35	1·865	..	34516	1·605	9·600
5	47	10	2·34	1·91	·215	31828	1·4353	9·901
6	45	15	2·995	2·693	..	37356	1·349	12·362
7	59	0	4·05	3·772	..	47212	1·7078	12·34
8	64	4	4·06	3·75	·150 nearly	49900	1·9015	11·71
9	96	8	6·366	6·106	·1298	91402	2·547	16·021
10	63	5	6·187	..	·0939	60075	1·799	14·908

The method of failure of some of the tubes in these experiments will be seen in the accompanying Plate.

It will be observed, that as regards form, other circumstances remaining constant, that a large square □ with thin plates is the weakest form experimented upon; the next in order is the rectangle , which is nearly doubled in its resistance to crippling by the addition of a division across the centre, thus ; the square divided into four compartments  gave still better results, while the circular form was much the strongest of all.

To illustrate the value of thickness Mr. Hodgkinson has arranged the following table:

TABLE XI.

Resistance of Tubes, Rectangular in Section, to a force of Compression in the direction of their Length.

Rectangular 8 × 4.		Tubes 8-inch square.		Tubes 4-inch square.	
Thickness of Plates.	Strength per square inch of Section.	Thickness of Plates.	Strength per square inch of Section.	Thickness of Plates.	Strength per square inch of Section.
Inch.	Tons.	Inch.	Tons.	Inch.	Tons.
..	..	..	..	·03	4·9
·061	6·79	·06	5·9	·06	8·6
..	..	..	..	·083	11·24
..	..	·139	9·1	·134	9·64
..	..	..	..	·134	10·36 in 7 ft. 6 in. tube.
..	..	·219	11·48	..	..
·264	12·015	..	..	..	..

From the above it is seen, that in the crushing of rectangular tubes the strength, instead of being nearly as the third power of the thickness, as in the case of flexure of plates, is so much reduced, that, to produce double the strength, even three or four times the thickness of metal is required: thus,—

	Thickness.		Tons.
Plates of....	$\left\{\begin{matrix}\cdot 03\\ \cdot 134\end{matrix}\right\}$	give strengths	$\left\{\begin{matrix}4\cdot 9\\ 10\cdot 36\end{matrix}\right\}$
and Plates of....	$\left\{\begin{matrix}\cdot 061\\ \cdot 264\end{matrix}\right\}$	give strengths	$\left\{\begin{matrix}6\cdot 79\\ 12\cdot 015\end{matrix}\right\}$

It also appears that when the thickness of the plates is the same, the strength of smaller tubes is greater than that of larger, as may be seen from the strengths of tubes of equal thickness placed opposite each other in the above table.

None but very thick plates resist with 12 tons per square inch, but the hollow cylinders gave better results, some of them attaining the limit of 16 tons per square inch.

If we compare together Experiment 8, rectangular form, and Experiment 7, cylindrical form, the importance of form is very evident, for we have, with the same length,

	Weight of Tube.		Breaking-weight.
Rectangular form	82 lbs.	......	43,673 lbs.
Cylindrical form	59 lbs.	......	47,212 lbs.

It was important to ascertain in what manner the strength depended on the length of the cells, and for that purpose shorter portions of each of the 10-feet tubes were again crushed for this purpose.

The following tables contain these results, and more accurate details of all the experiments, of which a general summary only has been given in the preceding abstracts.

As employed in the top of the tube of moderate width, the sides entirely prevent the top from flexure as a long pillar, and the destruction of the cells is thus confined to the crushing of the material in some short length of the top; the following experiments prove, that, within considerable limits, the strength of these cellular pillars is nearly independent of length. The form of section in each of the experiments in Table XII. is given in Table IX., corresponding numbers being used in each table.

TABLE XII.

Resistance of Rectangular Tubes or Cells of Wrought Iron to a force of Compression applied in the direction of their length. The ends of the Tubes being flat and crushed between two parallel discs of Steel.

No. of Experiment.	Length of Tube.	Weight of Tube.	External Dimensions of Tube.	Thickness of Plates of Tube.	Weight laid on Tube.	Decrease of Length from that Weight.	Weight with which Buckling, or Perceptible Undulation, was observed.	Weight of greatest Resistance.	Area of Section of Tube.	Weight per Square Inch at which Buckling, or Undulation, was observed.	Weight per Square Inch at greatest Resistance of Tube.	Remarks.
	Ft. in.	Lbs. oz.	Inches.	Inch.	Lbs.	Inch.	Lbs.	Lbs.	Sq. Inch.	Tons.	Tons.	
1	10 0	21 2⅓	4·1 × 4·1	·03 nearly.	..	..	..	5534	·504	..	4·902	
	5 0	..	,,	,,	..	..	..	5803	,,	..	5·140	
	2 6	..	,,	,,	..	..	..	6251	,,	..	5·537	
2	10 0	43 14⅓	4·1 × 4·1	·06 nearly.	..	..	..	19646	1·020	..	8·5986	
3	10 0	60 6⅓	4·25 × 4·25	·083	..	..	29290	37354	1·484	8·814	11·237	
	5 0	..	,,	·085	..	..	..	35850	1·52	..	10·529	
	2 6	..	,,	·085	..	..	..	41674	1·52	..	12·24	
4	10 0	80 6	4·25 × 4·25	·134	..	..	46314	51690	2·3947	8·634	9·636	
	7 6	..	,,	,,	28715 37675 46635	·062 ·079 ·095	..	55562	,,	..	10·358	
	1 8·1	..	4·44 × 4·44	·136	..	..	..	73034				

5	10 0	65 8	8·175 × 4·1	·061	..	..	13209	23289	1·532	3·85 Or more observable with 6·176 tons.	6·786	
	7 8	..	,,	,,	..	..	..	24843	,,	..	7·239	
	2 4	..	,,	,,	..	..	..	24395	,,	..	7·108	
6	10 0	231 0	8·5 × 4·75	·264	118315 172075 189995	·116 ·168 ·231	..	197163	7·326	..	12·015	
7	10 0	233 0	8·4 × 4·25	·260 & ·126	64555 100395 189995 198955	·110 ·144 ·276 ·289	99916 ?	206571 = 92·2 tons.	6·89 nearly.	6·474 ?	13·3845	
8	10 0	82 0	8·1 × 4·1	·059	..	..	37401	43673	1·885	8·857	9·877	
	7 7⅓	..	,,	,,	..	..	37387	45451	,,	8·854	10·764	
	3 8	..	,,	·06 nearly.	..	..	..	41259	,,	..	9·772	Sunk by buckling, as usual.
	1 7⅓ nearly.	..	,,	,,	..	..	..	49035	,,	..	..	
9	10 0	290 0	8⅓ × 4¾	¼ nearly.	100395 118315 154155 172075 189995 207915 not broken.	·080 ·099 ·139 ·168 in 10 minutes. ·197	..	.. (Not broken with 207915 lbs. = 92·819 tons.)	8·3466	..	..	After the tube had borne 207915 lbs. or 11·12 tons per square inch, the experiment was discontinued, no buckling having been observed.

Resistance of Rectangular Tubes of Wrought Iron to a Force of Compression applied in the direction of their Length—(continued).

No. of Experiment.	Length of Tube.	Weight of Tube.	External Dimensions of Tube.	Thickness of Plates of Tube.	Weight laid on Tube.	Decrease of Length from that Weight.	Weight with which Buckling, or Perceptible Undulation, was observed.	Weight of greatest Resistance.	Area of Section of Tube.	Weight per square Inch at which Buckling, or Undulation, was observed.	Weight per square Inch at greatest resistance of Tube.	Remarks.
	Ft. in.	Lbs. oz.	Inches.	Inch.	Lbs.	Inch.	Lbs.	Lbs.	Sq. Inch.	Tons.	Tons.	
10	10 0·3	91 8	8·1 × 8·1	·06 nearly.	..	..	15897	27545	2·070	3·428 Or more obviously with 4·267 tons.	5·926	
	7 8	..	,,	,,	..	..	..	27531	,,	..	5·938	
11	10 0	162 0	8·37 × 8·37	·139	28715 46635 73515 91435	·024 ·041 ·077 ·110	82475	100395 = 44·81 tons.	4·9262	7·474	9·098	
12	10 0	296 0	8·5 × 8·375	·2191	28715 64555 136235 163115 181035 198955 198955	·018 ·043 ·098 ·128 ·155 ·201 ·216 in 5 minutes.	..	.. 198955 = 88·8 tons.	7·7367	..	11·48	With 154155 lbs. scales were peeling off the surface of the tube. The area of section in this tube, as well as in others, includes the lap-over at the joints. It sunk by buckling 14 to 18 inches from one end.

13	10 0	333 0	8·5 × 8·4	·245 & ·238	136235 145195 163115 181035 198955 216875 225835 225835 225835	·117 ·125 ·144 ·162 ·181 ·205 ·224 in 5 minutes. ·228 in 10 minutes. ·229 in 15 minutes.	..	.. (Not broken with 225835 lbs. = 100 tons 16 cwt. 43 lbs.)	8·4665	..	..	With 225835 lbs. there was no certain evidence of undulations on the surface, this weight being upwards of 100 tons, the experiment was discontinued.
14	10 1	157 4	8·1 × 8·1	·0637		..	56630	70070	3·551	7·119	8·809	It failed by becoming wrinkled near to the top.
	9 10	..	"	"	28715 64555 77995	·049 ·097 ·123	46635	82027	"	5·863	10·312	This tube was cut out of the preceding one; the injured part, two or three inches long, being at one end, was cut away. This failed by wrinkling, nearly as before.
	5 0	..	"	"	20436 46196 77108 82260	·015 ·036 ·063 ·068 nearly.	..	82411	"	..	10·361	This and the following tube were both cut out of the last; both failed by buckling, or wrinkling; this about 7 inches from the end, and the other about 4 inches.
	2 4	..	"	"	..	..	..	85771	"	..	10·783	

TABLE XIII.—*Results of Experiments to determine the Resistance of Cylindrical Tubes of Wrought Iron to a Force of Compression applied in the direction of their Length; the ends of the Tubes being turned perfectly flat, and the Experiments performed as before.*

No. of Tube.	Length of the Tube.	Weight of the Tube.	External Diameter of the Tube.	Internal Diameter of the Tube.	Thickness of the Plates of the Tube.	Weight laid on the Tube.	Decrease of Length through that Weight.	Deflection of Tube in Middle through that Weight.	Weight of greatest Resistance.	Area of Section of the Tube.	Weight per Square Inch of greatest Resistance of the Tube.	REMARKS.
	Ft. in.	Lbs. oz.	Inches.	Inches.		Lbs.		Inch.	Lbs.	Inch.	Tons.	
1	9 11	15 8	1·495	1·292	..	1614	..	·05	6514	·4443	6·55	After bearing the weights 2790 lbs. and 5142 lbs. the bar was unloaded, and the deflections were ·00 and ·01 respectively.
						2790		·08				
						Unloaded.		·00				
						3182		·09				
						3966		·126				
						4750		·175				
						5142		·21				
						Unloaded.		·01				
						5534		·27				
						6122		·40				
						6318		·49 and ·50 in 1 minute.				
	5 0	..	,,	,,	..	11844	..	0	13860	,,	13·92	
						12516		+				
						13860		·04				
	2 6	..	,,	,,	..	..	..	..	15204	,,	15·277	

2	9 11	20 15½	1·964	1·755	..	3182 4750 6318 7886 9454 Unloaded. 11022 12590	..	+ ·01 nearly. ·02 ·03 ·04 + ·06 ·07	14158	·6104	10·35	It sunk by flexure with 14158 lbs. in about two minutes.
	5 0	..	,,	,,	..	..	..	..	20332	,,	14·866	It bore this weight a short time, and gradually sunk by flexure.
	2 6	..	,,	,,	..	..	..	..	22572	,,	16·509	With 21228 lbs. the tube emitted sounds indicative of crushing; no other perceptible change. With 22572 it sunk by flexure.
3	9 11	28 4½	2·49	2·275	..	4358 5142 9846 12982 16118 19254 22390	..	+ ? ·02 ·025 ·03 ·037 ? ·04 ·07	23958	·8045	13·294	With 23958 lbs. the deflection increased rapidly, and the tube sunk.
	5 0	..	,,	,,	..	5100 19284 26452 28244	..	0 ? + ? ·015	28244	,,	15·67	
	2 6	..	,,	,,	..	..	..	..	29364	,,	16·29	

Resistance of Cylindrical Tubes of Wrought Iron to a Force of Compression applied in the direction of their Length—(continued).

No. of Tube.	Length of the Tube.	Weight of the Tube.	External Diameter of the Tube.	Internal Diameter of the Tube.	Thickness of the Plates of the Tube.	Weight laid on the Tube.	Decrease of Length through that Weight.	Deflection of Tube in Middle through that Weight.	Weight of greatest Resistance.	Area of Section of the Tube.	Weight per Square Inch of greatest Resistance of the Tube.	REMARKS.
	Ft. in.	Lbs. oz.	Inches.	Inches.	Inch.	Lbs.	Inch.	Inch.	Lbs.	Inches.	Tons.	
4	10 0	53 14	2·35	1·865	. .	5100	. .	·03 ?	34516	1·605	9·600	The accompanying Plate may be taken as a representation of the form of a flexible pillar, when considerably bent; the ends of the pillar being flat and parallel, and pressed between two plane surfaces, which in the apparatus employed were necessarily parallel.
						Unloaded.		·0				
						19284		·06				
						Unloaded.		·005				
						19284 repeated.		·07				
						22868		·08				
						28244		·11				
						30932		·14				
								·145 in 1 minute.				
								·15 in 1 minute.				
						31828		·175				
						33620		·21				
								·22				
						34516		·24				
	2 5·9	14 12	2·383	1·891	·255, ·255, ·235, ·240; Mean ·246	43947	. .	0	54666	1·651	14·78	With 43947 lbs. scales were peeling off. It sunk by flexure.
						46635	·1 nearly	0				
	2 5·9	12 2	2·343	1·923	·210, ·210; Mean ·210	44843	·1 nearly	0	53770	1·407	17·06	With 44843 lbs. scales were perceived to be peeling off. It sunk by flexure.

	2 5·1	12 15	2·373	1·911	·241 ·241 ·225 ·219 } Mean ·231	48427 52011 55595	. . ·1 nearly. ·35	0 + increased.	57354	1·554	16·476	With 48427 lbs. scales were peeling off. It sunk by flexure.
5	10 0	47 10	2·34	1·91	·215	9828 15852 19284 22868 26452 30932 31828		·03 ·04 ·05 ·07 ·11 ·19 ·21 ·30	31828	1·4353	9·901	
	5 0	. .	2·35	1·91	. .	. .	. .	. .	43180	1·4721	13·094	Sunk by bending in the middle, and near to each end.
	5 0	. .	2·335	1·925	·205 nearly.	. .	. .	. .	41164	1·3718	13·392	Sunk by flexure. This tube was not quite straight.
	2 6	. .	. .	. .	. .	. .	. .	. .	52588	1·4353	16·357	With 37356 lbs. the scales began to peel off. With 44972 lbs. there was very little deflection, and the scales were peeling off fast, indicative of great crushing.
	2 6	. .	2·335	. .	. .	. .	. .	. .	50796	1·4353	15·799	
	2 5·1	11 2	2·343	1·939	·195 ·210 } Mean ·202	. .	. .	. .	53770	1·3587	17·665	Sunk by flexure.

No. of Tube.	Length of the Tube.	Weight of the Tube.	External Diameter of the Tube.	Internal Diameter of the Tube.	Thickness of the Plates of the Tube.	Weight laid on the Tube.	Decrease of Length through that Weight.	Deflection of Tube in Middle through that Weight.	Weight of greatest Resistance.	Area of Section of the Tube.	Weight per Square Inch of greatest Resistance of the Tube.	Remarks.
	Ft. In.	Lbs. Oz.	Inches.	Inches.	Inch.	Lbs.	Inch.	Inch.	Lbs.	Inches.	Tons.	
6	9 11¼	45 15	2·995	2·693	..	5100	·021	0	37356	1·349	12·362	
						6892	·028	·01				
						10476	·040	·03				
						14060	·055	·062				
						17644	·067	·069				
						21228	·079	·075				
						24812	·090	·080				
						28396	·102	·089				
						31980	·118	·098				
						35564	·129	·115				
						,,		·140				
						37356	·172	·240				
	7 6	..	3·035	2·717	·168	28715	·077	..	42122	1·414	13·299	
						33195	·090					
						37675	·103					
						42122	·162					
							·181 in 5 minutes.					
	2 4¼	10 7	3·00	2·712	·153	51115	·65	..	52874	1·414	16·693	With 43051 lbs. the tube was slightly bent and the scales were peeling of.

7	9 11¼	59 0	4·05	3·772	..	5100	·015	0	47212	1·7078	12·34	
						19436	·051	0				
						33772	·083	0 ?				
						Unloaded.	·006	..				
						37356	·091	·01				
						40940	·101	·02				
						Unloaded.	·009	·0 nearly				
						40940 repeated.	·102	·025				
						44524	·119	·055				
						„	..	·060 in minutes.				
						44524 repeated.	·127	·080				
						44524	..	·085 in 5 minutes.				
						46316	..	·09 +				
						„	·131	·095 in 5 minutes.				
							·139	·10 in 7 minutes.				
						„	..	·120 in 10 min.				
						47212	..	·150				
						„	..	·30				
						„	..	·40				
	7 6	..	4·052	3·79	..	28715	·060	..	53770	1·613	14·88	With 46635 lbs. repeated, and a decrement of length of ·113, there was no apparent deflection.
						37675	·080					
						46635	·108					
						..	·111 in 10 min.					

No. of Tube.	Length of the Tube.	Weight of the Tube.	External Diameter of the Tube.	Internal Diameter of the Tube.	Thickness of the Plates of the Tube.	Weight laid on the Tube.	Decrease of Length through that Weight.	Deflection of Tube in Middle through that Weight.	Weight of greatest Resistance.	Area of Section of the Tube.	Weight per Square Inch of greatest resistance of the Tube.	REMARKS.
	Ft. In.	Lbs. Oz.	Inches.	Inches.	Inch.	Lbs.	Inch.	Inch.	Lbs.	Inches.	Tons.	
						Unloaded.	·023					With 51115 lbs. the scales were peeling off, and there was very obvious crushing in various parts of the tube.
						46635 repeated.	·113					
						51115	·151					
						53355	·261 in 5 minutes.					
8	9 11¼	64 4	4·06	3·75	·150	5100	·014	0	49900	1·9015	11·71	
						19436	·047	·01 ?				
						26604	·066	·02				
						Unloaded.	·001 nearly.	0				
						33772	·082	·03				
						40940	·099	·038				
						Unloaded.	·009	·005 perhaps.				
						44524	..	·04				
						..	·118	·078 in 7 minutes.				
						Unloaded	·023	·045				
						46316	..	·11				
						..	..	·12 in 5 minutes.				

						48108	..	·22				
						..	·174	·24 in 5 minutes.				
						49900	..	·31				
						..	..	·35 in 1 minute.				
9	10 0	96½	6·366	..	·1298	28715	·050	..	91402	2·547	16·021	This tube had a row of rivets running up its side to unite its edges together; all the preceding ones were united by soldering, or otherwise, without rivets. The tube was unsound in one part near to the middle, and was rendered stronger there by a patch laid upon it. It was dinged in other part. With 73515 lbs. the tube emitted sounds, but shewed no apparent deflection. The tube failed at a p'ace about 1 foot from the top on one side, and about two inches from the top on the opposite side. It was previously quite sound at those places.
						46635	·082					
						Unloaded.	·002					
						..	·001					
						64555	·120					
						73515	·148					
						82475	·190					
							·197 in 10 min.					
						Unloaded.	·052					
						82475 repeated.	·197					

No. of Tube.	Length of the Tube.	Weight of the Tube.	External Diameter of the Tube.	Internal Diameter of the Tube.	Thickness of the Plates of the Tube.	Weight laid on the Tube.	Decrease of Length through that Weight	Deflection of Tube in Middle through that Weight	Weight of greatest Resistance	Area of Section of the Tube	Weight per Square Inch of greatest Resistance of the Tube	Remarks
	Ft. In.	Lbs. Oz.	Inches	Inches	Inch	Lbs.	Inch	Inch	Lbs.	Inches	Tons	
						Unloaded	·023					With 51115 lbs. the [illegible] and there was [illegible] cracking [illegible] various parts of the tube
						40035 repeated.	·113					
						51115	·151					
						53355	·201 in 5 minutes					
	11	64 4	4·06	3·75	·150	5100	·014	0	40000	1·0015	11·71	
						19436	·047	·017				
						25604	·088	·02				
						Unloaded.	·001 nearly.	0				
						33772	·082	·03				
						40940	·090	·038				
						Unloaded.	·009	·005 perhaps.				
						44524	..	·04				
						..	·118	·078 in 7 minutes.				
						Unloaded	·023	·045				
						46316	..	·11				
						..	..	·12 in 5 minutes.				

	48108	..	·22				
	..	·174	·24 in 5 minutes.				
	49900	..	·31				
	..	..	·35 in 1 minute.				
[illegible]	[illegible]	·050	..	91402	2·547	16·021	This tube had a row of rivets running up its side to unite its edges together; all the preceding ones were united by soldering, or otherwise, without rivets.
	[illegible]	·082					The tube was unsound in one part near to the middle, and was [illegible]
	[illegible]	[illegible]					
	69612	·103	·045				

No. of Tube.	Length of the Tube.	Weight of the Tube.	External Diameter of the Tube.	Internal Diameter of the Tube.	Thickness of the Plates of the Tube.	Weight laid on the Tube.	Decrease of Length through that Weight.	Deflection of Tube in Middle through that Weight.	Weight of greatest Resistance.	Area of Section of the Tube.	Weight per Square Inch of greatest resistance of the Tube.	REMARKS.
	Ft. In.	Lbs. Oz.	Inches.	Inches.	Inch.	Lbs.	Inch.	Inch.	Lbs.	Inches.	Tons.	
	7 6	..	6·366	..	·1298	28715	·035	..	106122	2·547	18·60	This tube was part of the preceding one, the injured part having been cut away. When the tube was bearing 100395 lbs., bright spots were observed about the heads of some of the rivets in the patch laid on near to the middle. A slight deflection and deformation of the tube was likewise observable.
						64555	·079 ·080 in 5 minutes.					
						73515	·090 ·091 in 5 minutes.					
						82475	·103 ·104 in 10 min.					
						91435	·120 ·122 in 10 min.					
						100395	·181 ·188 in 5 minutes. ·294 in 10 min.					
10	10 0	63 5	6·187	..	·0939	10795	·029	..	60075	1·799	14·908	This tube, like the last, had a row of rivets running up the side. With 42155 lbs. the scales were perceived to be peeling off; and with 46635 lbs. in a much higher degree.
						28715	·072					
						46635	·145					
						55595	·226 ·233 in 10 min.					

						57835	·245 ·265 in 10 min.					The circumference of the centre of the plate, the lap-over at the joint included, was 20·625 inches. With 57835 lbs. there was little or no sign of flexure in the tube, and no sign of crippling. The tube sunk by crippling 2½ feet from, and near to, the bottom.
	5 0	..	6·175	..	·101	28715 46635 Unloaded. 46635 repeated. 55595 64555	·040 ·062 ·003 ·063 ·075 ·097 ·100 in 5 minutes.	..	69002	1·799	17·123	Tube, part of the last. The circumference of this tube was 19·4 inches, and the lap-over, where riveted, increased it to 20·65 inches.
	2 6	..	6·125	..	·098	73515	·12 nearly.	..	74411	,,	18·465	This tube was cut out of the former one.
11	7 5·3	71 12	4·00	3·511	·2435	26604 48108 Unloaded. 55276 69612	·035 ·052 ·005 ·072 ·103	0 + ·01 ·045	74988	2·879	11·628	

No. of Tube.	Length of the Tube.	Weight of the Tube.	External Diameter of the Tube.	Internal Diameter of the Tube.	Thickness of the Plates of the Tube.	Weight laid on the Tube.	Decrease of Length through that Weight.	Deflection of Tube in Middle through that Weight.	Weight of greatest Resistance.	Area of Section of the Tube.	Weight per Square Inch of greatest Resistance of the Tube.	Remarks.
	Ft. In.	Lbs. Oz.	Inches.	Inches.	Inch.	Lbs.	Inch.	Inch.	Lbs.	Inches.	Tons.	
12	7 5·3	72 8	4·00	3·505	·25	40940	·047	0	76780	2·897	11·832	With 69612 lbs., scales were peeling off, indicating crushing.
						51692	·061	+				
						62444	·077	+				
						71404	·105	·03 in another direction.				
						73196	·110	·03				
						Unloaded.	·028					
						73196 repeated.	·113	·028 in another direction.				
						76780	·299					
13	7 5·3	72 10	4·00	3·504	·2425	40940	·048	0	79916	2·873	12·418	With 62444 lbs., scales were observed to be peeling off.
						55276	·066	0				
						62444	·079	0				
						69612	·097	+ under ·005				With 76780 lbs., the tube emitted sounds, and scales peeled off slightly; but there were no other signs of crushing.
						Unloaded.	·020	0 nearly.				
						69612 repeated.	·099	·01				

								73196 Unloaded. 76780 Unloaded. 76780 repeated.	·108 ·113 in 5 minutes. ·032 ·132 ·046 ·137 ·138 in 5 minutes.	·015 ·01 ·05 ·06 ·07				
14	5	0	47	12	3·995	..	·2455	28715 37675 Unloaded. 46635 Unloaded. 55595 Unloaded. 64555 Unloaded. 73515 Unloaded. 82475 Unloaded.	·028 ·036 ·008 ·047 ·010 ·053 ·013 ·064 ·017 ·077 ·023 ·128 ·074	..	86922	2·895	13·404	This tube was cut from one of the preceding ones, 7 feet 5·3 inches long.
15	5	0	47	8½	3·995	..	·241	28715 46635 64555 Unloaded.	·023 ·036 ·050 ·002	..	98122	2·848	15·381	With 82475 lbs., there was no perceptible deflection. With 95916 lbs., there was considerable deflection.

No. of Tube.	Length of the Tube.	Weight of the Tube.	External Diameter of the Tube.	Internal Diameter of the Tube.	Thickness of the Plates of the Tube.	Weight laid on the Tube.	Decrease of Length through that Weight.	Deflection of Tube in Middle through that Weight.	Weight of greatest Resistance.	Area of Section of the Tube.	Weight per Square Inch of greatest Resistance of the Tube.	REMARKS.
	Ft. In.	Lbs. Oz.	Inches.	Inches.	Inch.	Lbs.	Inch.	Inch.	Lbs.	Inches.	Tons.	
						73515	·059					
							·061 in 5 minutes.					
						Unloaded.	·004					
						82475	·073					
						Unloaded.	·012					
						91435	·126					
							·143 in 5 minutes.					
						Unloaded.	0·76					
	2 4·3	23 1	4·	..	¼ nearly.	127275	·60 nearly.	..	136202	..	..	With 94123 lbs., the scales were peeling off. It sunk by flexure.
	2 4·3	22 11	4·026	..	..	118315	·55	..	138442	..	..	With 82475 lbs., the scales were peeling off. With 118315 lbs., it was bent perceptibly. It sunk by flexure. The two last Tubes were cut out of those 7 feet 5·3 inches long.

The reader will observe the great care with which every change is noted by Mr. Hodgkinson as the experiments proceeded. As the strain increased, and the crushing strain was approached, the tubes began to sink rapidly under the load, and were destroyed, as regards all practical purposes, previous to the addition of the ultimate weight.

In some of the cylindrical tubes of small diameter, failure took place, partly from flexure, and in accordance with the laws of long columns. In such cases, the straining force in the direction of the tube is under 8 or 9 tons per square inch. The strength of a hollow cylindrical wrought-iron pillar, whose length is such that it would yield by bending before it sustains sufficient pressure to injure the material, is, we have seen, as

$$\frac{D^{3 \cdot 59} - d^{3 \cdot 59}}{l^n}.$$

Where $n=2$ in very long pillars, but in shorter ones the value of n will be reduced to any degree, or so that $n=o$ and l^n is constant in very short columns, shewing the weakness of wrought-iron to sustain compression. Thus, in the cylindrical tubes, $1\frac{1}{2}$ inches diameter, we have

Length.		Breaking-weight.	
10 ft.	....	6·55 tons	per square inch.
5 ft.	....	13·92 ,,	
2 ft. 6 in.	....	15·27 ,,	

In most of the foregoing experiments, however, they did not fail by flexure, and with all the square tubes which were compressed to a high degree, the length has but little influence, for tubes 10 feet, 5 feet, and 2 feet 6 inches long, all bear nearly the same pressure. Thus, in Experiment 3 we have,—

Length.	Breaking-weight.	
10 ft.	11·237 tons	
5 ft.	10·529 „	per square inch.
2½ ft.	12·24 „	

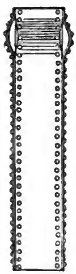
In Experiment 5 we have, similarly,

Length.	Breaking-weight.	
10 ft.	6·786 tons	
7 ft. 8 in.	7·239 „	per square inch.
2 ft. 4 in.	7·108 „	

And in Experiment 10, on circular tubes, the tube being 6 inches diameter, we have,—

Length.	Breaking-weight.	
10 ft.	14·9 tons	
5 ft.	17·1 „	per square inch.
2½ ft.	18·4 „	

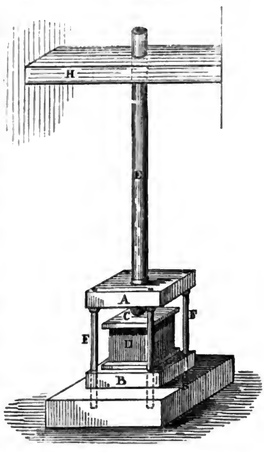
The above results are evidently directly applicable in the construction of the top of a beam, and furnish most valuable information in the design of the cells for tubular bridges. And, lastly, the following experiment was made to ascertain the resistance to crushing of a single square cell similar to those employed in the top of the bridges :



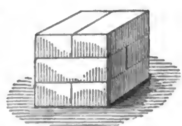
The cell was 8 feet long, 18 inches square, the plates were half an inch thick, and united at the corners by angle-iron. It was crushed vertically, by means of a hydraulic press. The pressure by which the cell was crushed was 680 tons, or 13·6 tons per square inch of section. The side-plates buckled and undulated, and at length the cell failed by the bulging of the sides, as in the sketch.

The fears occasioned by the first experiments were thus entirely dispelled, and Mr. Stephenson proceeded to act with confidence on such sound data.

To complete the subject of the crushing of material, we have yet to describe some experiments made at the works at the Britannia Bridge on the crushing of the limestone, sandstone, and brick, employed in the construction. These experiments are the more to be depended on as they were made with direct weight, without the intervention of a lever; the quantity of material always at command afforded valuable facilities for such experiments, and they were conducted with great care by Mr. L. Clark. The apparatus will be understood from the following sketch: — The scale A, on which the weights were placed, rests immediately upon a cast-iron disc with a similar disc beneath it, between which the specimens were placed; the platform was kept in a horizontal position by means of the vertical wrought-iron rod E, passing through a fixed ring at H, and care was taken, in loading the platform with plates, which were piled equably around this vertical rod, to avoid all uneven strain, by keeping the rod always loose in the ring, so that it was easily moved by the hand.



The limestones and sandstones were crushed in cubes which were placed, in some cases, between thin deal boards to equalise the pressure. The bricks were built into cubes in cement, so that six



bricks were used in each experiment, breaking joint as in the preceding figure.

The bricks were not of a hard description, being manufactured on the spot.

Results of Experiments made with actual Weight on Materials used in the Britannia Bridge, January 1848.

(BRICKWORK.)	Lbs. per Sq. Inch.
No. 1. 9-Inch Cube of Cemented Brickwork (Nowell and Co.), No. 1 (or best quality), weighing 54 lbs., set between deal boards.	
Crushed with 19 tons 18 cwt. 2 qrs. 22 lbs. .... =	551·3
No. 2. 9-Inch Brickwork, No. 1, weighing 53 lbs., set in cement.	
Crushed with 22 tons 3 cwt. 0 qrs. 17 lbs. ...... =	612·7
No. 3. 9-Inch Brickwork, No. 3, weighing 52 lbs., set in cement.	
Crushed with 16 tons 8 cwt. 2 qrs. 8 lbs. ...... =	454·3
No. 4. 9¼-Inch Brickwork, No. 4, weighing 55½ lbs., set in cement.	
Crushed with 21 tons 14 cwt. 1 qr. 17 lbs. ...... =	568·5
No. 5. 9-Inch Brickwork, No. 4, weighing 54½ lbs. set between boards.	
Crushed with 15 tons 2 cwt. 0 qrs. 12 lbs. ...... =	417·
Mean................	521

NOTE.—The three last cubes of common brick continued to support the weight, although cracked in all directions: they fell to pieces when the load was removed. All the brickwork began to shew irregular cracks a considerable time before it gave way.

The average weight supported by these bricks was 33·5 tons per square foot, equal to a column 583·69 feet high of such brickwork.

(SANDSTONE.)

	Lbs. per Sq. Inch.
No. 6. 3-Inch Cube Red Sandstone, weighing 1 lb. 14¾ oz., set between boards (made quite dry by being kept in an inhabited room).	
Crushed with 8 tons 4 cwt. 0 qrs. 19 lbs.	= 2043·
No. 7. 3-Inch Sandstone, weighing 1 lb. 14 oz., set in cement (moderately damp).	
Crushed with 5 tons 3 cwt. 1 qr. 1 lb.	= 1285·
No. 8. 3-Inch Sandstone, weighing 1 lb. 15½ oz., set in cement (made very wet).	
Crushed with 4 tons 7 cwt. 0 qrs. 21 lbs.	= 1085·
No. 9. 6-Inch Cube Sandstone, weighing 18 lbs., set in cement.	
Crushed with 63 tons 1 cwt. 2 qrs. 6 lbs.	= 3924·8
No. 10. 9½-Inch Cube Sandstone, weighing 58½ lbs., set in cement (77½ tons were placed upon this without effect, = 2042 lbs. per inch, which was as much as the machine would carry).	
Average crushing weight..........	2185

All the sandstones gave way *suddenly*, and without any previous cracking or warning. The 3-inch cubes appeared of ordinary description; the 6-inch was fine-grained, and appeared tough and of superior quality. After fracture the upper portion generally retained the form of an inverted square pyramid about 2½″ high, and very symmetrical, the sides bulging away in pieces all round. The average weight of this material was 130 lbs. 10 oz. per cube foot, or 17 feet per ton.

The average weight required to crush this sandstone is 134 tons per square foot, equal to a column 2351 feet high of such sandstone.

(LIMESTONE.)

	Lbs. per Sq. Inch.
No. 11. 3-Inch Cube Anglesey Limestone, weighing 2 lbs. 10 oz. set between boards.	
Crushed with 26 tons 11 cwt. 3 qrs. 9 lbs.	= 6618·

This stone formed numerous cracks and splinters all round, and was considered crushed, but on removing the weight about two-thirds of its area were found uninjured.

	Lbs. per Sq. Inch.
No. 12. 3-Inch Limestone, weighing 2 lbs. 9 oz., set between deal boards.	
Crushed with 32 tons 6 cwt. 0 qrs. 1 lb.	= 8039·
This stone also began to crack and splinter externally with 25 tons (or 6220 per inch), but ultimately bore as above.	
No. 13. 3-Inch Limestone, weighing 2 lbs. 9 oz., set in deal boards.	
Crushed with 30 tons 18 cwt. 3 qrs. 24 lbs.	= 7702·6
No. 14. 3 Separate Inch Cube Limestone, arranged in a triangle, weighing 4½ oz., set between deal boards.	
Crushed with 9 tons 7 cwt. 1 qr. 14 lbs.	= 6995·3
All crushed simultaneously.	
Average................	7579
	7338·5

All the limestones formed *perpendicular* cracks and splinters a considerable time before they crushed. Weight of the material from above = 165 lbs. 5 oz. per cubic foot, or 13½ feet per ton.

The weight required to crush this limestone is 471·15 tons per square foot, equal to a column 6433 feet high of such material.

(SINGLE BRICKS, OF DIFFERENT QUALITIES.)	Lbs. per Sq. Inch.
No. 15. A single Brick, No. 1 (Nowell and Co.), weighing 8 lbs., bedded flat in cement.	
Crushed with 17 tons 19 cwt. 1 qr. 7 lbs.	= 1022·
No. 16. A Single Brick, No. 3.	
Crushed with 13 tons 11 cwt. 1 qr. 6 lbs.	= 750·
No. 17. A Single Birkenhead Brick, 9×4½″, weighing 7 lbs. 14 oz., bedded in cement.	
Crushed with 32 tons 2 cwt. 0 qrs. 17 lbs.	= 1775·8
No. 18. A Single Buckley Mountain Brick, 9½″×4½″, weighing 9 lbs. 2 oz., bedded in cement.	
Crushed with 40 tons 13 cwt. 0 qrs. 15 lbs.......	= 2130·3

These last experiments not being on cubes, only serve for comparison among themselves. The bricks were completely crushed into powder. The cement used in all the above

experiments invariably began to crack away round the edges as soon as a very moderate weight was applied.

The enormous weight which timber is capable of bearing was practically shewn in many operations in the construction of the tubes. The piles of the Conway platform carried, for a considerable time, 10 tons per foot section.

The tubes rested, on some occasions, on beech wedges under each extremity, with a pressure of 30 tons per superficial foot. They were frequently supported on a pile of soft deal planking under each end, 6 feet high, with a pressure of 20 tons per foot.

On the occasion of the failure of the press, the extremity of the tube fell on to a bed of soft deal planks, piled loosely on each other about 4 feet high, and about 12 feet by 5 feet in superficies; the tube being about $1\frac{1}{2}$ inches above the planks, and compressing them through a space of about 7 inches, the bulk of the weight of half the tube, or about 1000 tons, was at the same time supported by an internal column of deal, 14 inches high and 14 inches square, which continued to support a great part of the weight, although crushed as in the following sketch.

In raising the tube the vertical pressure on the masonry of the abutments amounted to 500 tons upon 10 square feet, or 50 tons per foot superficial, being about one-eighth of its crushing weight in the experiments, exclusive of joints, while a small surface of sheet-lead immediately beneath the clam beams in the presses supported the whole weight of the tube without oozing away.

CHAPTER III.

EXTENSION AND TENSILE STRENGTH OF MATERIALS.

THE immediate effect of transverse strain in any beam is to extend the lower portions, or all those horizontal layers of the system which lie below the neutral line. We have, therefore, now to investigate the effect of direct longitudinal extension on various materials preparatory to the consideration of the simultaneous extension and compression which characterises the solid beam.

The subject will be much simpler than that we have just left, because lateral dimensions, or length and depth, are no longer important elements in its consideration. A pillar compressed in the direction of its length is in a state of unstable equilibrium, and hence the complexity of the laws that regulate its strength; but a rod pulled in the direction of its length is in a state of stable equilibrium, and the nature of the adhesion of its particles is all we have to investigate, its strength being dependent alone on its area of section, and not on its form.

We have no occasion, therefore, to limit ourselves in illustrating the subject to the consideration of the behaviour of a cubic inch, but of a bar of any length one square inch in section.

We find, on suspending a weight from such a bar, that it is extended longitudinally and reduced in lateral dimensions, the amount of its extension being proportionate to its length

and to the weight suspended, and, from numerous experiments, this extension of a rod, 1 inch square of wrought-iron, is found to be about at the rate of one-tenthousandth of the length of the rod for every ton of direct tensile strain.

Moreover, as regards its elasticity, all we have said of the compression of this material holds equally good in its extension, *i. e.* on removing the weight the rod is found to be permanently lengthened, or has retained a permanent set, which permanent set is proportional to the square of the weight with which the rod has been strained, and the lateral dimensions are similarly permanently diminished. The rod is thus in a new condition after having been strained, it has different elastic properties from those it possessed originally, and extends less under a given weight; and for practical purposes, where the amount of extension is the limit to its use, it is a stronger rod. This is, moreover, literally true, for the bar thus strained, though less in section, will ultimately break with the same weight as the new bar of larger section. An interesting example has been given in page 305.

The extension of this material is remarkably uniform up to 15 or 16 tons on the square inch, in proportion to the weight, that is, with 8 or 10 tons the extension is eight or ten times as great as with a single ton. We have the following experimental results in breaking a new wrought-iron bar, 10 feet long and 1 inch square, by a suspended weight increased ton by ton, the bar and weights being reduced to preserve the uniformity of the table.

It will be observed, on comparing columns 2 and 3, that up to 12 tons the observed and calculated extensions are almost identical by assuming the extension to be $\frac{8}{10000}$th of the length per ton per square inch. Beyond this strain the observed extension rapidly exceeds this uniform rate.

Tons.	Observed Extension in terms of the Length.	Computed Extension assumed uniform at $\frac{8}{100000}$ of the Length per Ton per Sq. Inch.	Corresponding Extension in fractional parts of the Length computed at $\frac{8}{100000}$ per Ton per Sq. Inch.	Observed Permanent Set in terms of the Length.	Observed Permanent Set in fractional parts of the Length.
1	·0000689	·00008	$\frac{1}{12500}$		
2	·000156	·00016	$\frac{1}{6250}$		
3	·000238	·00024	$\frac{1}{4166}$	·00000213	$\frac{1}{468750}$
4	·000319	·00032	$\frac{1}{3125}$	·00000283	$\frac{1}{350877}$
5	·000399	·00040	$\frac{1}{2500}$	·00000356	$\frac{1}{281030}$
6	·00048	·00048	$\frac{1}{2083}$	·00000427	$\frac{1}{233918}$
7	·00056	·00056	$\frac{1}{1786}$	·00000497	$\frac{1}{201005}$
8	·00064	·00064	$\frac{1}{1562}$	·00000650	$\frac{1}{153846}$
9	·00072	·00072	$\frac{1}{1389}$	·00001201	$\frac{1}{83275}$
10	·00080	·00080	$\frac{1}{1250}$	·00001334	$\frac{1}{74953}$
11	·000896	·00088	$\frac{1}{1136}$	·00003392	$\frac{1}{29484}$
12	·00102	·00096	$\frac{1}{1040}$	·00008368	$\frac{1}{11950}$
13	·00128	·00104	$\frac{1}{961}$	·0002598	$\frac{1}{3848}$
14	·00218 in ten minutes ·00231	·00112	$\frac{1}{893}$	·0011075	$\frac{1}{903}$
15	·00416	·00120	Beyond this weight the permanent set is greater than the computed extension as above.	·002976	$\frac{1}{336}$
16	·00443	·00128		·003175	$\frac{1}{315}$
17	·00934 in ten minutes ·01015	·00136		·008750	$\frac{1}{114}$
18	·01024 in ten minutes ·01212	·00144		·009170	$\frac{1}{109}$
19	·01785 in ten minutes ·02017	·00152		·018590	$\frac{1}{54}$
20	·02124 in ten minutes ·02146	·00160		·019790	$\frac{1}{50}$
21	·02429 in ten minutes ·02472	·00168		·022310	$\frac{1}{45}$
22	·03400 in ten minutes ·03425	·00176		·031933	$\frac{1}{31}$

By taking the observed extension per ton from the first 12 tons, the weight at which wrought-iron begins to be perceptibly extended and damaged, we find the mean extension to be $\frac{1}{12440}$th of the length of the bar per ton, which is rather less than we have derived from other experiments. Thus, for the modulus or the weight that would stretch the bar to double its length: since one ton extends it $\frac{1}{12440}$th of its length, 12,440 tons, or 27,865,600 lbs., would double its length, and is therefore the modulus of elasticity.

In such experiments a short time is requisite to allow the rod to adjust itself after each load, and when the load becomes great, the rod continues to sink for some time after the load is deposited, especially if assisted by any vibration.

It is nearly true, and very convenient in practice to assume both the extension and compression to take place at the rate of one ten-thousandth of the length for every ton of direct strain per square inch of section, in which case the modulus becomes 22,400,000 lbs.

The elasticity and ultimate strength vary considerably with the quality of the iron, and the elasticity, from some experiments made at the Britannia Bridge, appears even more variable than the ultimate strength.

The above results are reduced from an experiment made on a round half-inch drawn rod, and failure took place at last at a weld, the ultimate resistance being unusually large for welded iron, which can *never* be depended on. Iron rolled into bars is rendered more fibrous than when rolled into plates, and is generally stronger, and some experiments appear to prove that plates, drawn in the direction in which they were rolled, are stronger than when torn asunder across the grain. It is difficult to conceive the nature of the fibrous texture induced by rolling, but it is beautifully evident in an illustration of the compression of a rivet-head in a subsequent chapter, and in the twist of an ordinary gun-barrel;

we may imagine, for illustration, that the ultimate particles of the material are drawn into a fibrous form, and brought into closer contact by the act of rolling, and to some extent by the stretching of a bar.

This consideration appears to account for the observed fact that surface is intimately connected with strength, several smaller wires, or thin plates, being stronger than a single wire, or thick plate, of the same sectional area; the action of wire-drawing the fibres being more complete in thin plates than in thicker masses, and results arrived at experimentally from small rods will be found too high for application to heavier bars.

It will be seen, from the extensions of the bar, that although the ultimate resistance was 23 tons per square inch, yet the increase of its permanent set becomes so rapid and irregular after about 12 or 15 tons, that this weight would be the limit of its practical utility, especially when employed in the construction of a beam.

With respect to the ultimate strength of wrought-iron, the following experiments were made at the Britannia Bridge on the rolled plates and rivet-iron enployed in that construction. The plates were drawn in the direction in which they were rolled, except when otherwise mentioned, and the experiments were made with direct weight suspended from the bar or plate.

The specimens were carefully reduced to a uniform neck in the following form, the sectional area of the neck being always one square inch:—

These bars were suspended from a shackle, and broken by direct weight placed on a scale. The ultimate extension was

measured on the fractured bar from punch marks previously made on the neck.

TABLE XV.

Experiments on the ultimate Strength of Boiler-Plate.

No.	Description of Plate.	Breaking-weight per Square Inch.	Ultimate Extension in parts of the Length.
1	Plate $\frac{11}{16}$th inch thick, neck $1\frac{1}{2}$ inch long. Selected as bad iron, fracture bright, and crystalline, brittle, broke readily with a blow from a hammer	22	—
2	From the same plate	21	$\frac{1}{40}$
3	Plate $\frac{1}{2}$ inch, neck 6 inches. Selected as bad iron, containing two laminæ of crystalline metal, one-third of the whole section	18	$\frac{1}{100}$
4	Plate $\frac{1}{2}$ inch, neck 5 inches. Selected us a good plate, about $\frac{1}{10}$th of the section crystalline	19	$\frac{1}{48}$
5	Plate $\frac{1}{2}$ inch, neck 4 inches. Iron perfectly uniform and fibrous, supported the weight 15 minutes	21	$\frac{1}{8}$
6	Plate $\frac{11}{16}$th inch thick, neck 5 inches. Iron good, $\frac{1}{40}$th of the section crystalline ..	19	$\frac{1}{36}$
7	Plate $\frac{1}{2}$ inch, neck 5 inches. Iron fibrous except $\frac{1}{30}$th of the section	18	$\frac{1}{23}$
8	Plate $\frac{1}{2}$ inch, neck 50 inches	19·6	$\frac{1}{57}$
9	Plate $\frac{5}{8}$ inch, neck 50 inches	19·3	$\frac{1}{33}$
10	Plate $\frac{1}{2}$ inch, neck 7 inches	19·6	$\frac{1}{19}$
11	Plate $\frac{1}{2}$ inch, neck 7 inches	20·2	$\frac{1}{24}$
12	Plate $\frac{1}{2}$ inch, neck 50 inches	18·7	$\frac{1}{59}$

From the mean of the above experiments the ultimate tensile strength of boiler-plate appears to be 19·6 tons; and it is worthy of remark that the ultimate strength is remarkably constant, although the iron comes from different makers

from Staffordshire, Derbyshire, and Shropshire. The ultimate extension, on the contrary, is extremely irregular; indeed some of the brittle, crystalline irons, selected as bad, which fractured suddenly without much increase of length, actually supported more weight than the more fibrous and ductile iron.

The ultimate strength of wrought-iron, derived from experiments on 1-inch bars, has been usually taken at 25 tons per square inch. This conclusion is evidently erroneous as regards boiler-plate; indeed this strength was not obtained from any iron used at the works.

The best scrap rivet-iron, made by Messrs. Mare at their London works, the quality of the iron being unusually good, and the fracture beautifully fibrous, broke on an average with 24 tons per square inch, or 18·84 tons per circular inch. The length of the round rods experimented upon was 60 inches, the diameter $\frac{7}{8}$ths of an inch, and the mean ultimate extension (which was uniform) was $\frac{1}{8}$th of the length. The rods diminished very visibly in diameter before they were fractured, and drew into a conical neck at the point of failure.

In all these cases the iron was drawn in the direction of the fibre. In order to test whether the strength is influenced from this cause, two plates were selected, and from each plate two specimens were cut out, of similar form to those used in the experiments above. One specimen in each pair was cut out in the direction of the fibre, and the other across the fibre; in other respects they were precisely similar in form and dimensions. The following were the results :—

	Tons.	Tons.
Ultimate strength when drawn in the direction of the fibre	Exper. 1, 19·66.	Exper. 2, 20·2
Ultimate strength when broken across the fibre............	Exper 1, 16·93.	Exper. 2, 16·7

The ultimate extension was also twice as great when the plate was broken in the direction of the fibre.

Thus the mean ultimate strength from the two experiments was 19·93 tons per square inch when the bar was broken in the direction of the fibre, and 16·8 tons per square inch when broken across the fibre, the difference in the strength being 18 per cent in favour of using plates exposed to tensile strain in the direction in which they are rolled. With irons of different quality this result will probably be much varied.

To recapitulate the foregoing results; we may generally assume the ultimate tensile strength of wrought-iron bars at 24, and of wrought-iron plates at 20 tons per square inch, and its ultimate useful strength at 12 tons per square inch; and within this latter limit its extension may be taken at $\frac{8}{100000}$ths of the length per ton per square inch of section, and its permanent set may be obtained from the table given. Thus, the first extension of the chain used for raising the tubes was ·7968 inch, and the permanent set ·008 inch, but having once used them with this strain, and this permanent set being obtained, on employing them again under the same strain, the extension will only be ·7968 inch − ·008 inch, or ·789 inch, and the permanent set will be no further increased. Metal, however, requires a very considerable time to adapt itself to additional strain, or to return again to the diminished length, at which it will remain constant after strain, analogous to the length of time found requisite, with astronomical instruments, to allow them to adapt themselves to great change of temperature before they will remain in adjustment. These same laws are applicable both to the compression and the extension of wrought-iron.

Extension and tensile Strength of Cast-iron.

When a cast-iron bar is strained by a suspended weight, not only is its ultimate strength much less than that of wrought-iron, viz. about one-third, or 7 tons, instead of 20 or 24, but its elasticity is also twice as great, *i. e.* it is extended

about two-tenthousandths of its length per ton per square inch instead of one-tenthousandth, while it differs in this respect, that its extension is not in the exact proportion of the strain, *i. e.* 6 tons per square inch do not extend it six times as much as one ton, but only 5·6 times as much, which property is analogous to what we have seen in its compression. We shall now deduce these laws experimentally, and include them in general formulæ; and, secondly, compare together the tensile and compressive elasticity and strength of this material. For all information on this subject we are entirely indebted to the elaborate investigations of Mr. Hodgkinson, as given in the Report of the Government Commission, to which the reader is referred for more complete detail; and among other valuable contributions, his paper on this subject must certainly rank as one of the most remarkable productions of laborious and patient experimental research which modern mechanical science can boast.

For the purpose of direct comparison with the table of the extension and set of a wrought-iron rod, given at p. 373, the following table of the extension and permanent set of a cast-iron rod, 10 feet long and 1 inch square, drawn in the direction of its length, has been reduced from Mr. Hodgkinson's experiments.

TABLE XVI.

Extension of Cast-iron.

Tons.	Extension per ton.	Total Extension.	Total Permanent Set.
1	·01976	·01976	·00579
2	·02027	·04155	·001860
3	·02171	·06515	·003954
4	·02318	·09274	·007543
5	·02479	·12397	·012619
6	·02727	·16363	·020571
6½	·02815	·18297	·023720

The increase of the deflection per ton as the weight increases is thus very evident.

Since, therefore, the extension of cast-iron is not uniform in proportion to the weight, we cannot deduce a modulus of elasticity calculated on the assumption of equal extension for equal weight, it appears that cast-iron is extended about $\frac{1}{4262}$ of its length by 1 ton of tensile strain per square inch of section, and about $\frac{5 \cdot 6}{4262}$ of its length by 6 tons. We shall, therefore, for practical purposes, assume the extension at $\frac{1}{5000}$ per ton per square inch, or double that of wrought-iron, as this proportion is easily remembered and very convenient for application.

The following formula will be found also very correct, and convenient for deducing the permanent set from the extension of cast-iron, viz.,

$$\text{Permanent set} = \cdot 0193\, l + 64\frac{2}{l}$$

l being taken in inches.

The mean ultimate tensile strength of cast-iron we shall find is 7 tons nearly per square inch of section, its ultimate extension being $\frac{1}{600}$th of its length, and this weight would compress a bar through $\frac{1}{775}$th of its length.

With respect to the ultimate tensile strength of cast-iron, very different conclusions have been arrived at by various experimenters, who have drawn most erroneous conclusions from the fracture of cast-iron beams. It is certain that the strength of this material is considerably dependent on the amount of surface. When cast in thick bars, the external portions, becoming rapidly cool, form a solid shell around the internal part, which is thus prevented from contraction, and cools in a state of tension. The effect of this is to crystallise the internal portion of the metal, the crystals being larger in proportion to their distance from the surface, while the outside skin, as it is termed, is close and extremely hard, and considerably stronger than the interior, either as regards

extension or compression; experiments have accordingly been made with metal cut out of the interor of large castings, and the falling off in their strength has been found very considerable. Hence results drawn from small bars or beams cannot be depended on when applied to larger castings.

The castings employed for ascertaining directly the tensile strength of cast-iron by Mr. Hodgkinson were cruciform in section. And as these were objected to by some writers, he made also comparative experiments with bars rectangular and circular in section. The results from the cruciform section were somewhat better than from other forms, probably on account of the greater extent of surface, as explained above.

The sectional area of the cruciform bars in the following abstract, Table XVII., was from 4 to 4½ square inches. For direct comparison, the crushing strength of the various irons mentioned is also given, as derived from the crushing of cylinders ¾ inch diameter, and from ¾ to 1½ inches in height. The ratio of the tensile to the crushing strength is also given in the last column.

The specific gravity of the specimens out of the thinner part of the castings was found smaller than out of the thicker part, the mean from the former being 7·036, and from the latter 7·082.

The mean tensile strength from all these irons is 7·291 tons per square inch. The mean crushing strength is 40·853. The mean ratio of tensile to crushing resistance is 1 : 5·636.

To obtain the extension and permanent sets of cast-iron with great accuracy, experiments were made with rods 50 feet long suspended in a lofty building; the bars were round, and their sectional area was equal to 1 square inch. An abstract of these experiments is given in Table XVIII.

Table XVII.—*The ultimate Tensile and Crushing Forces of various Denominations of Cast-iron in common use.*

Description of the Iron.	Tensile Strength per square Inch of Section.	Height of Specimen.	Crushing Strength per square Inch of Section.	Ratio of the Powers to resist Tension and Compression.
	Tons.	Inch.	Tons.	Mean.
Low Moor Iron, No. 1 ..	5·667	$\frac{3}{4}$ $1\frac{1}{2}$	28·809 25·198	1 : 4·765
Low Moor Iron, No. 2 ..	6·901	$\frac{3}{4}$ $1\frac{1}{2}$	44·430 41·219	1 : 6·205
Clyde Iron, No. 1	7·198	$\frac{3}{4}$ $1\frac{1}{2}$	41·459 39·616	1 : 5·631
Clyde Iron, No. 2	7·949	$\frac{3}{4}$ $1\frac{1}{2}$	49·103 45·549	1 : 5·953
Clyde Iron, No. 3	10·477	$\frac{3}{4}$ $1\frac{1}{2}$	47·855 46·821	1 : 4·518
Blaenavon Iron, No. 1 ..	6·222	$\frac{3}{4}$ $1\frac{1}{2}$	40·562 35·964	1 : 6·149
Blaenavon Iron, No. 2, first sample	7·466	$\frac{3}{4}$ $1\frac{1}{2}$	52·502 45·717	1 : 6·577
Blaenavon Iron, No. 2, second sample	6·380	$\frac{3}{4}$ $1\frac{1}{2}$	30·606 30·594	1 : 4·796
Calder Iron, No. 1	6·131	$\frac{3}{4}$ $1\frac{1}{2}$	32·229 33·921	1 : 5·394
Coltness Iron, No. 3	6·820	$\frac{3}{4}$ $1\frac{1}{2}$	44·723 45·460	1 : 6·611
Brymbo Iron, No. 1	6·440	$\frac{3}{4}$ $1\frac{1}{2}$	33·399 33·784	1 : 5·216
Brymbo Iron, No. 3	6·923	$\frac{3}{4}$ $1\frac{1}{2}$	33·988 34·356	1 : 4·936
Bowling Iron, No. 2	6·032	$\frac{3}{4}$ $1\frac{1}{2}$	33·987 33·028	1 : 5·555
Ystalyfera anthracite Iron, No. 2	6·478	$\frac{3}{4}$ $1\frac{1}{2}$	44·610 42·660	1 : 6·735
Yniscedwyn anthracite Iron, No. 1	6·228	$\frac{3}{4}$ $1\frac{1}{2}$	37·281 35·115	1 : 5·811
Yniscedwyn anthracite Iron, No. 2	5·959	$\frac{3}{4}$ $1\frac{1}{2}$	34·430 33·646	1 : 5·712
Mr. Morries Stirling's Iron, denominated second quality	11·502	$\frac{3}{4}$ $1\frac{1}{2}$	55·952 53·329	1 : 4·751
Mr. Morries Stirling's Iron, denominated third quality	10·474	$\frac{3}{4}$ $1\frac{1}{2}$	70·827 57·980	1 : 6·149

TABLE XVIII.

Longitudinal Extension of Bars of Cast-iron.

Name of Iron.	Number of Experi-ments.	Mean Area of Section.	Weights, per Square Inch, laid on with their corresponding Extensions and Sets; the last, in each case, being the largest, where all were observed together.			Mean Breaking-Weight, per Square Inch of Section.	Mean Ultimate Extension.
			Weights.	Extensions.	Sets.		
		Inch.	Lbs.	Inch.	Inch.	Tons.	Inch.
Low Moor Iron, No. 2	2	1·058	2117 6352 10586 14821	·09500 ·3115 ·5740 ·9147	·00345 ·0250 ·06425 ·12775	7·325	1·085 or $\frac{1}{553}$rd of the length.
Blaenavon Iron, No. 2	2	1·0685	2096 6289 10482 13627	·09422 ·3065 ·5770 ·8370	·00268 ·01675 ·0575 ·11475	6·551	·9325 or $\frac{1}{643}$rd of the length.
Gartsherrie Iron, No. 3.. ..	2	1·062	2109 6328 10547 14766 15820	·09225 ·3117 ·5862 ·9452 1·0487	·001 + ·01450 ·0475 ·11325 ·13812	7·567	1·167 or $\frac{1}{514}$th of the length.
Mixture of Iron, composed of Leeswood No. 3, and Glengarnock, No. 3, in equal proportions.	3	1·063	2107 6322 10536 12643	·0914 ·2967 ·5349 ·6702	·00376 ·01823 ·04321 ·06417	6·6125	·8095 or $\frac{1}{741}$st of the length.

In all these experiments, both on compression and extension, the deflections and permanent sets were most carefully observed and recorded by Mr. Hodgkinson, his principal object being to deduce formulæ for the expression of the relation between the extensions and compressions of bars of cast-iron, and the weights producing them, in order to supply data for completing the complicated theory of an abstract beam composed of a material whose elasticity is imperfect.

The relation between the extension and compression of bars of cast-iron 10 feet long and 1 inch square, and the weights producing them, is included in the following formulæ:

$$\text{Extension} \quad w = 116117e - 201905e^2$$
$$\text{Compression} \quad w = 107763d - 36318d^2$$

w, d, and e, representing the weight, compression, and extension in inches.

For a bar of any other length, l inches, the formulæ become,

$$\text{Extension} \quad w = 13934040\frac{e}{l} - 2907432000\frac{e^2}{l^2}$$
$$\text{Compression} \quad w = 12931560\frac{d}{l} - 522979200\frac{d^2}{l^2}$$

And in order to determine experimentally the error produced by employing these formulæ, the following summary of experiments on compression and extension was prepared, the error in parts of the true weight being given in the last column.

TABLE XIX.—*The Extensions of Rods* 10 *feet long and* 1 *inch square, deduced from numerous Experiments, and compared with observed Compressions of Bars of the same Irons and the same size, cast with them for comparison, together with Formulæ for computing the Weights from the Extensions and Compressions.*

EXTENSION.						COMPRESSION.					
Number of Experiments.	Weights laid on, with the corresponding Extensions and Sets in Inches.				Error in parts of the Weight, when it is computed from the Formula $w=116117e-201905e^2$	Number of Experiments.	Mean Weights laid on, with corresponding Mean Compressions, Sets, and Ratios of Weights to Compression.				Error in parts of the Weight, when it is computed from the Formula $w=107763d-36318d^2$
	Weights (w.)	Extensions (e.)	Sets.	$\frac{w}{e}$.			Weights (w.)	Compressions in Inches (d.)	Sets in Inches.	$\frac{w}{d}$.	
9	1053·77	·0090	..	117086	$-\frac{1}{42}$	8	2064·745	·01875	·00047	110120	$-\frac{1}{36}$
9	1580·65	·0137	·00022	115131	$-\frac{1}{37}$	8	4129·49	·03878	·00226	106485	$-\frac{1}{816}$
9	2107·54	·0186	·000545	113309	$-\frac{1}{119}$	8	6194·24	·05978	·00400	103617	$+\frac{1}{52}$
9	3161·31	·0287	·00107	110150	$+\frac{1}{640}$	8	8258·98	·07879	·00645	104823	$+\frac{1}{1330}$
9	4215·08	·0391	·00175	107803	$+\frac{1}{257}$	8	10323·73	·09944	·00847	103819	$+\frac{1}{312}$
9	5268·85	·0500	·00265	105377	$+\frac{1}{163}$	8	12388·48	·12030	·010875	102980	$+\frac{1}{248}$
9	6322·62	·0613	·00372	103142	$+\frac{1}{177}$	8	14453·22	·14163	·01405	102049	$+\frac{1}{179}$
9	7376·39	·0734	·00517	100496	$+\frac{1}{123}$	8	16517·97	·16338	·01712	101102	$+\frac{1}{139}$
9	8430·16	·0859	·00664	98139	$+\frac{1}{135}$	8	18582·71	·18505	·02051	100420	$+\frac{1}{161}$
9	9483·94	·0995	·00844	95316	$+\frac{1}{134}$	8	20647·46	·20624	·02484	100114	$+\frac{1}{629}$
9	10537·71	·1136	·01062	92762	$+\frac{1}{221}$	8	24776·95	·24961	·03220	99263	$-\frac{1}{176}$
9	11591·48	·1283	·01306	90347	$-\frac{1}{674}$	8	28906·45	·29699	·04300	97331	$-\frac{1}{273}$
9	12645·25	·1448	·01609	87329	$-\frac{1}{195}$	7	33030·80	·35341	·06096	93463	$+\frac{1}{64}$
6	13699·83	·1668	·02097	82133	$+\frac{1}{268}$	7	37159·65	·41149	·08421	..	..
4	14793·10	.1859	·02410	79576	$-\frac{1}{80}$	..	..	..	..	..	..

Tensile Strength of other Materials.

The tenacity of good Baltic fir is very remarkable; sound rods of this material will bear an ultimate tensile strain of 5 tons per square inch, the specific gravity being from $\frac{1}{2}$ to $\frac{3}{4}$ of that of water, so that it will be about half or three-fourths immersed when floating in that liquid; the weight may be taken generally at about 45 lbs. per cubic foot, or nearly one-tenth that of wrought-iron, while the ultimate strength amounts to one-fourth. Thus a tension rod of wrought-iron will be $2\frac{1}{2}$ times as heavy as a tension-rod of Baltic fir of the same strength and of four times the sectional area.

Its elasticity is about one-fifteenth that of wrought-iron, *i. e.* a bar one inch square is extended only $\frac{1}{150000}$th of its length per ton of direct tensile strain.

The difficulty of connecting timber longitudinally is a complete bar to the use of deal in cases of tensile strain in greater lengths than it is naturally produced; the balks imported vary from 40 to 60 and 70 feet in length, being from 12 to 16 inches square.

In practice, wrought-iron should not be strained beyond 10 tons per square inch, and deal $2\frac{1}{2}$ tons per square inch.

Round iron rods are extensively used in construction as being more convenient for bolts where nuts are required, circular inches are reduced to square inches by multiplying them by ·7584; thus, a round rod 2 inches diameter contains 4 circular inches, and $4 \times \cdot 7854 = 3 \cdot 1416$ square inches.

Instead of multiplying by ·7854, it is frequently more convenient mentally to multiply by $5\frac{1}{2}$ and divide by 7; thus, as before,

$$4 \times \frac{5\frac{1}{2}}{7} = \frac{22}{7} = 3 \cdot 1428 \text{ square inches,}$$

which is a close approximation.

But even this reduction is excessively inconvenient, and it is far more simple to derive a unit for circular inches. Now 20 tons per square inch is very nearly equivalent to 16 tons per circular inch, which is a most convenient unit, and the following approximations will then be found very useful.

An ordinary round rod of wrought-iron 1 inch in diameter bears tensilely 16 tons, and weighs 8 lbs. per yard.

(1.) For a round rod of any diameter, the square of the diameter, taken in quarter inches, is the breaking-weight in tons.

(2.) Half this quantity is the weight in lbs. per yard. Thus, the breaking-weight of a round bar, 5 inches, or 20 quarter inches, in diameter, will be 20×20, or 400 tons. And the actual weight will be half 400, or 200 lbs. per yard.

N.B. A rod will be perceptibly damaged by half this strain, which can never be safely exceeded, one-third being sufficient in practice.

The strength of chain cable is thus easily arrived at, the strength of a link being double that of the bar from which it is forged.

It is usual technically to denominate chains by their diameter, thus a five-eighth chain is a chain made from a bar five-eighths of an inch in diameter.

The following approximations will be found very convenient in estimating the weight and strength of chains, the ultimate tensile strength of the material being taken, as before, at 16 tons per circular inch, or 20 tons per square inch of section.

(1.) The square of the diameter in eighths will be the weight of the chains in lbs. per fathom.

(2.) The square of the diameter in eighths, divided by 2, will be the breaking-weight in tons. Thus, the breaking-weight of a five-eighth chain will be half 25 tons $= 12\frac{1}{2}$ tons, and the actual weight will be 25 lbs. per fathom of 6 feet.

N.B. A chain will be perceptibly damaged by half this strain, which can never be safely exceeded, one-third being sufficient in practice.

The strength of ropes is very uncertain, their size is generally denominated technically by their circumference in inches; and the following approximations will be very correct for ordinary tarred hempen rope.

(1.) The square of the circumference in inches, divided by 10, will give the practical strength in tons, which will be about half their breaking-weight.

(2.) The square of the circumference, divided by 4, will roughly give the weight in lbs. per fathom. Thus, the useful strength of a 5-inch rope will be $\frac{25}{10}$, or 2·5 tons, the ultimate strength being 5 tons, and the weight of a tarred 5-inch rope will be $\frac{25}{4}$, or $6\frac{1}{4}$ lbs. per fathom.

A rope 10 inches in circumference, and a chain $\frac{6\cdot3}{8}$ths diameter, will each bear practically about 10 tons, taking half the breaking-weight, and the weight of the former will be 25 lbs., and of the latter $39\frac{1}{2}$ lbs. per fathom.

In the operations of floating, the largest ropes used have been 12 inches in circumference; the weight of these ropes untarred when immersed in water was about one-fifth of the weight out of water, whereas the weight of a chain is diminished very little by being immersed; a circular column of water one inch diameter weighs 2 lbs. per fathom, or 1 lb. per yard; the weight of water displaced by a 12-inch rope is 27 lbs. per fathom, the weight of the rope being 33 lbs. per fathom. The weight of a 12-inch white rope was 33·3 lbs. per fathom; the weight of a 12-inch tarred rope was 41·4 lbs. per fathom; the weight of a 9-inch white rope, purchased as 8-inch, was 16·2 lbs. per fathom.

In an experiment made with 12-inch white rope it required 6 lbs. per fathom to float it.

CHAPTER IV.

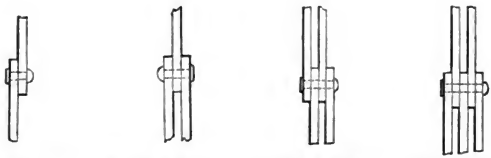
RIVETING AND THE SHEARING OF IRON EXPOSED TO TRANSVERSE STRAIN.

We have seen that where an ordinary long beam is exposed to transverse strain, failure takes place by the horizontal crushing or extension of the fibres, while the vertical strain tending to shear off the ends of the beam may be safely neglected; as the beam, however, becomes shorter, the horizontal strain is diminished and at length disappears, and under these circumstances failure takes place solely from the vertical shearing of the material in a transverse direction. If our formulæ for the strength of beams were perfectly general, they would include the failure of a beam under these conditions, which we have seen is not the case. The laws of shearing have thus to be separately investigated.

The most familiar examples of this kind of strain occur in the rivet which unites the two blades of a pair of scissors, or the rivet on which the blade rotates in an ordinary pocket-knife. In the former of these examples the evident tendency of the strain is to shear the rivet in one place only, and this is called a single shear; but in the knife the rivet must be sheared in two places before the blade can escape; this is consequently a case of double shearing. In the chains of a suspension-bridge, or the chain employed for raising the tubes, which consists of alternately 8 plates and 9 plates, the pin must be sheared in sixteen places; or, generally, if n be the

number of plates combined, and the pin that unites them fails, the number of places through which it must be sheared will be $n-1$, as in the following figures:—

1 2 3 4



2 plates, 1 shear. 3 plates, 2 shears. 4 plates, 3 shears. 5 plates, 4 shears.

The pin in these cases is supposed to be well fitted in vertical holes through the plates.

It is, therefore, only necessary to ascertain the force requisite for shearing a single pin of the given section to ascertain the strength of the whole joint.

Two simple laws determined from experiments are sufficient for finding the practical strength of any ordinary pin or rivet made of wrought-iron.

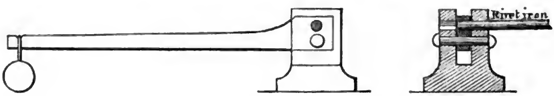
First, the ultimate resistance to shearing is proportional to the sectional area of the bar torn asunder.

Secondly, the ultimate resistance of any bar to a shearing strain is nearly the same as the ultimate resistance of the same bar to a direct longitudinal tensile strain.

These laws are derived from the following experiments.

The object was, first, to ascertain the force required to *shear* any given section of wrought-iron.

For this purpose a machine was made, as represented below, consisting of a wrought-iron lever $\frac{3}{4}$-inch thick and 7



feet long, working in a slot in a block of cast-iron; a stout pin passing through both formed the fulcrum of the lever. The bars of iron to be tested were inserted through a hole in one of the cheeks of the iron block a little above the pin, so that the end of the bar reached nearly across the slot; weights were then suspended from the opposite end of the lever till the protruding portion of the bar gave way; a piece ¾-inch in length being cut off the end of the bar at each experiment. In a second series of experiments the bar was thrust through both cheeks of the cast-iron block, and it had consequently to be sheared in two places simultaneously, a section ¾-inch in length being cut out of the middle of the bar. These two methods of fracture are analogous to the single shearing and double shearing, as explained above. The distance between the fulcrum of the lever and the bar was 6 inches, and the lever was 6 feet long, the leverage being as 12 to 1; allowance was made for the weight of the lever, and the machine was oiled to diminish friction.

The experiments were made on rods of rivet-iron ⅞-inch diameter, perfectly fibrous, and of excellent quality; area ·6013 square inch.

Experiments on the Single Shearing of Bars of Rivet-iron, Diameter ⅞-inch.

Experiment.	Weight in Tons per sq. inch of section.
1. Weight required to shear the bar = 2956 lbs.	26·1
2. Ditto	23·9
3. Four bars, each loaded with the same weight, but the additions to the weight were too sudden	26·1
4. Six bars of iron of a different quality	25·9
Mean	24·15

Experiments on the Double Shearing of Bars of Rivet-iron, as above, Diameter $\frac{7}{8}$-inch.

Experiment.		Weight in Tons per sq. inch of section.
5.	Area of the two sections 1·2026	22·9
6.	Ditto	21·6
7.	Ditto	21·6
8.	Diameter $\frac{31}{32}$-inch	22·5
9.	Ditto	22·5
10.	Ditto	22·5
11.	Ditto, bar $\frac{7}{8}$-inch diameter	21·6
12.	Ditto ditto	21·6
	Mean	22·1

The mean result from these experiments gives 23·3 tons per square inch as the weight requisite to shear a single rod of rivet-iron of good quality. The ultimate tensile strength of these same bars was also found to be 24 tons; hence their resistance to single shearing was nearly the same as their ultimate resistance to a tensile strain.

To avoid any anomalies from the use of the lever, or from the fitting of the pin loosely in the hole, two plates $\frac{5}{8}$-inch thick were now riveted together by a single rivet $\frac{7}{8}$-inch diameter, and the rivet was sheared by suspending actual weights from the plate; the rivet thus sustained 12·267 tons, or 20·4 tons per square inch.

Three plates were then united by a similar rivet, and the rivet was sheared in two places by the centre plate. The ultimate weight suspended from the rivet was 26·8 tons, or 32·3 tons per square inch of section.

Value of Friction produced by the Cooling of Red-hot Rivets.

In all the above experiments the object in view was to ascertain the resistance of a pin or rivet to a shearing

strain, and we have seen that in riveting two plates together to resist a tensile strain, the sectional area of the rivets should be equal to that of the plates themselves, if we depend solely on the shearing of the rivet; but as rivets are usually closed in a red-hot state, it is evident that the shortening of the rivet as it cools down must tend to draw together the plates united, and before they can slip on each other the friction thus induced must be overcome simultaneously with the shearing of the rivet itself; hence the value of the rivet is greater than the value determined above by the amount of friction produced by its contraction in cooling.

The contraction of a wrought-iron rod in cooling is about equivalent to $\frac{1}{10000}$th of its length from a decrease of temperature of fifteen degrees Fahrenheit, and the strain thus induced is about one ton for every square inch of sectional area in the bar. Thus, if a rivet, one inch in section, were closed at a temperature of 900 degrees, it would in cooling decrease in length $\frac{60}{10000}$ths of its length, and if its elasticity and strength remained perfect would produce a tension of 60 tons. The ultimate strength of rivet-iron, however, being only 24 tons, the rivet would in cooling be permanently elongated, and would continue when cool to exert a tension of 24 tons, provided its elasticity remain uninjured by the strain. Thus, if the rivet were not in contact with the plates, excepting at the head and tail, the plates would be held together by a pressure of 24 tons, and this friction would have to be overcome before the rivet came into action as a mere pin.

The following experiments were made to ascertain the value of friction induced by this cooling and consequent contraction of the rivets, and the force requisite to slide the plates over each other. For this purpose three $\frac{5}{8}$-inch plates were riveted together as in the last experiment, but the hole in the centre-plate was oval, and very much larger than the rivet, being $2\frac{1}{2}$ inches in its longest diameter. Weights were

suspended from the centre plate until it slipped and bore upon the rivet, which was $\frac{7}{8}$-inch diameter; it supported 5·59 tons before it began to slide, which it did abruptly.

The experiment was repeated with the addition of an $\frac{1}{2}$ inch plate of iron riveted on each side between the heads of the rivet and the plates, making the shank of the rivet $2\frac{7}{8}$-inch long; 4·47 tons caused the plates to slide.

The last rivet having been found faulty, the experiment was repeated exactly as before, and the plates sustained 7·94 tons before they slipped.

In the next experiment a $\frac{7}{8}$-inch rivet was inserted through two $\frac{5}{16}$ plates with large holes, with a $\frac{5}{16}$ washer on each side next the rivet-head. This combination supported 4·73 tons before it gave way.

These results are very important, for even the strain of $4\frac{1}{2}$ tons, which was the smallest weight that occasioned sliding of the plates in the above experiments, is a greater strain than is supported by any of the rivets in the tubes, from which it may be inferred, that the strain on the rivets is not wholly a shearing, but to a great extent, a tensile strain, and that the tubes would not deflect any further if all the holes were much too large for the rivets, so as not to be in contact anywhere except at the heads. This singular inference is supported by the consideration that the actual deflection of the tubes is the same as that indicated by theory for a tube formed of one welded piece of iron without joints.

It has been proved in practice that when rivets are made more than 6 or 8 inches in length the head is frequently drawn off by the cooling of the rivet. The reason is not very obvious, for, although at first sight it might appear that the great amount of contraction of a long rivet is sufficient cause, it must be remembered that in *proportion to the length* the contraction is the same whatever be the length of the rivet, so that, whether a rivet be 6 feet long or 6 inches,

the contraction is proportionately the same in both cases; that is to say, if the temperature be 900, then the decrease of length is $\frac{60}{10000}$th of the length for both rivets, and there is no reason why a long bar should be injured any more than a short one by the same proportionate extension.

In order to put this to the test of experiment, some red-hot rivets 8 feet long were inserted in some castings of great strength, which, therefore, could not yield to the tension. These rivets on cooling remained in all cases perfectly sound, and had merely undergone a permanent extension proportionate to the temperature. On carefully measuring the amount of extension from marks made on the bar, it was found to be extremely irregular on different portions of the bar, arising, probably, from unequal cooling, though the bar was wholly exposed to the air throughout its length, except at the head and tail, which were cooled rapidly by throwing water on them, to prevent their yielding to the strain.

In the construction of the A^1 beams some experiments were made on rivets 12 inches long, and most of them broke at the head in cooling, and it was found necessary to cool the centre part of the rivet artificially previous to inserting them, the head and tail alone remaining red-hot. In this manner the contraction was avoided, and the rivets remained sound.

The subject requires further investigation. It is probable that the head is somewhat damaged by the hammering, especially if continued too long, as this portion invariably fails. The rapid cooling produced by throwing water on the head and tail of the 8-feet rivets may have protected them, while the whole of the shank, which remained red-hot, yielded to the strain.

It is, however, evident that considerable strength is obtained from the friction produced by the cooling of a rivet, and hence it is not requisite that the area of the rivet should

equal that of the plates united, as with a simple loose pin. This close union of plates is a most important characteristic of riveted work, the joints being as immovable as the most perfect weld. This valuable quality is perfectly demonstrated by the deflection of the large tubes, which is precisely equal to the deflection calculated on the assumption of the material being everywhere solid; and we have seen that this would have been the case without the contact of a single rivet in any of the holes.

This close union of the plates entirely prevents the oxidization of the plates at the joints, the rust is entirely superficial, and such work is fortunately as durable as it is immovable.

Thus, also, by judicious riveting the friction may in many cases be nearly sufficient to counterbalance the weakening of the plate from the punching of the holes, so that a riveted joint may be nearly equal in strength to the solid plates united.

CHAPTER V.

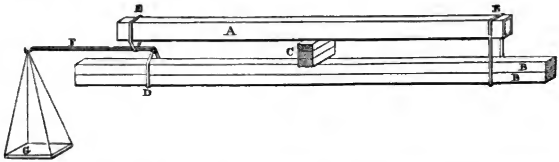
EXPERIMENTAL INQUIRIES ON THE TRANSVERSE STRENGTH OF BEAMS AND TUBES.

THE principal object in most of the following inquiries was to submit to the test of experiment the theoretical laws which govern the strength of beams, in order to apply them with confidence in the construction of the proposed bridge; also to investigate the best forms of section, and furnish data for the proper distribution of the material as regards the relative thickness of the top, bottom, and sides of tubular beams.

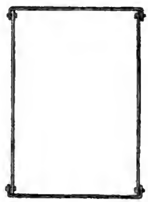
Most of the experiments on wrought-iron tubes were made for Mr. Stephenson by Mr. Hodgkinson; and the deductions drawn from them by that gentleman will be found in more detail in the "Report of the Government Commission on the Application of Iron to Railway Structures." Other experiments on wrought and cast-iron tubes were made for Mr. Stephenson's private information at his works at Newcastle, where they were conducted with great accuracy and care by his assistant, Mr. John Hosking, resident engineer at the High Level Bridge; the remainder of the following experiments were made by the Author.

Mr. Hodgkinson's experiments on wrought-iron tubes were confined to tubes rectangular in section, and were made with simple plates without cells on the top, the forces of resistance on the top and bottom being thus placed as far asunder as possible.

The large tubes were broken through the intervention of a lever, as in the following sketch. The tube A was



placed upside down, its top resting on a cushion placed on the balks of timber B B, the one end being strapped down by a wrought-iron strap to the balks of timber, the other extremity was drawn down by the loaded lever F, the deflections being measured along the top; the apparatus, of which this is a mere outline, was strengthened by struts, and the lever was most carefully constructed and employed; cast-iron boxes were inserted at the extremities, immediately beneath the straps, to prevent crushing of the ends. It is evident that the breaking-weight at the centre will be double the amount of weight applied through the lever, but this reduction is made in the tables where the tubes are described as if in their natural position, and broken by a weight at the centre.



All the plates on the tubes of less than 30 feet span were in one length, in the remainder the transverse joints were not near the centre; the joints were double-riveted, the angles being formed without angle-iron by the bending over of the top and bottom plates, as in the sketch.

The principal experiments made by Mr. Hodgkinson are tabulated in four series.

The first series, Table XX., is devoted to tubes uniform throughout, with plates of equal thickness on the top, bottom, and sides. It will be observed that tubes of the same length,

breadth, and depth, are placed in juxtaposition for comparison, the only variation being in the thickness of the plates. And, again, in determining the dimensions of the models, it will be observed, that they are always so formed as to be similar to each other, though differing widely in lineal dimensions: the object of which will be seen hereafter.

The broken tubes in the preceding experiments were now thoroughly repaired, and again broken by applying the weight, not at the centre as before, but at intermediate parts, as indicated by the column headed "Rectangle of the Segments," and the deflections were also taken at that spot.

These tubes were of equal thickness throughout, and form the second series, Table XXI.

In the third series, Table XXII., the thickness of the plates, instead of being uniform throughout, is decreased towards the ends in the ratio indicated by theory for a beam of similar strength at every part, *i. e.* in the ratio of the rectangle of the segments at every point; near the ends, however, the thickness was not reduced to less than half the thickness at the centre; the weights were applied at the centre.

The fourth series, Table XXIII., contains experiments on tubes which had been previously broken, and, being thoroughly repaired, were again broken, not by applying weight at the centre, but at intermediate points, indicated by the column headed "Rectangle of the Segments." These tubes were reduced in thickness towards the ends.

Mr. Hodgkinson was strongly impressed with the conviction that cast-iron might be advantageously used on the top of the large tubes; and to test its applicability to this purpose, he obtained permission from Mr. Stephenson to make several experiments on tubes in which this material was employed for resisting the compression in the top. These experiments are given in detail, but are not tabulated.

TABLE XX.

First Series.—Experiments on the Transverse Strength of Rectangular Tubes of Wrought-Iron.

Tubes of equal thickness throughout, Weight applied at the Centre.

No. of Experiment.	Length of Tube.	Weight of Tube.	Distance between Supports.	Weight of Tube between Supports.	External Dimensions.	Thickness of Plates of Tube.	Weights laid on Tube in middle.	Deflections from those Weights.	Breaking Weight, or Weight of greatest resistance.	Cause of Fracture, and Remarks.	Form of Section of Tube.
	Ft. in.	Lbs. oz.	Ft. in.	Lbs.	Inches.	Inch.	Lbs.	Inch.	Tons.		
1 (See note.)	4 $3\frac{1}{4}$	4 15	3 9	4·34	3·0 × 1·9	·03	224 336 448 560 672	·05 ·07 ·10 ·13 . .	·3	Crippling of top of tube, after bearing weight a short time.	. .
2	4 $2\frac{1}{8}$	10 12	3 9	9·65	3·0 × 1·95	·061	224 448 896 1792 2240 2464 2520	·019 ·035 ·092 ·225 ·345 ·435			□
3 (See note.)	8 2	38 11	7 6	35·5	5·8 × 3·8	·065	1006 1576 2136 2696 "	·09 ·14 ·21 ·31 ·32 in 5 minutes		Top wrinkled and bent down.	□

4	8	2	78	13	7	6	72·36	6·0 × 3·9	·1325	1006 2136 4376 6616 7736 8296 8856 9416	·055 ·12 ·24 ·42 ·59 ·73 ·81 ·96		Crushed at top, and torn at bottom, commencing at a rivet-hole. One of the top rivets failed with 7736 lbs.	
5 (See note.)	31	6	Cwt. 10	qrs. 1	30	0	Cwt. 9 qrs. 3 lbs. 1	24 × 15	·124, or ⅛ nearly.	5685 11285 12405	·49 1·20 1·32		Sunk down gradually, the top side in the middle becoming corrugated or contracted into wrinkles.	
6	31	6	24	1	30	0	23 0 11	23·75 × 15·5	·272, or ¼ inch nearly.	5685 11369 22737 Unloaded. 22737 28421 Unloaded. 34105 Unloaded. 39789 45473 51157	·21 ·40 ·74 ·045 ·72 ·91 ·07 1·11 ·11 1·22 1·53 2·5 nearly.		Bulged out at side near top, in consequence of the crushing of the top. 28421 lbs. Slight wrinkling at top.	

Experiments on the Transverse Strength of Rectangular Tubes of Wrought-Iron (continued).

No. of Experiment.	Length of Tube.	Weight of Tube.	Distance between Supports.	Weight of Tube between Supports.	External Dimensions.	Thickness of Plates of Tube.	Weights laid on Tube in middle.	Deflections from those Weights.	Breaking Weight, or Weight of greatest resistance.	Cause of Fracture, and Remarks.	Form of Section of Tube.
	Ft. in.	Cwt. qrs.	Ft. in.	Cwt. qrs. lbs.	Inches.	Inch.	Lbs.	Inch.	Tons.		
7	31 6	44 3	30 0	42 2 13	24 × 15·5	·525 or ½ inch nearly.	5685	·12	57·54	Cracked at top under shackle, and torn at a rivet-hole at bottom.	
							11285	·22			
							44885	·78			
							Unloaded.	·08			
								·09 in an hour.			
							56085	1·00			
							Unloaded.	·17			
							67285	1·29			
							78485	1·73			
							Unloaded.	·555			
							84085	. .		Sinking for some minutes. Failed at the rivet-holes, half-way between the middle and the end.	
							Tube repaired, experiment repeated.				
							84085	1·26			
							89685	1·35			
							100885	1·67		100885. Slight sinking in the middle under the strap.	
							106485	1 73			
							Unloaded.	·37			
							106485	1·88			
							117685	2·43			
							,,	2·53 in 10 minutes.			

							120485 126085 128885	2·69 3·03 in 5 minutes. 3·48		Crack at a rivet-hole at bottom.	
8 Same tube as in Experiment 6, but with thicker top plate for 15 feet in the middle. (See note.)	31 6	. .	33 0	. .	23·75 × 15·75	·437 at top, and ·272 at bottom and sides.	28085 33685 39285 Unloaded. 44885 Unloaded. 50485 Unloaded. 56085 Unloaded. 61685 Unloaded. 67285 Unloaded. 72885	·70 ·85 1·00 ·03 1·16 ·055 1·30 09 1·44 1·45 1·75 ·285 2·00 ·59 2·25	32·538	With this weight, 72885 lbs., the tube broke at top under shackle, in consequence of crushing of the packing, which was of oak and ash. It was torn likewise at bottom, beginning at a rivet-hole.	

TABLE XXI.

Second Series.—Experiments on the Transverse Strength of Rectangular Tubes of Wrought-Iron.

Tubes of Equal Thickness throughout Broken by Weights at various parts of their Length.

No. of Experiment.	Length of Tube.	Weight of Tube.	Distance between Supports.	Weight of Tube between Supports.	External Dimensions.	Thickness of Plates of Tube.	Rectangle of Segments of Tube, at place where the weights were laid.	Weights laid on Tube.	Deflections from those Weights.	Breaking Weight, or Weight of greatest resistance.	Cause of Fracture, and Remarks.	Form of Section of Tube.
	Ft. in.	Cwt. qrs. lbs.	Ft.	Cwt. qrs. lbs.	Inches.	Inch.	Ft. Ft.	Lbs.	Inches.	Tons.		
9 Same tube as that in No. 5, repaired. (See note.)	31 6	10 1 0	30	9 3 1	24 × 16	·124, or ⅛ nearly.	7½ × 22½	7580 15345 16540	·36 ·71 ·80	8·3	Bulged out at top and sides.	
10 Same tube as in No. 6, repaired. (See note.)	31 6	26 0 14	30	. .	24 × 16	·272, or ¼ in. nearly.	14 × 16	20632 Unloaded. 30605 Unloaded. 40685 Unloaded. 50730 57414 Unloaded.	·53 ·04 ·83 ·07 1·20 ·23 1·66 2·32 1·05	26·1	Top much wrinkled in centre.	
11 Same tube repaired.	31 6	26 0 14	30	. .	24 × 16	·272, or ¼ in. nearly.	6 × 24	38276 58436 65156 71876 78596	·47 ·78 ·90 1·04 1·26	36·6	Top bulged out, and sides bent in near to point of application of weight. Slightly wrinkled on compressed side.	

12 Same tube repaired, and the strain applied near to the other end.	31 6	26 0 14	30	..	24 × 16	·272, or $\frac{1}{4}$-in. nearly.	8 × 22	63748	..	34·3	Wrinkled at top and torn at bottom.	..
13 Same tube as that in No. 7, repaired. (See note.)	31 6	44 3 0	30	42 2 13	24 × 15·5	·525, or $\frac{1}{2}$-in. nearly.	14 × 16	42578 88658 100178 111698	·67 1·28 1·44 1·59	54·8	With this weight the tube broke at a joint.	
14 Same tube repaired.	31 6	44 3 0	30	42 2 13	24 × 15·5	·525, or $\frac{1}{2}$-in. nearly.	6 × 24	28425 99345 110545 121745 132945 144145 155345 166545 175505	·17 ·61 ·72 ·86 ·97 1·15 1·27 1·43 1·67	88·8	With this weight the tube broke by tearing through 12 rivets with which it was united, 24$\frac{1}{2}$ inches from the place where the weight was laid on.	..
15 Same tube as that in No. 9, repaired. (See note.)	31 6	..	30	..	24 × 16	·124, or $\frac{1}{8}$-in. nearly.	16 × 14	10659 13179 14859 16539 18219 19899 21579	·51 ·66 ·74 ·82 ·93 1·04 1·22	10·94	Failed 8 feet from the end. Top bulged out and sides bent in.	

TABLE XXII.

Third Series.—Experiments on the Transverse Strength of Rectangular Tubes of Wrought-Iron.

Tubes reduced in Thickness towards the Ends. Weight applied at the Centre.

No. of Experiment.	Length of Tube.		Weight of Tube.			Distance between Supports		External Dimensions.	Thickness of Plates of Tube in middle.			Weights laid on Tube in Middle.	Deflections from those Weights.	Sets.	Breaking Weight, or Weight of greatest Resistance.	Cause of Fracture, and Remarks.	Form of Section of Tube.
									Top.	Bottom.	Each Side.						
	Ft.	in.	Cwt.	qrs.	lbs.	Ft.	in.	Inches.	Inch.	Inch.	Inch.	Lbs.	Inches.	Inch.	Tons.		
16 (See note.)	31	6	20	3	13	30	0	24 × 16 nearly.	·372	·244	Intended to be ⅛ each.	11786	·37	..	26·1	It failed with this weight, 26·1 tons, at the riveting in middle at bottom. Sides in middle much distorted.	
												14618	·48	·01			
												21786	·69	·04			
												28954	·97	·09			
												32538	1·11	·18			
												39706	1·43	·29			
												46874	1·84	..			
												50458	2·19	..			
												54042	2·63	..			
17 Same tube repaired and laid on its side. (See note.)	..		..			30	0	..	..	..	..	11370	·82	·06	13·8	The top part of the tube, which, in the former experiment, was the side, and very thin, became wrinkled.	
												15850	1·24				
												20330	1·67				
												24810	2·20				
												29290	2·75 nearly.				
18 (See note.)	31	8	36	3	8	30	0	24 × 16½	¾	½	¼	11370	·18	·01	50·48	Tearing at rivet-holes, through sound metal, about 2 ft. 8 in. from middle.	As in Experiment 16; but the plates thicker.
												32874	·525	·073			
												54378	·865	·16			
												75882	1·305	·305			
												86634	1·547	·415			
												97386	1·945	·645			
												108138	2·64				

19 Same tube repaired and strength-ened in for-mer place of fracture. (See note.)	31	8	37	0	0	30	0	24 × 16½	¾	½	¼	47210 ,, repeated. 65130 83050 100970	·69 ·70 ·95 1·22 1·58	0·3	53·92	..	..
20	31	6	40 nearly.			30	0	24 × 16	½	½	½	11746 40114 58034 75954 93874 102450 111410 ,, repeated	·16 ·70 1·05 1·50 1·99 2·197 2·672 2·724	·0 ·17 ·29 ·47 ·74	54·3	Bottom plate torn at a rivet-hole near to middle.	
21 (See note.)	47	0	Tons 3	cwt. 1	qrs. 0	45	0	36 × 24	5625	·397	·214	11370 47210 65130 83050 92010 ,, repeated in an hour after rain. 100970 118890 127850 136810 146768	·24 ·85 1·09 1·47 1·63 1·62 1·77 2·09 2·34 3·10 ..	.. ·13 3·5	65·5	..	
22 Same tube tried again.	..		..			..		..	..	..	..	..	..	..	62·32	..	..

Experiments on the Transverse Strength of [illegible]

No. of Experiment.	Length of Tube.	Weight of Tube.	Distance between Supports.	External Dimensions.	Thickness of Plates of Tube in middle. Top.	Bottom.	Each Side.	Weights laid on Tube in middle.	Deflections from those Weights.	Set.	Breaking Weight [illegible]	[illegible]
	Ft. In.	Tons cwt. qrs.	Ft. In.	Inches.	Inch.	Inch.	Inch.	lbs.	Inches.			
23 (See note.)	46 10½	6 13 0	45 0	35½ × 24	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
								[illegible]	[illegible]	[illegible]		[illegible]
								[illegible]	[illegible]	[illegible]		[illegible]
								„ repeated	[illegible]			[illegible]
								„	[illegible] in 5 minutes			[illegible]
								[illegible]	[illegible]			
								[illegible]	[illegible]			
								„ repeated	[illegible]			
								[illegible]	[illegible]			
								[illegible]	[illegible]			
								[illegible]	[illegible]			[illegible]
								„	[illegible]			
								„	[illegible]			
								[illegible]	[illegible]			
								[illegible]	[illegible]			
								[illegible]	[illegible] from straight line			The top of tube [illegible]
								257080	[illegible]			

24 Same tube altered, with new plates and thinner sides in middle (See note.)	46 10½	..	45 0	36 nearly × 24	¾	¾	⅛	22674	·23	·02	102·77	With this weight, 102·77 tons, the experiment was discontinued, as the tube would sink to any degree.	
										·03 in 10 minutes.			
								,, repeated.	·225				
								57170	·80	·26			
										·25 after 1 hour.			
								93010	1·36	·47			
								,, repeated.	1·41				
								128850	1·96				
								,, repeated.	1·99				
								146770	2·27				
								164690	2·65				
								182610	2·97				
								200530	4·00			200530 lbs. left on three days, slight wrinkling sides at the middle. Top crushing, and scales peeling off. Ultimately top in the middle quite straight, although literally crushed. Sides wrinkled perceptibly.	
								,, repeated.	4·10				
								209490	4·33				
								218450	5·38				
								,, repeated.	5·69				
								227410	6·04				
								,,	6·09 in 3 minutes.				
								,,	6·27 in 7 to 10 minutes.				
								,,	6·51 in 15 minutes.				
								,,	6·58 in 20 minutes.				
								,, repeated.	7·33				

TABLE XXIII.

Fourth Series.—Experiments on the Transverse Strength of Rectangular Tubes of Wrought-Iron.

Tubes reduced in Thickness towards the Ends. Broken by weights at different parts of their length.

No. of Experiment.	Length of Tube.	Weight of Tube.	Distance between Supports	External Dimensions.	Thickness of Plates of Tube in Middle. Top.	Thickness of Plates of Tube in Middle. Bottom.	Thickness of Plates of Tube in Middle. Each Side.	Rectangle of the Segments of Tube at place where the Weights were laid.	Weights laid on Tube.	Deflections from those Weights.	Sets.	Breaking Weight, or Weight of greatest Resistance.	Cause of Fracture, and Remarks.	Form of Section of Tube.
	Ft. In.	Cwt. qrs. lbs.	Ft. In.	Inches.	Inch.	Inch.	Inch.	Feet.	lbs.	Inches.	Inch.	Tons.		
25 Same tube as that in No. 16 repaired. (See note.)	31 6	20 3 13	30 0	24 × 16 nearly.	·372	·244	Intended to be $\frac{1}{8}$	6 × 24	7366	·14	·02	22·1	Side wrinkled. The wave shewing the line A C increased. (See note.) Line A C very distinctly marked.	
									13846	·26				
									18326	·34	..			
									27286	·48				
									36246	·67	..			
									40726	·80				
									45206	·98				
									47446	1·11				
26 Same tube as that in No. 17 repaired, and laid on its side, the weight being applied in another place nearer to one end than before (See note.)	..	..	30 0	..	..	..	..	10 × 20	8527	·52	·03	16·7	Top wrinkled near to fulcrum. The part now broken had not been injured by the preceding experiments.	
									15247	·92	·04			
									21967	1·39	·07			
									,, repeated.	1·40				
									25327	1·64				
									28687	1·93	·20			
									,, repeated.	1·95				
									32047	2·28				
									35407	2·81				
										2·84 in 5 minutes.	·91			

27 Same tube as that in No. 20 repaired. (See note.)	31 6	40 0 0 nearly.	30 0	24 × 16	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{6}$	6 × 24	25071	·165	·005	61·46	With 61·46 tons the tube sunk, bulging out at the top and sides, and cracking at a rivet-hole near to the bottom.	
									47471	·35	·03			
									69631	·515				
									,, repeated.	·52	·065			
									80831	·625				
									92031	·74	·14			
									103231	·88				
									114431	1·08	·38		Slight change in form of tube at the shorter end, and the plate near to the application of the weight slightly bulged out at the side. Several rivets springy.	
									125631	1·29	. .		Top of tube distorted.	

NOTES ON THE EXPERIMENTS IN THE PRECEDING TABLES.

Exper. 1.—This and the following tube differ only in the thickness of the plates used, those of the latter being about double the thickness of those of the former. Each tube was made out of a single plate, and the rivets were placed longitudinally along one side only.

Exper. 3.—The tubes in the third and fourth experiments, like those in the first and second, differ only in the thickness of the plates; those in No. 4 being intended to be double those in No. 3, and the dimensions of those tubes are nearly double those of the former.

Exper. 5.—These three tubes (in Experiments 5, 6, 7) were intended only to differ in the thickness of their plates; the second and third being each double the strength of the preceding one. The tubes were likewise designed to be similar in every respect to those in the former experiments. Thus, the tube in Experiment 6 had a span or distance between the supports four times as great as that in Experiment 3, and eight times as great as that in Experiment 1; and their thickness, ·272, ·065, ·03, were designed to be in that proportion. In like manner the tubes in Experiments 7, 4, and 2, having their spans as 8, 2, 1, had their thicknesses ·525, ·132, ·061, or nearly in the ratio of their spans.

Exper. 7.—The tube having failed with 84085 lbs. through insufficient riveting at a joint, half-way between the middle and the end, it was afterwards secured with a plate $\frac{3}{4}$-inch thick in that part, and united with an extra row of rivets. The deflections, commencing with 1·26, from 84085 lbs., were now taken from that which the tube had assumed after repairs.

Exper. 8.—The thickness of the top plate in this tube was now increased from ·272 to $\frac{7}{16} = \cdot 437$ inch for 15 feet in the middle, and the joint was strengthened by an additional plate and better riveting in that part.

Exper. 9.—This tube, in the former experiment, No. 5, when the weight was applied in the middle, failed with 5·54 tons.

Exper. 10.—This tube, in Experiment 6, broke, when the weight was applied in the middle, with 22·84 tons.

Exper. 13.—This tube, in Experiment 7, broke with 57·55 tons.

EXPER. 15.—This tube is the same as that in Experiments 5 and 9, but it had here the top increased in thickness from $\frac{1}{8}$th to $\frac{3}{8}$ths of an inch, for 15 feet in the middle, to ascertain the force necessary to break the tube by tension, or tearing asunder the bottom. It failed, however, by compression as before. Strength increased from 5·54 tons to 10·94 tons.

EXPER. 16.—The sides of the tube were rendered crooked in the middle by the weight, particularly towards the ends.

EXPER. 17.—This experiment was made to ascertain the comparative power of a tube of this form to bear a load applied vertically and on its side; and, supposing the tube not to have been injured in its bearing power on the side by the former experiment, which strained it vertically, we see that the power of a tube of this form to resist a vertical and a side strain is as 26·1 to 13·8. This experiment having been made to determine in some degree the power of a tube to resist the action of the wind, will be repeated further on, where the result will be somewhat modified.

EXPER. 18.—The plates in this tube were intended to be of double the thickness of those in the first tube. It maintained its form much better than that did.

EXPER. 19.—The tube broke with the weight 53·92 tons in the middle, and the fracture was through the sound plate, $2\frac{1}{4}$ to 10 inches from the middle, and about 2 feet from a joint. The tube maintained its vertical position in every part up to the time of fracture, but the rust peeled off the plate next to one end, shewing that part to be overstrained with 4 or 5 tons less than the breaking-weight.

EXPER. 21.—In this tube the sides near to the ends were stiffened by means of small vertical stays. The weight here given as the breaking-weight was not absolutely so; the experiment was discontinued on account of the supports giving way. In another experiment, tried on the 9th September, upon the same tube, it failed with 139600 lbs. = 62·32 tons. The fracture commenced at the bottom, about 15 inches from the middle, tearing the sound plate across from the bottom to the top. It was 2 feet $8\frac{1}{2}$ inches from any riveted part, and 3 feet 8 inches from that at the bottom. The fracture was a complete tearing through of the sound material.

EXPER. 23.—The tube during the experiment became very much bent, both from extension in the bottom and compression in the top, but no undulation was observable in the top.

The area of section of the tube in the middle was 96·4 square inches nearly.

Exper. 24.—Same tube as the last, but the plates in the middle at top and bottom replaced with others $\frac{3}{4}$-inch thick, as before, and the plates at the sides in the middle reduced to half their former thickness, or $\frac{3}{8}$-inch each. The only difference between the tube in its present state and as it was before is in the reduction of the thickness of sides in the middle.

The area of the section of the tube in the middle in Experiment 23 was 96·4 square inches, and with its sides, as now reduced in thickness, it is 70·9 square inches nearly. Its strength before was 114·77 tons, and it is now 102·77 tons. But 96·4 : 70·9 : : 114·77 : 84·4 tons, the weight it would have borne if its form had not been changed. But the strength by the improved form was increased to 102·77 tons as above.

This tube, when tried 23d September, required 114·76 tons to break it; its top, bottom, and sides, were each then $\frac{3}{4}$-inch thick. It has now new plates of the same thickness at the top and bottom in the middle, and plates of half the thickness at the sides. Its area of section is reduced from 96·4 to 70·9 inches; but the resistance for an equal area is increased from 84·4 to 102·77.—*See* Experiments 23 and 24, Third Series.

Exper. 25.—With the weight 7366 lbs., the side of the tube, near to the ends, became wrinkled as in the line A C in the adjoining figure.

Exper. 26.—*Resistance to a Side Strain as from the Action of the Wind.*—This tube was made to be equally strong to bear a load in every part, and therefore we will take a mean between the breaking-weight on the side in Experiment 17 and this, though the points of application were different. The breaking-weight in Experiment 17 was 13·8 tons, and in the present experiment 16·7 tons, mean 15·25.

The tube, when subjected to a vertical pressure in Experiment 16, required 26·1 tons to break it. Hence the power of the tube to resist a vertical strain is to its power to resist a strain on its side, as from the wind, as 26·1 to 15·25 nearly.

Exper. 27.—This tube was broken, in Experiment 5 with 54·3 tons laid on the middle. The strength was, therefore, greater towards the ends than in the middle. The tube formed of thinner plates in Experiments 16 and 25 was weaker towards the ends than in the middle.

Experiment 28.

Tube with Cast-Iron riveted to the Top.

The tube which had been used in Tables XX. and XXI., Experiments 7 and 13, was repaired with the following modification.

Wrought-iron straps were added to strengthen the bottom. They were 15 square inches in section, and the sides were stiffened by pillars of angle-iron placed vertically.

To strengthen the top two cast-iron straps were riveted along the half-inch plates of which it was composed, the section of each being $4{\cdot}5 \times 1{\cdot}1$, or 5 square inches, and their combined area was 10 inches; they extended 22 feet 6 inches along the top, the tube being 29 feet 6 inches between the supports; the thickness of the plates was half-inch throughout, and the tube was 24 inches deep and 16 broad externally.

The weights were not placed at the centre; the distance from one end was 14 feet, and from the other 15·5 feet; the rectangle of the segments was therefore $14 \times 15{\cdot}5$ feet.

The following Table contains the result of this experiment:—

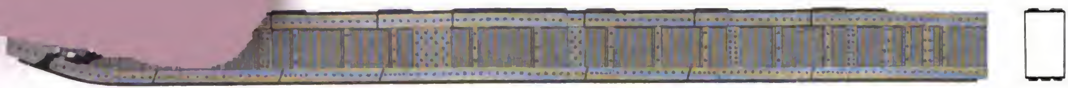
Weights laid on Tube.	Deflections from those Weights.	Remarks on the Results.
Lbs.	Inches.	
44925	·5	
61978	·75	
96084	1·27	
130190	1·7	
164296	1·8	
181349	2·0	
198402	2·2	The timber on which the tube was supported became crushed.
215455	2·35	
232508	. .	Experiment discontinued in order that the timber might be strengthened.
232508	2·85	Experiment renewed.
238476	2·9	Rivets of the wrought-iron strap yielding slightly to the strain.
249560	3·3	Wrought-iron near to cast much compressed, and the scales peeling off in large flakes.
266613	4·4	With this weight the tube shewed no crippling or other defect, though it was slowly yielding to the pressure, and emitted loud sounds.
273526		

The tube broke by tearing asunder the wrought-iron straps at the bottom. The fracture commenced at the nearest rivet-hole to the middle. The cast-iron had shewn no signs of weakness, and, after the experiment, remained sound, excepting one of the straps, which was cracked partly through, the crack beginning at the bottom. The breaking-weight was 122 tons 2 cwt.

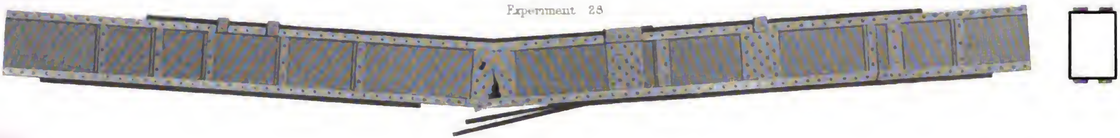
This tube has been broken in Experiment 7 with $57\frac{1}{2}$ tons laid on the middle, the distance between the supports being 30 feet, and with 54·8 tons laid on a point where the rectangle of the segments was 14×16 in Experiment 13.

TRANSVERSE STRENGTH OF TUBES

Experiment 30



Experiment 28



Day & Son Lith.rs to the Queen

EXPERIMENT 29.—*Tube with Cast-Iron riveted to the Top.*

The tube used in the last experiment was again repaired, and, as the cast-iron 10 inches in section remained uninjured when opposed to 15 square inches of wrought-iron, the section of the wrought-iron was increased to 21 square inches, consisting of two straps $7 \times 1\frac{1}{2}$ inches. The weight was applied as before. The following was the result:—

Weights laid on Tube.	Deflections from those Weights.	Remarks on the Results.
Lbs.	Inches.	
54403	·54	
71456	·70	
88509	·81	
Repeated.	·80	
105562	·94	
119204	1·02	
122615	1·05	
139668	1·19	
173774	1·53	
207879	1·87	
212995	..	Experiment discontinued on account of the supports becoming crushed.
		Experiment resumed.
207879	2·03	
213848	..	Loud sound in tube.
224932	2·14	
241985	2·35	
259038	2·94	
267565	3·23	

With 267,565 lbs. both the wrought-iron straps and the bottom of the tube became cracked at rivet-holes near the middle; with 269,365 lbs., one of the wrought-iron straps tore asunder after one hour. It had previously been sound, and was bright throughout. The other strap was cracked. The fracture was 10 inches from the point of application of the weight. The cast-iron bars remained uninjured. The wrought-iron was of inferior quality, or it would have supported more. The tube retained its form, and the breaking-weight was 120 tons.

Experiment 30.—*Tube with Cast-Iron riveted to the Top.*

The tube, which had been broken in Experiments 21 and 22, in Table XXII., was repaired and strengthened by the addition of two cast-iron bars to the top, each $4\frac{1}{2} \times 1\frac{1}{16}$ inches in section, making a total section of 10 square inches. They were riveted with $\frac{3}{4}$ rivets, 6 inches asunder, and extended 37 feet along the tube, the length of the tube between supports being 45 feet, the depth 36 inches, and the breadth 24 inches. The bottom was strengthened by three straps of wrought-iron, the total sectional area of which was 21 inches, extending 46 feet along the tube.

The sides were stiffened by vertical pillars of $2\frac{1}{2}$-inch angle-iron, the thickness of the plates in the top, bottom, and sides, being ·562, ·397, and ·214 inch respectively. The tube was loaded at the centre.

The following was the result of the experiment:—

Weights laid on Tube.	Deflections from those Weights.	Remarks on the Results.
Lbs.	Inches.	
57170	·8	
75090	·95	
93010	1·1	
110930	1·50	
128850	1·75	Sides in plates nearest to the ends slightly buckling, notwithstanding the angle-irons.
146770	1·95	
164690	2·12	Buckling, as above, but little or none increased.
182610	2·25	
200530	2·62	Sides at ends more puckered, but straight in every other part. Rust peeling off in various parts.
218450	3·00	
236370	3·2	Rust peeling off near to bottom.
247122	. .	A stay placed in the middle of the tube having broken, caused a great shake in it, and injured it, perhaps.
253264		

The tube failed within 3 feet of the end by the distortion of the sides and failure of the wrought-iron, the plates being decreased in thickness towards the ends. The breaking-weight was 113 tons.

Thus, in Experiment 28 an addition of 22 cwt. to the original weight of the tube, which was 44 cwt. 3 qrs. more than doubled its strength.

In Experiment 30, with twice as much wrought-iron as cast-iron, the wrought-iron still failed, the strength being increased from 65·5 to 113 tons.

Experiments on the Strength of similar Tubes.

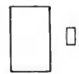
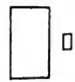
Some of those properties of beams which were more immediately concerned in deducing the strength of larger structures from experiments made on models have been alluded to in Section III., where we have seen that the strength of similar beams should be directly proportional to the square of their lineal dimensions; so, that if we find the strength of any given model to be W tons, then any other similar beam, which is n times as long, n times as broad, n times as thick, and n times as deep, should be n^2 times as strong, or its strength should be n^2 W tons.

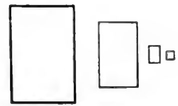
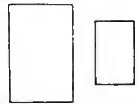
To determine experimentally whether this law would be interfered with by any anomaly in tubular beams, similar tubes, varying very considerably in lineal dimensions, were selected from the preceding experiments, and compared together in pairs, and the power of the lineal dimensions to which their breaking-weight was found proportional is calculated and given in column 7.

The weight of each tube is given in column 2; if the tubes had been made without covers or stiffening strips, or any extraneous additions, the weights would be found to be as the cube of the lineal dimensions, which, it will be seen, is only approximately the case.

TABLE XXIV.

Comparison of Results from Experiments on the Transverse Strength of similar Tubes in the preceding Tables to ascertain the Power of the Lineal Dimension to which the Breaking Weight was Proportional.

Distance between the Supports.	Weight of the Tubes between the Supports.	Breaking Weights, exclusive of the Weights of the Tubes.	Depth of the Tubes.	Breadth of the Tubes.	Thickness of the Plates of the Tubes.	Power N of the Lineal Dimensions on which the Strength depends.	Form of Section, and comparative Magnitude of the Tubes compared.
Feet. 30	42·62 cwt.	Tons. 57·5	Inches. 24 nearly.	Inches. 16 nearly.	Inch. ·525	1·858	
7·5	72·36 lbs.	4·454	6 ,,	4 ,,	·1325		
30	23·09 cwt.	22·84	24 ,,	16 ,,	·272	1·946	
7·5	35·53 lbs.	1·409	6 ,,	4 ,,	·065		
30	42·62 cwt.	57·5	24 ,,	16 ,,	·525	1·903	
3·75	9·65 lbs.	1·1	3 ,,	2 ,,	·061		
30	23·09 cwt.	22·84	24 ,,	16 ,,	·272	1·965	
3·75	4·34 lbs.	·3	3 ,,	2 ,,	·03		

45 3·75	130·36 cwt. 9·65 lbs.	114·76 1·1	36 ,, 3 ,,	24 ,, 2 ,,	·75 ·061	1·870	
45 7·5	130·36 cwt. 72·36 lbs.	114·76 4·454	36 ,, 6 ,,	24 ,, 4 ,,	·75 ·1325	1·874	
45 30	130·36 cwt. 39· nearly.	114·76 54·3	36 ,, 24 ,,	24 ,, 16 ,,	·75 ·50	1·846	
45 30	59·80 cwt. 20·46 cwt.	65·5 26·1	36 ,, 24 ,,	24 ,, 16 ,,	As 1, 2, 3, As 1, 2, 3, Similar tubes with different thicknesses of plates in each.	2·271	
						1·942 = mean, or 1·895 if we neglect the last result, 2·271.	

The strength of similar tubes is therefore nearly as the square of their lineal dimensions, or, more accurately, as the 1·9 power.

Conversely, the formula $W = \frac{a\,d}{l}\,c$ is proved experimentally to hold true for similar tubes. For, in similar tubes, a varies as n^2, and $\frac{d}{l}$ is constant, therefore $\frac{a\,d}{l}$ varies as n^2; but we have seen the strength varies as n^2, therefore the strength varies as $\frac{a\,d}{l}$, or $W = \frac{a\,d}{l}\,c$.

Experiments on the Strength of Beams loaded at different parts of their Length.

We have seen that the strain from a weight laid on a beam at any part of its length varies directly as the rectangle of the two parts into which the beam is divided by the point of application of the weight.

In order to submit this to the test of experiment, tubes 31 feet 6 inches long, 2 feet deep, and 1 foot 4 inches wide, were broken on supports 30 feet asunder, not by applying weights at the centre, but at various parts of the length, and the comparative breaking-weights were then compared with the theoretical breaking-weights calculated as above. Thus, in the first pair, in the following table, the breaking-weights are as $88{\cdot}8 : 54{\cdot}8 = \frac{162}{100}$, whereas the rectangles of the segments are as $224 : 144 = \frac{155}{100}$; so that the result agreed with theory in the ratio of 162 to 155.

TABLE XXV.

Thickness of the Plates.	Rectangle of the Segments where the weight was applied.	Breaking Weight at that point.	Inverse ratio of Rectangles of the Segments, or Ratio of Strengths according to theory.	Ratio of actual Breaking Weights.	Mean from the inverse ratio of the Rectangles of the Segments.	Mean from ratio of the Breaking-Weights.
Inch. $\frac{1}{2}$	6 × 24 = 144 16 × 14 = 224	Lbs. Tons. 199025 = 88·8 122738 = 54·8	$\frac{224}{144} = \frac{155}{100}$	$\frac{88·8}{54·8} = \frac{162}{100}$	$\frac{142}{100}$	$\frac{146}{100}$
$\frac{1}{4}$	14 × 16 = 224 6 × 24 = 144 8 × 22 = 176	58542 = 26·1 81996 = 36·6 76883 = 34·3	$\frac{224}{144} = \frac{155}{100}$ $\frac{224}{176} = \frac{127}{100}$	$\frac{36·6}{26·1} = \frac{140}{100}$ $\frac{34·3}{26·1} = \frac{131}{100}$	Whence it appears that the mean theoretic to the experimental ratio, from all the experiments, is as 142 to 146; and hence that the tubes may apparently be reduced in strength and thickness towards the ends, in the ratio indicated by theory.	
$\frac{1}{8}$	7·5 × 22·5 = 169 15 × 15 = 225	18631 = 8·3 12405 = 5·54	$\frac{225}{169} = \frac{133}{100}$	$\frac{8·3}{5·54} = \frac{150}{100}$		

Comparison of Results on the Transverse Strength of Rectangular Tubes.

The theoretical laws for the reduction of similar tubes to any dimensions having been verified and somewhat modified by experiment, it was important to compare directly the relative strengths of all the different forms experimented upon, by reducing them all to the same span and the same weight; and, as the construction of a tube for a span of about 450 feet to weigh about 1000 tons was the constant object in view, it was more convenient to adapt all the tubes to these dimensions. More minute details of each model will be found in the preceding tables, from which the following is extracted. With respect to the reductions that have been made, we must observe, that the breaking-weight now given is not the actual weight laid on as previously given, but it is increased in each experiment by the addition of half the weight of the tube itself, since the strain at the centre of a beam from its own uniform weight is the same as though half that weight were placed at the centre. In the case of tubes diminishing in thickness towards the ends, six-tenths the weight of the tube has been added.

In reducing the tubes to a length of 450 feet, the breaking-weight has been assumed proportional to the square of the relative dimensions, as previously demonstrated. For example, in the first reduction the breaking-weight of the model was 58·605 tons, the length being 30 feet; hence, for a length of 450 feet, which is 15 times as great, the breaking-weight will be $58{\cdot}605 \times 15^2$ tons $= 13186$ tons. Again, for the actual weight of this enlarged model, since the weight of similar solids is as the cube of their relative dimensions, the weight will be $2{\cdot}13 \times 15^3$ tons $= 7192$ tons. For the purpose of

comparison, the weight of all the tubes is reduced to 1000 tons, the breaking-weight therefore is reduced in the same ratio. The strength of a tube is directly proportional to its sectional area, or the thickness of the plates, consequently any reduction of the sectional area, or weight, will reduce the strength or breaking-weight in the same ratio. Hence, since the weight in the above example is 7192 tons, and the breaking-weight, 13186, we have for the breaking-weight, when the weight of the tube is reduced to 1000 tons, the following analogy:

As 7192 : 1000 : : 13186 : 1833, the breaking-weight of the given tube so reduced in thickness as to weigh 1000 tons.

In the last column of the table, the strength has been assumed to vary as the 1·9 power of the lineal dimensions, as in p. 423, the breaking-weight has been reduced in the ratio of the weight as before, the operation being indicated at the head of the column.

The following table, therefore, furnishes a ready means of comparing the relative advantages of each form of construction. In order to determine the actual weight which these tubes thus enlarged would carry, we have merely to deduct 500 tons, in the case of tubes of uniform section throughout, and 600 tons in the case of tubes reduced in thickness towards the ends.

TABLE XXVI.

Results of Experiments on the Transverse Strength of Rectangular Tubes, each Tube being reduced to the dimensions required for the Britannia Bridge, and to 1000 tons weight.

TUBES MADE AND TRIED IN MANCHESTER.

No. of Experiment.	Distance between the Supports.	Weight of the Tube between the Supports (W).	Depth of the Tube throughout its length.	Breadth of the Tube throughout its length.	Thickness of Top, or compressed Plates of the Tube in middle.	Thickness of Bottom, or extended Plates of the Tube in middle.	Thickness of Plates of each side of the Tube in middle.	Observed Deflections of the Tube in middle.	Breaking Weight (w).	Breaking Weight, including Pressure from Weight W of the Tube (w').	Breaking Weight P of similar Tube, 450 feet long, $P = w' \times \text{length}^2$.	Weight of similar Tube, 450 feet long.	Strength of similar Tube, 1000 tons weight, the strength being reduced in the ratio of the weight, and supposed to vary as the square of the dimensions.	Strength of similar Tube 1000 tons weight, strength supposed to vary as the 1·9 power of dimensions.
										Here $w' = w + \frac{W}{2}$.	Here $P = w' \times 15^2 = 225\ w'$.	Weight $= 15^3 W. = 3375 W.$	Strength $= \frac{P \times 1000}{15^3\ W.}$	Strength $= \frac{P' \times 1000}{15^3\ W.}$
	Ft. in.	Cwt. Tons.	Inches.	Inches.	Inch.	Inch.	Inch.	Inches.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.
13	30 0	42·62 = 2·131	24	15·5	·525	·525	·525	3·03	57·54	58·605	13186	7192	1833	1398
6	30 0	23·09 = 1·154	23·75	15·5	·272	·272	·272	1·53	22·84	23·417	5269	3895	1352	1032
5	30 0	9·76 = ·488	24	15	·124	·124	·124	1·20	5·54	5·784	1301	1647	790	603
										Here $w' = w + \frac{6}{10} W.$				
20	30 0	39 nearly = 1·95	24	16	½	½	½	2·672	54·3	55·470	12481	6581	1896	1446
18	30 0	35·93 = 1·796	24	16½ nearly.	¾	½	¼	2·64	50·48	51·558	11600	6061	1914	1460
19	30 0	36·1 = 1·805	24	16½ nearly.	¾	½	¼	1·58	53·92	55·003	12376	6092	2031	1549
16	30 0	20·46 = 1·023	24	16	⅜	¼	⅛	..	26·1	26·714	6011	3453	1741	1328

										Here $w' = w + \frac{6}{10}W.$	$P = w' \times 10^2$ $= w' \times 100.$	Weight $= 10^3 W.$	Strength $= \frac{P \times 1000}{10^3 W.}$	Strength $= \frac{P' \times 1000.}{10^3 W.}$
23	45 0	130·30 = 6·515	35¼	24	¼	¼	¼	·71 6·78	114·76	118·669	11867	6515	1821	1447
21	45 0	59·77 = 2·988	36	24	$\frac{3}{16}$	⅜	$\frac{3}{16}$	3·10	.. 65·5	.. 67·293	.. 6729	.. 2988	.. 2252	.. 1789
22	45 0	59·77 = 2·988	36	24	$\frac{3}{16}$	⅜	$\frac{3}{16}$	..	62·32	64·113	6411	2988	2145	1704

LARGE TUBE MADE AND TRIED IN LONDON.

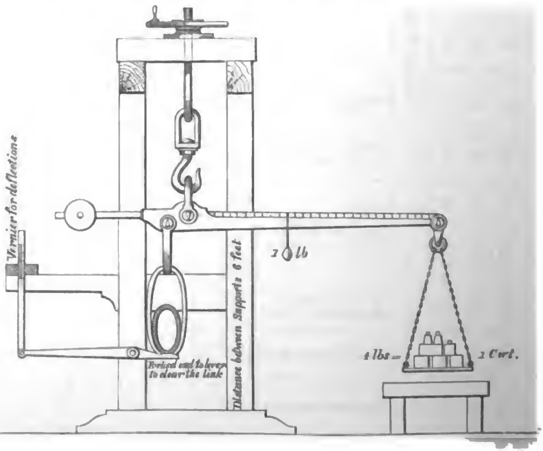
No. of Experiment.	Weight of the Tube.	Distance between the Supports.	Weight of the Tube between the Supports (W).	Depth of the Tube throughout its length.	Breadth of the Tube throughout its length.	Sectional area at top of Tube.	Sectional area at bottom of Tube.	Thickness of Plates of each side of the Tube in Middle.	Observed Deflections of the Tube in Middle.	Breaking Weight (w).	Breaking Weight, including Pressure from Weight, W, of the Tube (w').	Breaking Weight, P, of similar Tube, 450 feet long, $P = w' \times$ length².	Breaking Weight, P', of similar Tube, 450 feet long, if as below.	Weight of similar Tube, 450 feet long.	Strength of similar Tube, 1000 tons weight, the strength being reduced in the ratio of the weight, and supposed to vary as the square of the dimensions.	Strength of similar Tube, 1000 tons weight, strength supposed to vary as the 1·9 power of dimensions.
	Cwt.	Ft. In.	Cwt. Tons.	Inches.	Inches.	Inches.	Inches.	Inch.	Inches.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.
IV.	114·0	75 0	107·594 = 5·380	54	32	24·024	18·3	·1	4·25	66·1	68·93 nearly.	2481·5	2074·4	1162·1	2135	1785

Experiments on the Transverse Strength of Cast-Iron Tubes.

On account of the anomalies occasioned by the buckling of wrought-iron tubes, the following extremely interesting series of experiments was made by Mr. Stephenson, at his own works, on cast-iron tubes. They were superintended with great care by Mr. John Hosking.

The immediate object in view was to test the advantage of different forms of section, the sectional area and weight of material being as nearly as possible the same in each tube, and uniform throughout. They were, moreover, all of the same thickness, viz. three-eighths of an inch. The forms selected are drawn to a scale a quarter size in the following tables, the depth being twice the breadth in the oval and rectangular tubes.

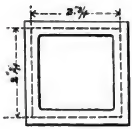
The distance between the supports was exactly 6 feet; they were broken by means of a well-adjusted lever by a link clasping the tube, as in the following sketch.



The tubes were all cast at the same time and of the same metal. The weight was placed gently in the scale first by additions of 7 cwt. up to 35 cwt., then by additions of 1 cwt. up to 40 cwt., then by $\frac{1}{4}$ cwt., and, ultimately, by single pounds, until the tubes failed by the tearing asunder of the bottom.

TABLE XXVII.

Comparative Transverse Strength of Cast-Iron Tubes of different Forms, but of constant Length, Weight, and Thickness.

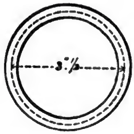
SQUARE  TUBES.

Length, 6 feet; depth and breadth, $2\frac{3}{4}$ inches; *thickness,* $\frac{3}{8}$ *inch.*

Exp.	Weight of Tube.	Weight applied at Centre.	Deflection.	Set.	Remarks.
	Qrs. lbs.	Cwt.	Inch.	Inch.	
31	2 23	7	·10	·10	Breaking-weight, 41 cwt. = 2·05 tons. The bottom rather under the thickness, and a small air hole at one angle.
		14	·205	·025	
		21	·325	·042	
		28	·45	·067	
		35	·595		
32	2 21	7	·095	·009	Breaking-weight, 43 cwt. 0 qrs. 13 lbs. = [illegible] tons. Full in thickness at the bottom, sound and clean.
		14	·21	·017	
		21	·33	·033	
		28	·46	·047	
		35	·605	·077	
		42	·77		
33	2 19	7	·10	[illegible]	Breaking-weight, 41 cwt. = [illegible] tons. Full in thickness at the bottom, fracture irregular.
		14	·21	[illegible]	
		21	[illegible]	[illegible]	
		[illegible]	[illegible]	[illegible]	

[illegible] ght, [illegible] breaking-weight, [illegible]

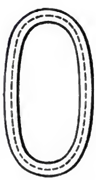
[illegible] ght = [illegible]

ROUND  TUBES.

Length, 6 feet; diameter, 3½ inches; thickness, ⅜ inch.

Exp.	Weight of Tube.	Weight applied at Centre.	Deflection.	Set.	Remarks.
	Qrs. lbs.	Cwt.	Inch.	Inch.	
34	2 23	7	·09		Breaking-weight, 41 cwt. 3 qrs. = 2·0875 tons. Barely ⅜ths at bottom, and slightly blown on one side.
		14	·185		
		21	·28		
		28	·387		
		35	·515		
35	2 24	7	·095	·009	Breaking-weight. 48 cwt. 1 qr. 9 lbs. = 2·416 tons. Rather full in thick.
		14	·185	·021	
		21	·28	·035	
		28	·39	·041	
		35	·515		
		42	·645		
36	2 23	7	·085		Breaking-weight, 47 cwt. 0 qrs. 18 lbs. = 2·358 tons. Sound and regular.
		14	·167		
		21	·25		
		28	·36		
		35	·46		
		42	·60		

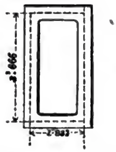
Average weight, 2 qrs. 23 lbs. 5 oz. Average breaking-weight, 2·287 tons; besides half its own weight = ·017.

OVAL  TUBES.

Length, 6 feet; depth, 4·66 inches; breadth, 2·33 inches; thickness, ⅜ inch.

Exp.	Weight of Tube.	Weight applied at Centre.	Deflection.	Set.	Remarks.
	Qrs. lbs.	Cwt.	Inch.	Inch.	
37	2 18	7	·07	·303	Breaking-weight, after a few seconds, 60 cwt. = 3·6 tons.
		14	·12	·008	
		21	·18	·021	
		28	·25	·030	Fracture rather unsound, seedy, and pinholed.
		35	·32	·042	
		42	·39	·057	
		49	·47	·070	
		56	·55		
		59	·59		
38	3 13	7	·07		Breaking-weight, 63 cwt. 2 qrs. 13 lbs. = 3·18 tons. Rather light in weight.
		14	·125		
		21	·205		
		28	·285		
		35	·372	·037	
		42	·465		
		49	·567		
		56	·69		
		63	·82		
39	2 18	7	·07	·009	Breaking-weight, 58 cwt. 1 qr. 12 lbs. = 2·918 tons.
		14	·14	·020	
		21	·21	·036	
		28	·28	·044	Fracture rather unsound, and pinholed, from the metal being poured too hot.
		35	·365	·055	
		42	·445	·070	
		49	·532	·088	
		56	·63		
		58			
40	3 2	7	·057	·002	Breaking-weight, 68 cwt. 1 qr. = 3·412 tons.
		14	·117		
		21	·182		
		28	·25		A little heavy and unequal in thickness at the sides. Bottom thickness right.
		35	·32	·047	
		42	·395		
		49	·47		
		56	·555		
		63	·64		
41	3 2	28	·245	·036	Breaking-weight, 64 cwt. = 3·2 tons.
		42	·395		Same as the last.
		56	·56	·092	
		63	·645		
42	3 5	28	·215		Breaking-weight, 71 cwt. 3 qrs. 18 lbs. = 3·595 tons.
		42	·345		Same as the last.
		56	·50	·07	
		63	·58		
		70	·68	·11	

Average weight, 2 qrs. 23 lbs. 11 oz. Average breaking-weight, 3·207 tons; besides half its own weight = ·018.

RECTANGULAR  TUBES.

Length, 6 feet; depth, 3·666 inches; breadth, 1·833 inches; thickness, $\frac{3}{8}$ inch.

Exp.	Weight of Tube.	Weight applied at Centre.	Deflection.	Set.	Remarks.
	Qrs. lbs.	Cwt.	Inch.	Inch.	
43	2 16	7	·075		
		14	·16		Breaking-weight, 44 cwt. = 2·2 tons.
		21	·265		
		28	·365		Tube sound and clear.
		35	·467	·046	
		42	·602		
44	2 18	7	·082		Breaking-weight, 45 cwt. 23 lbs. = 2·26 tons.
		14	·167		
		21	·26		
		28	·36		Sound and clear.
		35	·47	·035	
		42	·59		
45	2 17	7	·065	·0015	Breaking-weight, 49cwt. = 2·45 tons. Fracture sound, and clear.
		14	·14	·006	
		21	·215	·012	Tube broke on replacing the weight after taking the set.
		28	·305	·023	
		35	·4	·035	
		42	·5	·053	
		49	·615	·0825	
46	2 18	7	·065		Breaking-weight, 46 cwt. 2 qrs. 17 lbs. = 2·333 tons.
		14	·15		
		21	·225	·0125	
		28	·31	·0325	
		35	·405	·047	
		42	·505	·0715	Highly defective at the bottom.
		46	·57		

Average weight, 2 qrs. 17 lbs. Average breaking-weight. 2·3 tons; besides half its own weight = ·016.

We have, therefore, the mean breaking-weight of cast-iron tubes of different forms, but of similar weight and thickness, 6 feet long, as follows:

	Tons.	Depth. Inches.	
Square Tubes	2·152	2·75	Mean sectional area, 4·12 sq. inches.
Round Tubes	2·287	3·5	
Rectangular Tubes ..	2·3	3·66	
Elliptical Tubes	3·207	4·66	

Thus, with the same quantity of material, in the forms above described, the elliptical tube is considerably the strongest; and if these tubes were respectively enlarged to nearly sufficient dimensions for the Britannia Bridge, we should have as follows:

	Weight of Tube itself, 450 feet long.	Weight such a Tube would support at the centre, besides its own weight.	
	Tons.	Tons.	
Square Tube, 17·2 feet square	14502	4950	28 inches thickness. Sectional area, 23175 inches.
Round Tube, 21·9 feet diameter..	14941	5490	
Rectangular Tube, 22·9 feet deep .... 11·5 feet wide	13749	6153	
Elliptical Tube, 29·1 feet deep .... 14·6 feet wide	15010	10636	

It will be observed that the increase of the depth in the various models is in the order of the increase of strength. Now, with a constant length and sectional area, if there were no advantage in any particular form, the strengths should be precisely in the ratio of the depths. In order, therefore, to compare these tubes together as regards form, the depth must be taken into account. For this purpose it will be more simple to find a constant for each form of tube from the formula, $W = \frac{a\,d}{l}\,c$, or $c = \frac{W\,l}{a\,d}$.

The value of c, thus obtained, gives the relative advantage of each particular form of section; and we have—

For rectangular tubes,	$c = 10{\cdot}96$
For round tubes,	$c = 11{\cdot}42$
For elliptical tubes,	$c = 12{\cdot}01$
For square tubes,	$c = 13{\cdot}67$

It would thus appear with cast-iron tubes broken transversely, that with respect to form of section, when the tubes are taken of the same depth and sectional area, the square form is the strongest, and the rectangular tube the weakest; and the above will be found useful practical constants in estimating their respective strengths.

These experiments confirm a useful relation between the strength of tubes and solid bars of similar section, viz., that the strength is simply as the depth when the section and length are constant.

Transverse Strength of Wrought-Iron Welded Tubes without Rivets.

For direct comparison with the experiments on cast-iron tubes the following experiments on the importance of form in wrought-iron tubes, were made for Mr. Stephenson by Mr. John Hosking.

In this case the tubes were identical in everything except form. The round tube was made by Messrs. Russell and Co., by their patent process of welding. The diameter was exactly 4 inches, and the thickness $\frac{3}{16}$ths. The rectangular and oval form were made from the round tube with which they are compared, by hammering them when heated, with wooden hammers on a prepared mandril, care being taken not to stretch the iron. When finished they were all put in a furnace together, and heated gently to a dull red colour, and allowed gradually to cool. No weld could be discovered either by heating or hammering. The angles of the rectangle were slightly rounded to prevent injury to the iron.

A saddle 6 inches broad was placed on the centre of the tube, and from this the breaking-weight was suspended. The length between bearings was 6 feet. The weights were laid on quietly and by small additions as the breaking-weight was approached.

TABLE XXVIII.

	Experiment 47. Round Tube.		Experiment 48. Rectangular Tube.		Experiment 49. Oval Tube.	
						
Weight. Tons.	Deflection. Inch.	Set. Inch.	Deflection. Inches.	Set. Inch.	Deflection. Inches.	Set. Inch.
·35	·067	·004	·065	·005	·065	·0045
·7	·14	·0095	·12	·009	·122	·009
1·05	·215	·016	·177	·0135	·187	·013
1·4	·285	·0245	·232	·0205	·245	·018
1·45	·3 ..	..	·25 ..	..	·255	
1·5	·31 ..	..	255 ..	..	·262	
1·55	·32 ..	..	·265 ..	..	·27	
1·6	·335 ..	..	·272 ..	..	·277	
1·65	·355 ..	..	·28 ..	..	·285	
1·70	·367 ..	..	·29 ..	..	·295	
1·75	·39 ..	·0655	3 ..	..	·305	·0345
1·8	·442 ..	..	·31 ..	..	·317	
1·85	·567 ..	..	·32 ..	..	·327	
1·9	·845 ..	..	·33 ..	..	·34	
1·95	.. ..	..	·345 ..	..	·36	
2·0	.. ..	..	·36 ..	..	·37	
2·05	.. ..	..	·38 ..	..	·38	
2·1	.. ..	..	·405 ..	..	392	
2·15	.. ..	..	·43 ..	·0865	·41	·07
2·2	.. ..	..	·485 ..	..	·435	
2·25	.. ..	..	·54 ..	..	·457	
2·3	.. ..	..	·625 ..	..	·482	
2·35	.. ..	..	·705 ..	..	·522	
2·4	.. ..	..	·9 ..	..	·58	
2·45	.. ..	..	1·085 ..	..	·65	
2·5	.. ..	..	1·45 ..	..	·73	
2·55	.. ..	..	.. ..	..	·87	
2·6	.. ..	..	.. ..	..	1·035	
2·65	.. ..	..	2·2 ..	..	1·23	
	Tube failed suddenly with 2·6 tons.		Tube failed with 3·15 tons as before.		Tube failed with 3·46 tons.	

The tubes all gave way by the compressed side becoming first distorted. In the round and oval ones the sides were forced outwards, and the tubes became flattened at the centre of pressure. In the rectangular tube one side buckled inwards, and the tube yielded sideways in a corresponding direction, and became very much twisted. No apparent derangement of the tubes could be detected near the ends or points of bearing.

The section being constant, the deeper tube bears the greater weight, as might be expected, and to compare the relative advantage of form it will be necessary, as before, to eliminate the depth by deriving a constant for each form from the formula—

$$c = \frac{W\,l}{a\,d},$$

and, taking the depth, length, and sectional area in inches, the sectional area being 2·2457 square inches, we have—

		Tons.
Round tube	$c =$	20·916
Oval tube	$c =$	22·251
Rectangular tube........	$c =$	23·531

which gives the relative value of form independent of depth. These values are *in the same order* as those obtained in the preliminary experiments from tubes constructed of riveted plates, where we have (*see* pp. 114, 126)

		Tons.
For round tubes	$c =$	13·03
For oval tubes	$c =$	15·3
For rectangular tubes	$c =$	18·07

But not only are these values absolutely greater, but the relative difference between each form is materially lessened in the welded tubes, on account of the increased thickness of the plates and the absence of rivets.

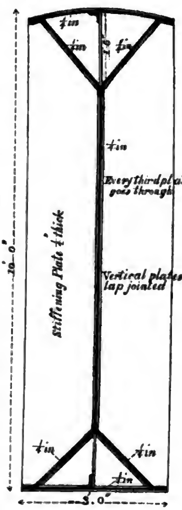
If we refer to page 193, we find the value of c to be

26·7 tons in the large model. This model with its thin sides was not, therefore, 14 per cent stronger than the rectangular tube, Experiment 48, in which the top, bottom, and sides were all of the same thickness.

EXPERIMENTS ON THE TRANSVERSE STRENGTH OF A WROUGHT-IRON GIRDER.

One of the most important advantages in the use of wrought-iron for girders arises from the security with which any moderate change of form may take place. Beams may, therefore, be safely constructed of much greater rigidity or depth than in cast-iron; and since the strength of a beam is as its depth when the area is constant, while the weight is in a much less ratio, this characteristic is of most important practical value. The following experiment illustrates the advantage that may be *practically* derived from great depth with this material. The details have been most kindly supplied by Mr. Brunel, by whom this magnificent experiment was made.

The figure will illustrate the novel section adopted in this girder:—

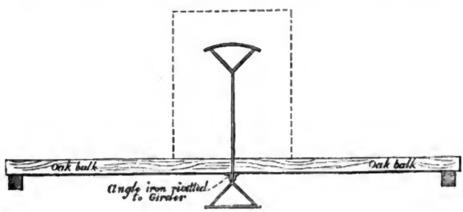


The thickness of the plates was one-quarter inch throughout; the total depth was 10 feet at the centre, and 6 feet at the ends, and the distance between the bearings, 66 feet. The top plate is slightly curved to resist any tendency to buckle. The total sectional area of each of the triangular sections, which may be considered the top and bottom flanges of

the beam, is 25 square inches. The connecting vertical rib consists of single $\frac{1}{4}$-inch plates, 4 feet broad, every third plate only extending the whole depth of the girder. The vertical rib is thus 7 feet deep, and is the most remarkable part of the girder, affording an interesting confirmation of the small quantity of material that is absolutely requisite in this element of a  girder. This thin plate is preserved in shape by two stiffening plates, placed 15 feet apart, at the centre of the girder, and at the same width as the top and bottom plates, as shewn in the section. Two similar stiffening plates, with additional side-plates, occur also at the extremities over the bearings. The vertical plates were lap-jointed. The bottom horizontal plates were connected by covers, with two rows of three-quarter rivets, arranged zig-zag, with 10 rivets in the front row. The bottom inclined plates were similarly connected with 5 rivets in the front row.

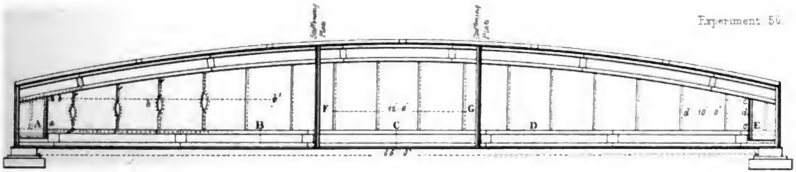
The weights were laid on this girder, as in the sketch below, by means of balks resting at one extremity on angle-iron riveted to the girder, and at the other on a platform independent of the girder, allowance being made for the leverage.

The dotted lines shew the position occupied by the load.

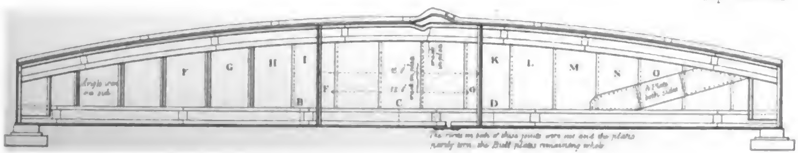


EXPERIMENTS BY Mr BRUNEL

Experiment 50.



Experiment 51



Experiment 50.

The following weights were laid on distributed over a distance of 12 feet 6 inches.

Actual Reduced Weight.	Central Deflection.	Mean Deflection at 12 ft. 6 in. on either side of centre.	Remarks.
Tons.	Inches.	Inches.	
10	·031	·023	
20	·125	·062	
30	·156	·125	
40	·187	·187	
50	·312	·281	50 tons were placed wholly on one side.
60	·343	·281	Commenced loading the other side.
70	·437	·312	
80	·468	·343	
90	·5	·406	
100	·625	·468	50 tons on each side of the girder, *c c* extended $\frac{1}{64}$ inch.
110	·75	·593	Vertical rib beginning to buckle between the bearings and stiffening plates.
120	·812	·656	Buckle increasing, *a a* shortened by buckle $\frac{1}{64}$ inch, *d d* extended $\frac{1}{32}$ inch. (See Plate.)
130	·875	·781	Buckle further increasing in the vertical rib, as above.
140	1·062	·843	Buckle in vertical rib as much as $\frac{3}{8}$ inch on either side centre line.
140	1·166	·812	Load remaining on 2 days.
150	1·125	·968	Buckle fast increasing, *a a* shortened $\frac{1}{32}$ inch, *d d* extended $\frac{1}{16}$ inch, vertical rib at centre not at all buckled; but undulation increasing near the ends
160	1·25	1·125	*a a*, shortened $\frac{1}{16}$ inch, *d d* extended $\frac{1}{8}$ inch. The top remained uninjured.
165	..	..	Girder failed by the collapse and tearing asunder of the vertical rib *b b b′*, the rivets being sheared and the plates torn, but no other part of the girder was damaged.

Experiment 51.

The girder was now repaired, and the vertical rib stiffened by the addition of angle-iron pillars at each joint in the vertical plates throughout the part that had failed; the vertical rib

at the other extremity was strengthened by two diagonal three-sixteenth plates, riveted on either side. (*See* Plate.)

The following weights were then laid on as before.

Actual Reduced Weight.	Central Deflection.	Mean Deflection at 12 ft. 6 in. from the centre.	Remarks.
Tons.	Inches.	Inches.	
10			
20	·093	·125	
30	·093	·156	
40	·187	·187	
50	·312	·343	50 tons all on one side.
60	·187	·343	
70	·437	·437	
80	·437	·437	
90	·5	·531	
100	·625	·562	50 tons on each side; commencement of buckle at plate *k*.
110	·687	·687	
120	·75	·781	
130	·875	·843	The plates *l m* began buckling.
140	·875	·937	The plate *i* began buckling.
150	·1	·984	The plates *g h* began buckling, 28 tons excess on one side.
160	1·062	1·125	Buckle of the above plates increasing, 38 tons excess on one side.
170	1·312	1·281	Buckle at *f*.
180	1·5	1·25	Plates *n o* slightly buckled.
188	..	..	Bottom plates torn asunder at a joint, the top web also crushing as in the sketch. The fracture was sudden, and no permanent buckle was left in the side-plates.

The constant, from the formula $c = \frac{W\,l}{a\,d}$, would, for this girder, be about 55 tons, and the strain per square inch in the bottom and top would be 13·7 tons at the time of failure. The bottom plates, in the large model tube before described, tore asunder with 16 tons per square inch, exclusive of rivets, but the iron of the latter was of unusually excellent quality;

perhaps a safe practical allowance for the ultimate strength of the riveted bottom of such girders would be 14 tons, in which case the constant would be 56 tons.

This experiment is the more interesting as the top and bottom failed simultaneously with similar sectional areas, so that the greater resistance of wrought-iron to compression is about counterbalanced by the weakening of the bottom by the rivets.

The resistance of the triangular cell with the curved top to buckling was most satisfactory when compared with cells of other form in Table IX., and the benefit of increased depth was effectually obtained by this light vertical rib with remarkable economy of material. This rib shewed no strain at the middle of the beam, the strain in the sides of tubes being nothing at the centre, and at a maximum at each extremity.

Experiments on the Transverse Strength of Cast-iron Bars.

We have seen that with cast-iron the permanent set is proportional to the square of the tension or compression. It would also follow, that the permanent set in a cast-iron beam, bent transversely, should be similarly proportional to the square of its deflection; and in order to test this experimentally, the deflection and set of five bars of Blaenavon iron No. 2, resting on friction rollers 13 feet 6 inches asunder, and $3 \times 1\frac{1}{2}$ inches in section, were most carefully observed, the bars were bent horizontally, and in the direction of their least dimension.

The mean breaking-weight of all the bars was 819 lbs., and the mean ultimate deflection 10·46 inches. The following process was employed by Mr. Hodgkinson to determine the power n of the dimensions to which the set was proportional.

Let D, d = the mean deflections from any two weights.

D^1, d^1 = the mean sets corresponding to those deflections.

n = the power assumed constant.

Then $$D^n : d^n :: D^1 : d^1$$

Or $$\left(\frac{D}{d}\right)^n = \frac{D^1}{d^1}$$

And $$n \log. \frac{D}{d} = \log. \frac{D^1}{d^1}$$

$$\therefore\ n = \frac{\log. \frac{D^1}{d^1}}{\log. \frac{D}{d}}$$

And taking the mean deflections and sets from various weights, the mean value of n was 1·92, or nearly 2. The set was therefore computed on this supposition, and compared with the observed set for every weight.

The mean value of the ratio $\frac{d^2}{d^1}$ was found to be 31·5; and in computing the set for any weight the formula employed was therefore

$$d^1, \text{ or permanent set} = \frac{d^2}{31 \cdot 5}.$$

The deflections in these experiments were in inches and the weights applied in lbs.

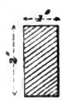
The deflection of cast-iron beams bent transversely was found, moreover, not proportional to the weight or load, 10 tons on a beam causing more than ten times the deflection due to a single ton, in accordance with what we have seen in speaking of its compressions and extensions longitudinally.

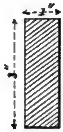
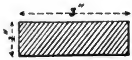
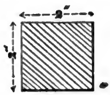
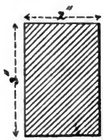
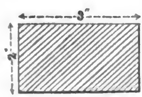
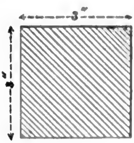
The ultimate strength of a number of bars varying considerably in dimensions was ascertained by Captain James, R.E., F.R.S.; and it was found that as the bars become larger the spongy crystalline texture of the central portion modifies the strength very considerably; and also that bars planed down from the centre of larger bars are comparatively very weak. Hence the strength of large beams cannot be com-

puted from the constants usually given by experimenters, derived generally from bars only 1 inch in section and 4 feet 6 inches long. The advantage of running metal over again for large castings, which is commonly the practice with founders, was also clearly shewn.

On the assumption that the strength of a rectangular bar varies directly as its area of section and depth, and inversely as its length, the value of c in the ordinary formula $W = \frac{a\,d^2}{l}\,c$ has been found by Barlow from inch bars to be equal to 30480 lbs. = 13·6 tons nearly, and 12 tons has been usually employed by practical men; so that a bar 1 square inch in section and 1 foot long has been assumed to carry 1 ton. In the following table the actual breaking-weight is placed opposite the breaking-weight calculated from the formula $W = \frac{a\,d^2}{l}$ 30480 lbs.; and it will be seen that in the larger castings there is a very great falling off from this strength.

Table XXIX.

Length of Bar. Ft.	In.	Section.	Iron.	Computed Breaking Weight. Lbs.	Mean Actual Breaking Weight. Lbs.
4	6		No. 3, Clyde	564	567
2	3	Ditto	Ditto	1129	1136
18	0		No. 2, Blaenavon	564	1218 572
13	6	Ditto	Ditto	752	603
9	0	Ditto			
4	6	Ditto			
			No, 3, Clyde	1129	780
			Ditto	2258	2113
2	3		Ditto	2258	2234

Length of Bar.		Section.	Iron.	Computed Breaking Weight.	Mean Actual Breaking Weight.
Ft.	In.			Lbs.	Lbs.
18	0		No. 2, Blaenavon	1270	961
13	6	Ditto	Ditto	1693	1287
9	0	Ditto	Ditto	2540	2008
4	6	Ditto	No. 3, Clyde	5080	4384
2	3		Ditto	3386	3084
13	6		Ditto	1505	1065
9	0	Ditto	Ditto	2258	1842
4	6	Ditto	Ditto	4515	3586
9	0		Ditto	5080	4034
4	6		Ditto	6773	5396
13	6		Ditto	5080	3217
6	9	Ditto	Ditto	10160	6731
Ditto		Ditto	Ditto, recast	10160	6949

The average breaking-weight of bars, 1 inch square and 3 feet long, as determined from a very extensive series of experiments by Mr. Robert Stephenson (*see* Government Report on Iron, page 390, *et seq.*), gives

Iron.	Ultimate Deflection.	Breaking Weight.
	Inch.	Lbs.
Hot Blast	·789	826
Cold Blast	·784	855
Mixtures of various Irons	——	898

The breaking-weight derived from these, for a bar 1 inch square and 1 inch long, would be—

	Tons.
Hot Blast	13·27
Cold Blast	13·29
Mixed Irons	14·43

the mean being 13·6, and agreeing exactly with Barlow.

But so great is the falling off with larger bars, that even with bars 3 inches square, as in the three last experiments in the table, the mean constant is only 9 tons; so that approximately for the strength of rectangular bars not exceeding 1 square inch of section, we have,

$$\text{Breaking-weight} = \frac{\text{sectional area} \times \text{depth}}{\text{length}}\ 13{\cdot}6 \text{ tons.}$$

And for larger bars the constant increases, so that for bars about 3 inches square we have,

$$\text{Breaking-weight} = \frac{\text{sectional area} \times \text{depth}}{\text{length}}\ 9 \text{ tons.}$$

The dimensions being all in inches.

[illegible] average [illegible] ent that the mean
long, as determined [illegible] -iron 1 inch long is
ments by Mr. [illegible] it is therefore 1·13
t on Iron, page [illegible] of bars 3 feet long
[illegible] onstant the deflection
[illegible] eight, and inversely as
[illegible] ate deflection of a cast-
[illegible] efore be $\frac{3}{27} \times \cdot 78 = \cdot 086$

[illegible]

[illegible] deflection of the wrought-
[illegible] fore, we estimate the useful
[illegible] tion, the wrought-iron bars
The [illegible] cast-iron, and hence the
uare and [illegible] wrought-iron is very little
[illegible] rs. But the cast-iron was
[illegible] ught-iron carried more than
[illegible] sufficiently bent to fall
[illegible] deflection is nearly propor-
[illegible] to this limit. Hence the
he [illegible] for the ultimate strength of
But [illegible] we define the limit to which
with [illegible]
in the [illegible]
[illegible]

[illegible] 53.

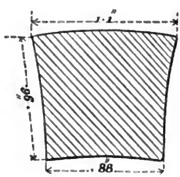
[illegible] feet 6 inches long, with
[illegible] then broken. This
[illegible] rucks, and was some-
[illegible] the constant ham-
[illegible] nly cracked on the
[illegible] deflections being

Transverse Strength of Rectangular Bars of Wrought-Iron.

To determine a constant for the strength of wrought-iron bars, three ordinary bars, 16 inches long, 1 inch square, with 12 inches clear between the bearings, were broken by weight suspended from the centre. The weight was increased gradually. The experiments were made at the Britannia Bridge, and the mean of the three, which corresponded very closely with each other, was as follows:—

EXPERIMENT 52.

Weight suspended.	Deflection.	Remarks.
Tons.	Inches.	
·22	003	The experiment in each case terminated by the bar slipping through between the supports when the deflection amounted to $2\frac{1}{4}$ inches.
·342	·010	
·465	·013	
·588	·023	
·711	·026	
·835	·046	
·958	·056	
1·057	·066	
1·177	·090	The centre section was carefully guaged after each experiment, and the upsetting of the iron was evident, as in the following central section, which is a mean from the three measurements.
1·300	·015	
1·424	·025	
1·547	·039	
1·670	·064	
1·833	·084	
1·948	1·05	
2·100	1·44	
2·200	1·83	
2·330	2·26	



Moreover the bar on the top, or compressed side, was shortened $\frac{10}{16}$ths of an inch, and was increased in length, on the extended side, $\frac{13}{16}$ths of an inch.

We have seen in the last experiment that the mean breaking-weight of an inch bar of cast-iron 1 inch long is 13·6 tons, and for a bar 12 inches long it is therefore 1·13 tons. Moreover, the ultimate deflection of bars 3 feet long was ·78 inch, and the section being constant the deflection of a bar will vary directly as the weight, and inversely as the cube of the length. The ultimate deflection of a cast-iron bar 12 inches long would therefore be $\frac{3}{27} \times \cdot 78 = \cdot 086$ inch.

Now, with this same weight the deflection of the wrought-iron bars was ·080 inch. If, therefore, we estimate the useful strength by the amount of deflection, the wrought-iron bars appear but little stronger than cast-iron, and hence the usual constant for the strength of wrought-iron is very little greater than for cast-iron bars. But the cast-iron was actually broken, whereas the wrought-iron carried more than double this weight before it was sufficiently bent to fall through the bearings, and its deflection is nearly proportionate to the weight close up to this limit. Hence the difficulty of giving any constant for the ultimate strength of this material in this form, unless we define the limit to which its deflection may be carried.

Experiment 53.

A bar 1½ inch square, and 4 feet 6 inches long, with 3 feet clear between the bearings, was then broken. This bar had been used as a rail for the trucks, and was somewhat damaged and altered in texture by the constant hammering of the wheels, and it consequently cracked on the lower side with 1 ton 18 cwt. 2 qrs., its deflections being as follows:—

Cwt.	Inch.
12	·12
19¼	·27
24	·35
31¼	·82
36	1·70
38½	2·38

The bar cracked suddenly, and continued to support the weight sixteen hours, the crack having considerably increased. The weight of the bar was 33 lbs.

Experiment 54.

A plate of iron 3½ inches deep, and 1½ inch thick, 7 feet long, and 4 feet 6 inches between the bearings, was kept vertical by angle-iron riveted over the bearings. It failed, however, by twisting sideways with 1 ton 16 cwt.

The deflection was as follows:—

Cwt.	Inch.	Cwt.	Inch.
3	·01	13	·14
4	·02	14	·15
5	·04	15¼	·19
6	·06	16¼	·21
7	·08	17¼	·22
8	·095	18¼	·23
9	·1	19¼	·24
10	·11	Increased in 16 hours	·27
11	·13	31	·33
12	·14	36	1·2

The plate buckled sideways, and the weights were removed.

Experiment 55.

Transverse Strength of Bars previously bent and straightened.

It is evident from these experiments that the practical use of wrought-iron bars is limited, not by their absolute strength, but by the amount of deflection which may be allowed in any particular application of this material. We have seen in a previous chapter, that as we increase the permanent set of wrought-iron, we diminish the subsequent extension and compression from any given load; and we have alluded to the fact that the tubes would have deflected less from any given load if the top and bottom had been previously compressed and extended by any artificial strain. It follows from this consideration, that if the compressed and extended portions of a wrought-iron bar could by any artificial means be permanently strained previous to its employment as a beam, that such a beam would deflect less than a new bar, and would be practically a stronger beam, since the strength is regulated solely by the bending of the bar.

In order to test this result the following experiments were made at the Britannia Bridge.

Four bars of wrought-iron, similar to the bar in Exp. 53, viz., $1\frac{1}{2}$ inch square, 4 feet 6 inches long, 3 feet between the bearings, and weighing 33 lbs. each, were thus prepared. The four bars were placed in an air-furnace until they attained a dull red heat; in this state two of them were arched, or curved, 3 inches; they were bent with a wooden mallet, so that the metal was not upset by the hammering; the two other bars remained straight, and thus the four bars were allowed gradually to cool.

The curved bars, when cold, were straightened; and being placed on the supports in the position in which they were straightened, they were loaded at the centre for comparison

with the remaining bars, which remained as they came from the furnace.

The result deserves attention, as being of constant practical application. It furnishes, moreover, a confirmation of the views explained in a previous chapter on the nature of permanent set arising from strain, and satisfactorily accounts for the many anomalies which characterise the conclusions arrived at by different authors from experiments on the elasticity of materials, in which the effect of previous strain has been overlooked.

Weight in Cwt.	Two new Wrought-Iron Bars, 1¼ in. square, 3 ft. between bearings.			Two Bars precisely similar, previously bent and straightened.		
	Deflection. Bar 1.	Deflection. Bar 2.	Mean Deflection.	Deflection. Bar 1.	Deflection. Bar 2.	Mean Deflection.
2·6				·01	·01	·01
3·73	·02	·05	·035	·03	·02	·025
5·42	·04	·06	·05	·05	·03	·04
7·1	·06	·08	·07	·06	·04	·05
8·8	·08	·09	·085	·08	·06	·07
10·48	·11	·11	·11	·1	·08	·09
12·16	·13	·13	·13	·12	·09	·105
13·85	·14	·15	·145	·14	·11	·125
15·5	·16	·17	·165	·16	·13	·145
17·2	·18	·19	·185	·17	·15	·16
18·9	·21	·2	·205	·18	·16	·17
20·6	·23	·22	·225	·19	·17	·18
22·3	·25	·25	·25	·21	·19	·2
23·9	·29	·29	·29	·24	·21	·225
25·5	·32	·39	·355	·26	·23	·245
27·1	·44	·71	·575	·28	·25	·265
28·7	·63	1·01	·82	·31	·27	·29
30·3	·95	1·55	1·25	·33	.29	·31
31·9	1·44	1·73	1·585	·37	·32	·345
33·9	1·92	2·3	2·11	·4	·35	·375
35·9	2·34	2·75	2·545	·45	·4	·425
37·9	2·96	3·5	3·23	·53	·47	·5
39·9	3·75	4·15	3·95	·63	·56	·595
41·9	4·61	5·68	5·145	·81	·77	·79
43·9				1·4	1·31	1·355
44·9				1·53	1·45	1·49
46·5				1·57	1·96 Signs of cracking.	1·765

Thus, if a new bar, $1\frac{1}{2}$ inch square, were used as a beam where no greater deflection than $\frac{8}{10}$ths of an inch could be allowed, the greatest load such a bar would bear would be 28·7 cwt.; but the bar previously strained would require 42 cwt. to deflect it the same amount, while with this weight the new bar would be bent upwards of 5 inches. The strained bar would, therefore, under such conditions, be 46 per cent stronger than the new bar, and as the weight increases the increase of strength becomes still more remarkable. In fact, as regards deflection, the strained beam may be considered a new material, of which the elasticity is quite different from that of the original beam, and this important change in the practical value of the bars is obtained without difficulty or expense.

The ultimate strength of a 3-feet bar of cast-iron, $1\frac{1}{2}$ inch square, would be 25·5 cwt., and the ultimate deflection ·29 inch. And with this same deflection the new bar would only bear 24 cwt., and the strained bar 28·8 cwt.

If we derive constants from the formula $c = \frac{l\,W}{a\,d}$, assuming the above to be the greatest deflection that can be admitted, we have for the new bar—

$$W = \frac{a\,d}{l}\ 15{\cdot}3 \text{ tons},$$

for the strained bar—

$$W = \frac{a\,d}{l}\ 22{\cdot}3 \text{ tons},$$

all the dimensions being in inches.

It has been usual to assume that wrought-iron would bear a certain strain without any ascertainable permanent set or injury, and the practical strength has been derived from this assumption. We have seen that such a limit is founded on erroneous assumptions, and its practical use may be safely much extended on these grounds, as well as on account of the

remarkable uniformity of its strength, and the gradual character of its fracture.

Experiment 56.

Transverse Strength of a Slab of Slate from the Penrhyn Quarries.

A slab of slate, 2 feet 10 inches broad, 4 inches thick, and 4 feet between the bearings, failed with 24½ tons distributed over 15 inches at the centre of the span.

A slab of cast-iron of the same dimensions would scarcely support five times as much, and would be above two and a half times as heavy. This material forms a valuable flooring for bridges.

Transverse Strength of Timber.

To avoid any anomalies in deducing the strength of large beams of timber from experiments on small battens, the following experiments were made on the transverse strength of whole balks of American red pine timber selected from the scaffolding employed in constructing the tubes.

These beams were exactly 12 inches square and 17 feet long, the distance between the bearings being 15 feet. They were broken by actual weight suspended on a scale from the centre of the beams.

Experiment 57.

Dry Timber from the butt end of the balk.
Weight of the beam, 5 cwt. 2 qrs. 5 lbs., or 36·5 lbs. per cubic foot.
Breaking-weight, 14·82 tons.

Experiment 58.

Dry timber from the top of the balk.
Weight of the beam, 5 cwt. 17 lbs., or 33·9 lbs. per cubic foot.
Breaking-weight, 13·24 tons.

Experiment 57.		Experiment 58.	
Weight.	Deflection.	Weight.	Deflection.
Cwt.	Inches.	Cwt.	Inches.
15·11	·05	16·13	·1
32·62	·15	37·22	·2
51·92	·25	48·57	·25
74·77	·40	75·25	·5
91·1	·50	85·75	·57
136·04	·77	136·9	1·0
156·06	·90	158·8	1·1
200·75	1·2	204·	1·52
217·85	1·37	215·03	1·67
236·85	1·5	237·5	2·0
253·32	1·7	250·5	2·25
273·31	2·2	257·25	2·5
282·6	2·7	gradually sinking.	
292·93	3·3	264·71	2·97
294·6	3·45	¼ min. before breaking.	3·1
296·32	4·0		

The nature of the fracture is accurately represented in the accompanying Plate.

The mean breaking-weight of these two balks was therefore 14 tons, and from the formula $c = \frac{l\,W}{a\,d}$, we have $c = 1·45$ ton; or, for the breaking-weight of any beam of such timber, we have $W = \frac{a\,d}{l}$, 1·45 ton, the dimensions being all in inches.

Experiment 59.

Similar timber to the last.

Red American deal from the centre of the balk.

Beam 6 inches square, 7 feet 6 inches between the supports.

Weight.	Deflection.	Weight.	Deflection.
Cwt.	Inch.	Cwt.	Inches.
5·9	·1	35·4	·55
11·7	·18	47·5	·79
17·5	·28	50·04	·85
24·03	·37	56·384	1·08
29·1	·45	65·785	1·35
			1·68

This beam failed with 3·289 tons in the same manner as the last, viz. by the tearing asunder of the bottom. Its strength from the constant derived from the mean of the two last experiments would be $W = \frac{36 \times 6}{90} 1{\cdot}45$ ton $= 3{\cdot}48$ tons, instead of 3·289, as above.

It will be observed, that in the foregoing experiments the deflection increases in a higher ratio than the weight as the breaking-weight is approached. The mean deflection per ton, as long as the deflection continued regular in Experiments 57, 58, was ·13 inch. Hence from the formula

$$C = \frac{\delta\, b d^3}{l^3\, W} \text{ we have } C = {\cdot}000462.$$

And for the deflection of any other balk of similar timber

$$\delta = \frac{l^3\, W}{b\, d^3}\, {\cdot}000462.$$

The deflection per ton in Experiment 59 should therefore be,

$$\delta = \frac{90^3 + 1}{6 + 6^3}\, {\cdot}000462 = {\cdot}26 \text{ inch nearly.}$$

It was rather greater than this quantity, viz. ·30 inch.

Experiments on the Resistance of Beams to Impact.

In such an extension of the theory of the beams as was involved in the construction of these bridges, it became imperative to inquire into every property of such structures, lest any phenomenon, hitherto unimportant in ordinary beams, should now rapidly rise into importance, and increase in some high ratio of the magnitude, the effect of isochronous vibration from wind or other causes, and the impact of trains in rapid motion, were always foremost among the theoretical apparitions that haunted the early history of the bridges.

Mr. Stephenson attached, however, little importance to these considerations, depending on the great weight of the structure itself for obviating any danger from impact, and on the fitful nature of gusts of wind, as affording no apprehension of continued isochronous motion. During the violent gales of February last, the heaviest that have occurred for many years, the tubes were but little affected, although one of them was resting at each end only on a pile of loose planks, and at an elevation of 100 feet, and was neither connected, laterally nor longitudinally, with the neighbouring tubes, which must nearly quadruple its lateral strength; its lateral motion amounted, under these circumstances, to about $1\frac{1}{2}$ inches. The blow struck by the gale was not simultaneous throughout the length of the tube, but impinged locally and at unequal intervals on all parts of the length which presented a broadside to the gale. It was impracticable to pass along the top of the tube, except by clinging to the windward edge; and even in this position the fitful nature of the gusts was disagreeably perplexing. The gale was diverted from its horizontal course, and descending obliquely into the water below, ploughed it up in clouds of spray for some distance from the tube. The maximum vibration did not occur during the greatest violence of the wind, but at the momentary lulls, when the tube, partially returning to its normal shape from its own elasticity, was again met by the succeeding wave. The tube, however, on no occasion attained any serious oscillation, but appeared to some extent permanently sustained in a state of lateral deflection, without time to oscillate in the opposite direction.

The impact from the passage of an ordinary train must, of course, be incomparable in effect with the blow of such a hurricane on a surface of 13,000 square feet in one span.

The strength of the top of the tube to resist impact trans-

versely was remarkably tested at Conway, during the process of raising the hydraulic press in the tower, when the tackle gave way, and the press, weighing 12 tons, descended from a height of 18 feet on to the top of the tube, slightly indenting the top plates, but doing no further injury. Again, in lowering the large hydraulic press at the Britannia Bridge, a fatal accident occurred by the slipping of the rope round a capstan; the press, weighing 12 tons, fell on to the stone shelf at the base of the Anglesey Tower from a height of 140 feet, and glided out 40 feet into the water, fracturing the shelf, but in no way injuring the press, which was afterwards used for raising the second tube. The bottom of the press which failed in raising the first tube in August 1849, weighing 2½ tons, fell from a height of 90 feet on to the top of the tube, indenting the plates, and fracturing the internal castings across the top. The most remarkable example of impact was, however, the fall of the tube itself on to the timber packing below, the total descent of the tube, including the crushing of the timber, being 8 inches, or rather more; and the tube was entirely supported by the central portions only of the bottom.

The weight of the tube is nearly 3 tons per foot run, and a train of locomotives would only weigh one ton per foot run; hence the ratio between the weight of the tube and of its passing load is so much greater than in any ordinary beam, that this consideration alone was sufficient to allay any apprehension from the effect of impact, which has in no well-established case been the cause of failure of even a lighter structure. Weight is, however, of primary importance in resisting impact. The resilience of a prismatic beam, that is, its power of resisting any transverse impulse, to use the words of Dr. Young, is simply proportional to the bulk or weight of the beam, whether it be shorter or longer, narrower or wider,

shallower or deeper, solid or hollow; thus a beam 10 feet long will support but half as great a pressure without breaking as a beam of the same breadth and depth, which is only 5 feet long; but it will bear the impulse of a double weight striking against it with a given velocity, and will require that a given body should fall from a double height in order to break it.

The resistance of beams to impact is directly as the product of their strength into their ultimate deflection, and since the strength is as $\frac{b\,d^2}{l}$, and the ultimate deflection $\frac{l^2}{d}$, the power of resisting impact is as

$$\frac{b\,d^2}{l} \times \frac{l^2}{d} = b\,d\,l,$$

or as their solid content or weight, $b\,d\,l$ representing the breadth, depth, and length respectively. The weight of similar beams being as the cube of their lineal dimensions, it follows that in similar beams the resistance to impact is as the cube of the lineal dimensions, and thus the resistance to impact of a large tube, as in the Britannia Bridge, is one thousand times greater than in a similar beam only one-tenth of its dimensions.

Among the profound investigations of Mr. Hodgkinson and others on the subject of impact and of long-continued impact, there are some facts of practical importance, although the subject is more replete with theoretical than practical interest.

To illustrate the importance of weight in structures intended to resist impact, Mr. Hodgkinson took cast-iron bars, 13 feet 6 inches long between supports, and 3 inches square, and broke them by letting a weight of 303 lbs. fall vertically upon the centre; and taking the mean of several experiments, the height requisite for the fall of this weight to break such a bar was about 30 inches. The weight of the bars was

400 lbs. These bars were now loaded with weights laid upon them, and equally distributed; and as the load on the beam was increased, so the height through which the ball fell to break the bar was also increased, until at length, when the bar was loaded with half its breaking-weight, the height of fall necessary was doubled, and became 60 inches.

The permanent sets, however, from the impacts on these loaded beams were much increased by the load, and were very great, but did not appear to injure their strength more than in ordinary cases: at the maximum they amounted to half the deflection.

By comparing the impacts and deflections, the deflections were found to be as the square root of the height fallen through, which is the velocity of impact. The following table will illustrate the above results:—

Additional Load on Beam in lbs.	Height of Fall necessary to break the beam.	Velocity of Impact answering to that height.
	Inches.	
None	28½	..
Lead, 4 lbs. weight in centre	33	13·301
28 lbs. in centre; no lead	42	15·005
166 lbs. spread over beam + 4 lbs. lead in centre	48	16·042
389½ lbs. spread over beam; 4 lbs. lead in centre	48	16·042
389 lbs. spread over; no lead	48	16·042
391·2 lbs. spread over; 4 lbs. lead in centre	66	18·810
956½ lbs. spread over; 4 lbs. lead in centre	60	17·935

The lead placed at the centre served to resist, to some extent, the jar of the blow.

Experiments on the Effect of repeated Impact on Beams.

Another inquiry, though not affecting beams of which the breaking-weight is so great compared with the load to

which they are subjected as in the tubes, was made the subject of experiments described in the "Report of the Commissioners on the Use of Iron," viz. the effect of continued or repeated impact on beams in cases where the deflection caused by the impact amounted to a considerable part of the ultimate deflection of such beams from dead weight; and it was ascertained that in cases where the deflection caused by each blow amounted to one-half the ultimate deflection from dead weight; that no cast-iron bar would stand 4000 such blows, although no bar was broken by 4000 blows, each causing a deflection of only one-third of the ultimate deflection. The same results were confirmed by the bending of bars by a revolving cam acting at their centre, and causing about four deflections per minute. Wrought-iron bars appeared to be exempt from these effects.

When the velocity of impact exceeds a certain limit, the material is broken by the blow not having time to accommodate itself by deflection to so rapid a change of circumstances; and an analogous phenomenon has been observed with respect to the deflection of a flexible beam from a weight passing over it with extreme rapidity. The investigation of this extremely interesting problem has occupied considerable attention; and although the results are extremely complicated, and have but little reference to the circumstances under which beams are usually employed in the construction of bridges, they are, nevertheless, under some circumstances, of considerable importance.

A weight capable of causing considerable deflection passing slowly over a very flexible rod causes a certain deflection of the same amount as though the weight were quietly placed on the centre. Now (the weight of the passing load being considerable as compared with that of the rod), if the velocity of its motion be increased, the deflection of the rod will also be increased; and at very high velocities the rod may be

broken by a weight much less than would be required to break it as a mere statical load. For example, a carriage loaded with 1120 lbs., placed at rest upon a pair of cast-iron bars 9 feet long, 4 inches broad, and $1\frac{1}{2}$ inches deep, produced a deflection of six-tenths of an inch; but when the carriage was caused to pass over the bars at the rate of 10 miles an hour, the deflection was increased to eight-tenths, and at 30 miles an hour became $1\frac{1}{2}$, or more than double the statical deflection. It was also observed, that the points of greatest deflection did not remain in the centre of the bar, but was removed nearer to the further extremity; and the bars were broken ultimately at points beyond their centres, and sometimes into four or five pieces.

The effect is analogous to passing the hand or a rod very rapidly down a piece of string suspended vertically, and bearing slightly against the string; the string having no time to deflect uniformly, undergoes an increasing partial deflection, and becomes folded round the rod.

Or the effect of the motion of the load over the beam may be considered, after passing the centre, as the effect of an impact against a flexible inclined plane, which has to raise the weight in the given time through a height equal to the amount of deflection.

When, however, as in most practical cases, the beam is of considerable rigidity, or considerable weight, as regards the passing load, no such effect can have much influence on the amount of deflection, or on the strength of the beam; and the effect will rapidly diminish as the beam increases in length, and with girders extremely rigid, it appears probable that the deflection may be even less as the velocity becomes greater.

CHAPTER VI.

DEFLECTION OF CONTINUOUS BEAMS.

In order to test the correctness of the results arrived at by the theoretical investigation of the deflection of continuous beams, as given in Chapter IV. of the last Section, it was considered advisable to make some experiments on wooden rods supported at several points of their length. The following are the details of the experiments, shewing also a comparison of the experimental with the computed results.

Experiment 1.

*A Continuous Beam supported at Four Points.**

A rod of yellow pine 38 feet long, and half an inch square, was supported on four bearings at equal distances from each other, in a horizontal line, and the deflection and pressures upon the various points of support were carefully observed. The rod was laid on the supports with each of the four sides alternately uppermost, and the results given below are the

* This experiment was communicated by Mr. Brunel. It was accompanied by theoretical formulæ and calculations, and by a drawing of the calculated and experimental curves, plotted to a large scale. For the sake of uniformity we have adopted our own formulæ for the calculations, as given in Section III.; the results are, however, very nearly identical with those given by Mr. Brunel.

means of the four observations. The c
accompanying Plate.

Deflection.

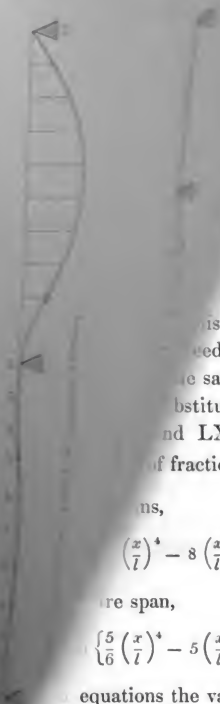
The deflections of the experime
dividing the length of each span int
measuring, at each division, the dist
the rod from a straight-edge fixed
In the following table column
observation, column 2 the mean
as *observed*, and column 3 the ca

	Poi Obs (or v
Support on the outer prop. .	
Centre of outer span	
Support on the inner pr	
Centre of middle sp	
Support . . .	

The c
tained by
follows.

·63 lbs., that of one span,
f μ is therefore $= \frac{0{\cdot}543}{152}$.
und by Equation V. (page
nd $d = \frac{1}{2}$ inch, whence,

$$= \frac{1}{192}.$$



of pine is given by Tredgold at
writers at much less: that of the
is instance was found to be about
ed with experiments on the longi-
e same material. Taking, therefore,
bstituting the values of the quantities
nd LX., putting for convenience the
f fractions $\frac{x}{l}$, of the length l, we obtain—

ns,

$$\left(\frac{x}{l}\right)^4 - 8\left(\frac{x}{l}\right)^3 + 3\left(\frac{x}{l}\right)\Big\}$$

re span,

$$\left\{\frac{5}{6}\left(\frac{x}{l}\right)^4 - 5\left(\frac{x}{l}\right)^3 + 11\left(\frac{x}{l}\right)^2 - \frac{21}{2}\left(\frac{x}{l}\right) + \frac{11}{3}\right\}$$

equations the values in column 3 are calculated.

Pressures on the Supports.

e pressures on the supports of the model beam were
red by a steel-yard, applied at each bearing of the beam
ccession. The distance from the fulcrum to the point of
port of the beam was always 18 inches, and the weight on
e steel-yard half a pound. The leverage, or distance of
his weight from the fulcrum, was carefully recorded as the
lever was applied to the different points of support.

the weight of the whole rod being 1·63 lbs., that of one span, $\mu\, l = 0{\cdot}543$ lb., and the value of μ is therefore $= \frac{0{\cdot}543}{152}$. The moment of inertia, I, is found by Equation V. (page 245); in this case $a = \frac{1}{4}$ inch, and $d = \frac{1}{2}$ inch, whence,

$$I = \frac{\frac{1}{4} \times (\frac{1}{2})^2}{12} = \frac{1}{192}.$$

The modulus of elasticity of pine is given by Tredgold at 1,600,000 lbs., but by other writers at much less: that of the rod experimented on in this instance was found to be about 1,200,000 lbs., which agreed with experiments on the longitudinal compression of the same material. Taking, therefore, E = 1,200,000, and substituting the values of the quantities in Equations LIX. and LX., putting for convenience the abscissæ in the form of fractions $\frac{x}{l}$, of the length l, we obtain—

For the side spans,

$$y = 2{\cdot}542 \left\{5\left(\frac{x}{l}\right)^4 - 8\left(\frac{x}{l}\right)^3 + 3\left(\frac{x}{l}\right)\right\}$$

For the centre span,

$$y = 15{\cdot}25 \left\{\frac{5}{6}\left(\frac{x}{l}\right)^4 - 5\left(\frac{x}{l}\right)^3 + 11\left(\frac{x}{l}\right)^2 - \frac{21}{2}\left(\frac{x}{l}\right) + \frac{11}{3}\right\}$$

From which equations the values in column 3 are calculated.

Pressures on the Supports.

The pressures on the supports of the model beam were measured by a steel-yard, applied at each bearing of the beam in succession. The distance from the fulcrum to the point of support of the beam was always 18 inches, and the weight on the steel-yard half a pound. The leverage, or distance of this weight from the fulcrum, was carefully recorded as the lever was applied to the different points of support.

The mean weight on each of the outside supports A and D, in terms of the leverage as above, was 7·53; the mean weight on each of the middle supports B and C was 21·78; giving the total weight of the rod

$$= \frac{1}{2} \times \frac{58{\cdot}62}{18} = 1{\cdot}63 \text{ lbs.}$$

The absolute values, therefore, of the pressures given by the experiment are as follows:—

		Lbs.
On support A	.... =	0·210
" " B	.... =	0·605
" " C	.... =	0·605
" " D	.... =	0·210
Total weight of rod ..		1·630

The *calculated* pressures may be obtained from Equation LVIII.; P_1 being the pressure on the supports A and D, and P_2 that on the supports B and C. Taking, then, as before, $\mu l = 0{\cdot}543$ lb., we have the following values:—

		Lbs.
On support A	.... =	·217
" " B	.... =	·598
" " C	.... =	·598
" " D	.... =	·217
		1·630

Experiment 2.

A Continuous Beam supported at Five Points.

The following experiment was made by the Author to imitate the conditions of the Britannia Bridge.

A rod of uniform red Memel deal, ½ inch square, and 33 feet long, was supported at five points.

The two central spans were 11 feet each, and the side spans 5′ 6″ each.

Each face of the rod was placed alternately uppermost, and the deflections were measured from a straight edge beneath the rod.

The mean observed deflection of one large, and one small, span, together with the calculated deflection, are given in the following table. The curve is plotted in the accompanying Plate.

	Number of Observation.	Distance from support ($=x$).	Observed Deflection.	Computed Deflection ($=y$).
		Inches.	Inches.	Inches.
Support A	0	0	·000	·000
	1	11	− ·010	− ·003
	2	22	− ·031	− ·016
Middle of small span	3	33	− ·044	− ·023
	4	44	− ·039	− ·035
	5	55	− ·038	− ·035
Support B	6	66	·000	·000
	7	12	+ ·076	+ ·090
	8	24	·209	·208
	9	36	·359	·318
	10	48	·374	·398
	11	60	·428	·436
Middle of large span	12	66	·437	·431
	13	72	·408	·418
	14	84	·348	·352
	15	96	·294	·256
	16	108	·100	·139
	17	120	·031	·041
Centre support C ..	18	132	·000	·000

The mean central deflection of the long span, as an independent beam, was found to be 1·72 inches.

The computed deflection is obtained from Equations LXII. and LXIII., page 289. The following are the values of the quantities:—

$$l = 66 \text{ inches,}$$

$$\mu l = \frac{1}{3} \text{ lb.}$$

$$I = \frac{1}{192} \text{ as before.}$$

The modulus of elasticity of this rod was found to be much greater than that of the preceding one, namely, $E = 2,220,000$, this value giving the computed deflection of the long span, as an independent beam = 1·72 inches, as above.

Substituting these values, in the above-named equations, we have, for the side spans—

$$y = \frac{1}{55,000,000}(x^4 - 66\,x^3).$$

For the centre spans—

$$y = \frac{1}{55,000,000}\left(x^4 - \frac{495}{2}x^3 + 13068\,x^2 + 287496\,x\right).$$

The pressures on the points of support are found from Equation LXI. as follows :—

On support	A	= 0·0833
,, ,,	B	= 0·5625
,, ,,	C	= 0·7084
,, ,,	D	= 0·5625
,, ,,	E	= 0·0833
Total weight of beam		2·0000

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